

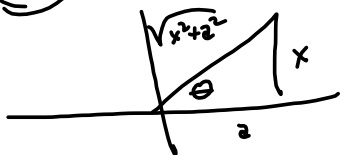
1420

WEEK 06 SOLNS

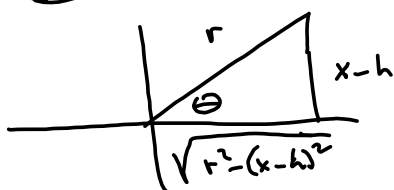
H. MILLS

① We write an algebraic expression equivalent to the given trigonometric expression.

② (5pts) $\csc(\arctan(\frac{x}{a})) = \csc \theta = \boxed{\frac{\sqrt{x^2+a^2}}{x}}$



③ (5pts) $\cos(\arcsin(\frac{x-h}{r})) = \cos \theta = \boxed{\frac{\sqrt{r^2 - (x-h)^2}}{r}}$



② Verify the identity algebraically.

② (5pts) $\frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$

$$\frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \frac{\cos \theta \cot \theta - (1 - \sin \theta)}{1 - \sin \theta}$$

$$= \frac{\frac{\cos^2 \theta}{\sin \theta} + (\sin \theta - 1) \frac{\sin \theta}{\sin \theta}}{1 - \sin \theta} = \frac{\cos^2 \theta + \sin^2 \theta - \sin \theta}{\sin \theta (1 - \sin \theta)} = \frac{1 - \sin \theta}{\cos \theta (1 - \sin \theta)}$$

$$= \csc \theta \quad \checkmark$$

③ (5pts) $\sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} = \frac{1 + \cos \theta}{|\sin \theta|}$?! I don't think so.

$$\sqrt{\left(\frac{1 + \cos \theta}{1 - \cos \theta}\right) \left(\frac{1 + \cos \theta}{1 + \cos \theta}\right)} = \sqrt{\frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}} = \frac{|1 + \cos \theta|}{\sqrt{\sin^2 \theta}}$$

$$= \frac{1 + \cos \theta}{|\sin \theta|}, \text{ b/c } 1 + \cos \theta \geq 0 \text{ (} \cos \theta \leq 1 \text{)}$$

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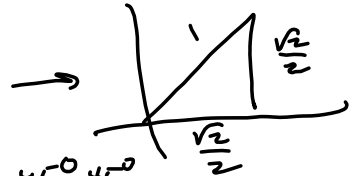
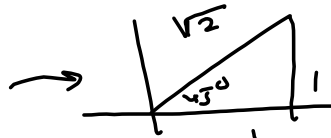
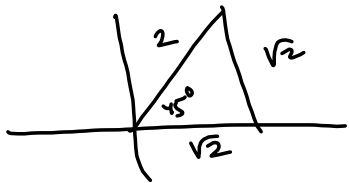
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3 solve the trig equations:

a) Spts $\frac{\sqrt{2}}{2} \csc \theta - 1 = 0 \rightarrow$

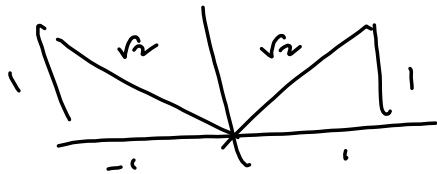
$\csc \theta = \frac{2}{\sqrt{2}}$

$\Rightarrow \sin \theta = \frac{\sqrt{2}}{2}$



Any way you "see" 45-45-90 right triangle is OK.

So, $\sin \theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



$x = \frac{\pi}{4}, \frac{3\pi}{4}$ on $[0, 2\pi)$. In general,

$x = \frac{\pi}{4} + 2n\pi$ or $\frac{3\pi}{4} + 2n\pi, n \in \mathbb{Z}$

b) Spts $\sin(2x)(2\sin(x)+1) = 0 \rightarrow$

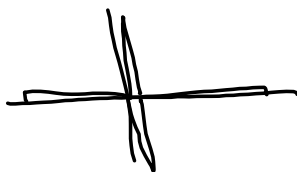
$\sin(2x) = 0$ or $2\sin(x) + 1 = 0$
 To find all $x \in [0, 2\pi)$,
 find all $2x \in [0, 4\pi)$

