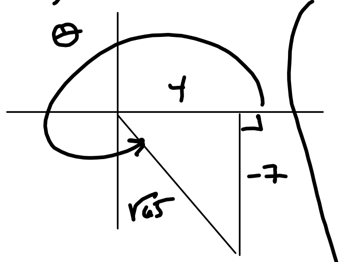
1420 Week 3 Assignment Solutions H. Mills

(1) we find the exact values of the 6 trig functions corresponding to the point $(4,-7)$.

5pts

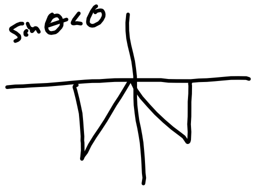


$4^2+7^2 = 16+49 = 65$

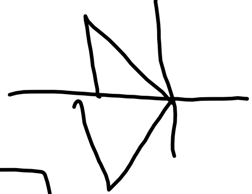
$$\boxed{\begin{array}{ll} \sin\theta = -\dfrac{7}{\sqrt{65}} & \csc\theta = -\dfrac{\sqrt{65}}{7} \\ \cos\theta = \dfrac{4}{\sqrt{65}} & \sec\theta = \dfrac{\sqrt{65}}{4} \\ \tan\theta = -\dfrac{7}{4} & \cot\theta = -\dfrac{4}{7} \end{array}}$$

(2) $\sin\theta < 0$ & $\cos\theta < 0$

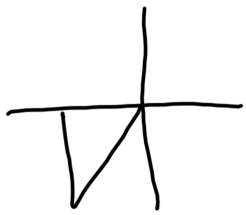
$\sin\theta < 0$



$\cos\theta < 0$



$\Rightarrow$



$\Rightarrow \boxed{\theta \in \text{QIII}}$

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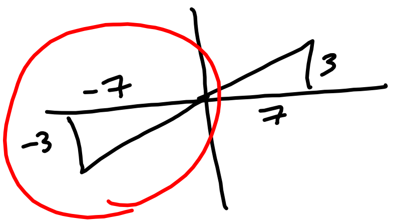
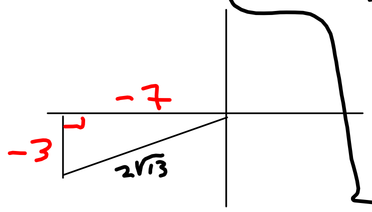
③ 5pts $\tan\theta = \frac{3}{7}$, $\cos\theta < 0 \Longrightarrow$





$$\boxed{\begin{array}{ll} \sin\theta = \frac{-3}{\sqrt{58}} & \csc\theta = -\frac{\sqrt{58}}{3} \\ \cos\theta = \frac{-7}{\sqrt{58}} & \sec\theta = -\frac{\sqrt{58}}{7} \\ \tan\theta = \frac{3}{7} & \cot\theta = \frac{7}{3} \end{array}}$$

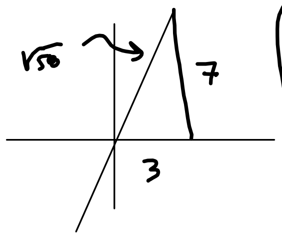
$7^2 + 3^2 = 49 + 9 = 58$

$2\,|\,58$
$2\,|\,26$
13

$\rightarrow$ No! $58 = 2 \cdot 29$, not $2 \cdot 26$.

$\sqrt{58} = 2\sqrt{13}$ No!

④ 5pts We find the value of the 6 trigs for a point lying on $y = \frac{7}{3}x$ in QI:



$$\boxed{\begin{array}{ll} \sin\theta = \frac{7}{\sqrt{58}} & \csc\theta = \frac{\sqrt{58}}{7} \\ \cos\theta = \frac{3}{\sqrt{58}} & \sec\theta = \frac{\sqrt{58}}{3} \\ \tan\theta = \frac{7}{3} & \cot\theta = \frac{3}{7} \end{array}}$$

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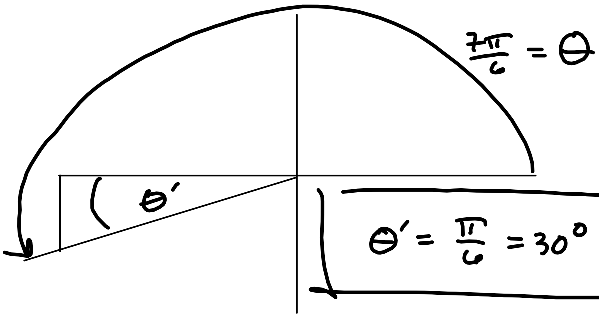
(5) We sketch $\theta = \frac{7\pi}{6}$ in standard position, and find and label the reference angle θ'

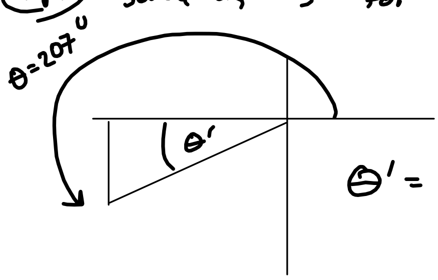


Check: $\frac{7\pi}{6} = \frac{7\pi}{6} \cdot \frac{180^\circ}{\pi}$

$= 7(30^\circ) = 210^\circ$

$\boxed{\theta' = \frac{\pi}{6} = 30^\circ}$ via $\frac{7\pi}{6} - \pi$ or $210^\circ - 180^\circ = 30^\circ$

(6) 5pts Same as #5 for $\theta = 207^\circ$



$\theta' = 207^\circ - 180^\circ = \boxed{27^\circ = \theta'}$

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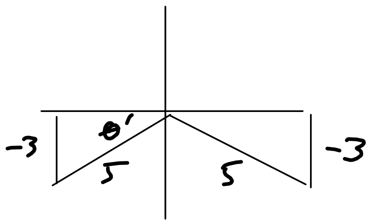
(7) We sketch and find 2 sol'ns to each of the following. We round to 4 places.

(a) (5pts) $\sin\theta = -\frac{3}{5}$

$\arcsin\left(-\frac{3}{5}\right) \approx \boxed{-36.8698976458^\circ \approx \theta_1}$



The angle terminating in Q III will be

$180^\circ - \arcsin(-3/5) \approx 180 + 36.8698976458^\circ$

$= 216.8698976458^\circ$

b/c the angle's negative, to add θ'. $\approx \boxed{216.8699^\circ \approx \theta_2}$

$\boxed{\approx 3.78509376239 \approx 3.7851}$

My work is cramped and a bit disorganized.

Other answers are possible:

$\theta_2 = -180^\circ + \arcsin(-3/5)$

$\approx -143.130102354^\circ$

$\approx \boxed{-143.1301^\circ} \approx \theta_2$

$\approx -2.49809154479 \approx \boxed{-2.4981 \approx \theta_2}$

$\theta_1 = 360^\circ + \arcsin(-3/5)$

$\approx 323.130102354^\circ$

$\approx \boxed{323.1301^\circ} \approx 5.63968419838$

$\boxed{\approx 5.6397 \approx \theta_1}$

I should've specified that the solutions must lie between 0 and 360 degrees, but I didn't. Woe is me.

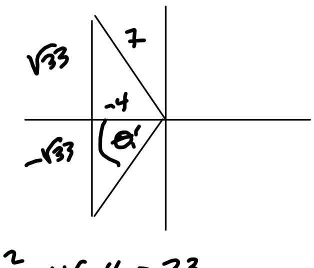
Radians answers:

$\theta_1 \approx -36.8698976458^\circ \approx -0.643501108793 \approx \boxed{-.6435}$

$\theta_2 \approx 323.130102354^\circ \approx 1.7951672353 \approx \boxed{1.7952}$

(b) (5pts) Let's see if we can clean this up.

$\sec\theta = -\frac{7}{4}$ $\left(\cos\theta = -\frac{4}{7}\right)$



$\theta_1 = \arccos\left(-\frac{4}{7}\right) \approx 124.849904579^\circ \approx 2.1790419057$

$\boxed{\theta_1 \approx 124.8499^\circ \text{ or } 2.1790}$

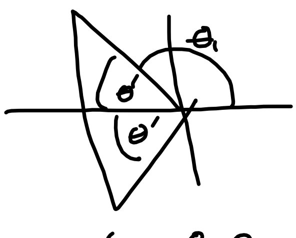
$7^2 - 4^2 = 49 - 16 = 33$

$\theta_1' = 180^\circ - \theta_1 \approx 55.150095421^\circ$

$\theta_2 = 180^\circ + \theta' = 180^\circ + (180^\circ - \theta_1) = 360^\circ - \theta_1$

$\approx 235.150095421^\circ \approx 4.10414340148$

$\boxed{\theta_2 \approx 235.1501^\circ \approx 4.1041}$



$\theta_1' = 180^\circ - \theta_1$

Notice I didn't use the rounded degrees to find the radians. I found the radians from the raw degrees and *then* rounded.

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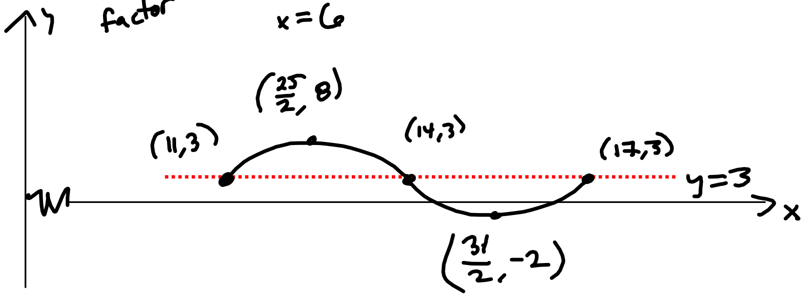
8 5pts We sketch one period of $f(x) = 5\sin\left(\frac{\pi}{3}x - \frac{11\pi}{3}\right) + 3$

$= 5\sin\left(\frac{\pi}{3}(x - 11)\right) + 3$

up & down 5 factor

$\frac{\pi}{3}x = 2\pi$

$x = 6$



9 NOTHING ASKED ?!

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10 5pts

On the longest day of the year, there are 22 hours of daylight. On the shortest day of the year, there are 2 hours of daylight. We model y = the # of hours of daylight as a function of x = the day of the year (as a number, starting with x = 1 corresponding to January 1st.

$a\cos(b(x-c))+d$

High = 22
Low = 2 } Midline: $y = 12 = d$

Amplitude $= \frac{22-2}{2} = \frac{20}{2} = 10$

Period = 365

$bx = 2\pi$

$365b = 2\pi \Rightarrow b = \frac{2\pi}{365}$

Start on the high point

The longest day of the year occurs on the summer solstice, which, in the northern hemisphere is June 21st.

J F M A M J
31 28 31 30 31 21

June 21st = Day 172 ⇒ x = 172 @ max height, so

$$\boxed{y = f(x) = 10\cos\left(\frac{2\pi}{365}(x-172)\right)+12}$$

Midline & growing longer:

Mar 21st:
J F M
31 + 28 + 21 = 80

Building Sine

$y = f(x) = 10\sin\left(\frac{2\pi}{365}(x-80)\right)+12$