$\sin(2x) = 0$



$2x = 0, \pi, 2\pi, 3\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$



All $\frac{\pi}{2}$ apart, so

$x = \frac{n\pi}{2}$

$2\sin(x) + 1 = 0$
 $2\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$



$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ on $[0, 2\pi)$

In general,

$x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in \mathbb{Z}$

$x = \frac{\pi}{2} + \frac{n\pi}{2}, n \in \mathbb{Z}$

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(4) Find $\sin \theta$, $\cos \theta$, $\tan \theta$ (exact).(a) (5pts) $\theta = 75^\circ = 45^\circ + 30^\circ$

$$\sin \theta = \sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ$$

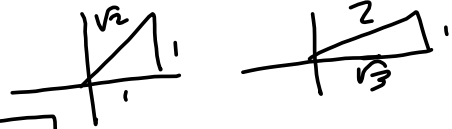
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}} \text{ OR } \frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\cos \theta = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}} \text{ OR } \frac{\sqrt{6}-\sqrt{2}}{4}}$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ OR } \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}}$$

(b) (5pts) $\theta = \frac{13\pi}{12} = \pi + \frac{\pi}{12} = \frac{2\pi}{12} + \frac{11\pi}{12} = \frac{3\pi}{12} + \frac{10\pi}{12} = \frac{\pi}{4} + \frac{5\pi}{6}$

$$\sin(u+v) = \sin\left(\frac{\pi}{4} + \frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right) + \sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)$$

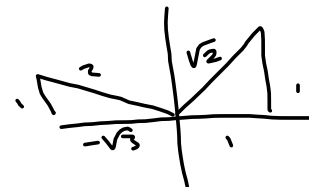
$$= \frac{1}{\sqrt{2}} \cdot \left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}} \text{ OR } \frac{\sqrt{2}-\sqrt{6}}{4}}$$

$$\cos(u+v) = \cos\frac{\pi}{4}\cos\frac{5\pi}{6} - \sin\frac{\pi}{4}\sin\frac{5\pi}{6} =$$

$$= \frac{1}{\sqrt{2}} \cdot \left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{2}\right) = \boxed{-\frac{\sqrt{3}+1}{2\sqrt{2}} \text{ OR } -\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{2}} \cdot \left(-\frac{1}{\sqrt{3}}\right)} = \boxed{\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{2}\sqrt{3}}} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+1} \text{ OR } \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}}$$



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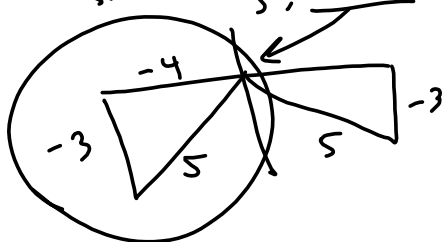
(5) $\sin(u) = -\frac{3}{5}$, $\cos(u) < 0$ and $\cos(v) = \frac{14}{17}$, $\tan(v) < 0$

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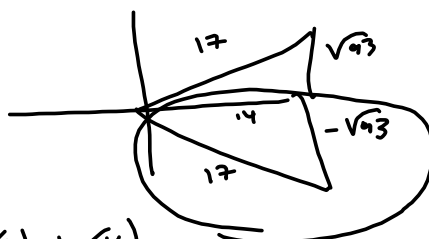
(5pts) Find $\cos(u+v)$

Let's find the quadrant for u & v

$\sin(u) = -\frac{3}{5}$, $\cos(u) < 0$



$\cos(v) = \frac{14}{17}$, $\tan(v) < 0$



$$\sqrt{17^2 - 14^2} = \sqrt{289 - 196} = \sqrt{93}$$

$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$$

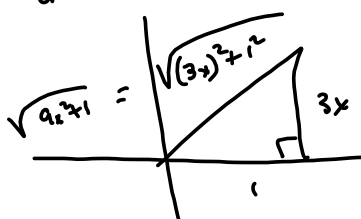
$$= \left(-\frac{4}{5}\right)\left(\frac{14}{17}\right) - \left(-\frac{3}{5}\right)\left(-\frac{\sqrt{93}}{17}\right)$$

$$= \frac{-56 - 3\sqrt{93}}{85}$$

(6) (5pts) $\sin(\arctan(3x) - \arccos(2x)) = \sin(u-v)$

$u = \arctan(3x)$

$v = \arccos(2x)$



$$\sin(u-v) = \sin(u)\cos(v) + \sin(-v)\cos(u)$$

$$= \sin(u)\cos(v) - \sin(v)\cos(u)$$

$$= \left(\frac{3x}{\sqrt{9x^2+1}}\right)\left(\frac{2x}{1}\right) - \left(\frac{\sqrt{1-4x^2}}{1}\right)\left(\frac{1}{\sqrt{9x^2+1}}\right)$$

$$= \frac{6x^2 - \sqrt{1-4x^2}}{\sqrt{9x^2+1}}$$

