MAT 1420-R11 Written Midterm Test – Spring, 2024 Print Name_____
130 Points Preparation for Written Midterm.
Covers Chapters 1 - 3

Do your own work. Show all your work. Circle final answers. Organize your work for efficient processing, left to right, top to bottom. It will be a 20% deduction from your final score if problems are not submitted in the order in which they were presented.

Paper will be provided. Do not take anything from the test with you. Turn in your work, cheat sheet, and any scratch pages to the proctor. You should not make a separate scratch sheet. EVERYTHING you do related to a given question should be included with the work on that question, including your scratch work.

Do all your work on the blank paper provided. All I'm looking for on test-questions pages is your NAME. No points given or taken for anything else you write on these test-question pages.

The only difference between this test and your Written Assignments is that you don't have to write out the question on a test. No annoying "Context" deductions.

When the question has a picture and you don't draw it, it will cost 20% of each problem. If you resist drawing the pictures, it will cost you.

1. Arc Length and Area of Sector. Suppose we have a circle of radius $r = 8$ cm.

   a. (5 pts) Find the *exact* arc length on the circle, that is intercepted by an angle of 1290^0.

   b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = 225^0$

2. Answer the questions about the equation $\sin(\theta) = -\frac{5}{7}$.

   a. (5 points) Sketch two triangles that satisfy $\sin(\theta) = -\frac{5}{7}$.

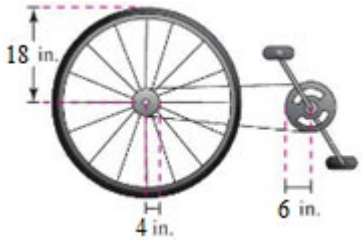
   b. (5 pts) Given $\cos(\theta)$ is negative, find the other five trigonometric functions of θ.

   c. (5 pts) Assuming $0 \le \theta < 2\pi$, find the *exact* value of θ. Then round your answer to 3 decimal places.

   d. (5 pts) Give *all* solutions to the equation $\sin(\theta) = -\frac{5}{7}$, in radians. Give *exact* answers.

3. (5 pts) Sketch one period of the graphs of $y = \cos(x)$ and $\mathrm{y} = \sec(\mathrm{x})$ on the same set of coordinate axes. Use a really thick line for the graph of cosine, to set it apart. Label highs, lows and midline intersections of the graph of the cosine.

4. (10 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 6 inches, 4 inches and 18 inches, respectively. A cyclist is pedaling at a rate of 2 revolutions per second. Find the *exact* speed of the bicycle in feet per second. Then round your answer to 3 decimal place.

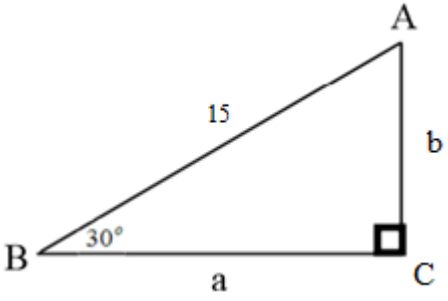


5. (5 pts) Sketch one period of the graph of $f(x)=3\sin\left(\frac{\pi}{6}x-2\pi\right)+7$.

   Clearly label all highs, lows, and intersections with the midline.

6. (5 pts) Find the cosine function $f(x)=a\cos\left(b(x-c)\right)+d$ that achieves its maximum height of $y=57$ centimeters at time $t=7$ seconds and its minimum height of $y=-7$ centimeters at $t=31$ seconds.

7. (5 pts) Solve the triangle in the figure on the right. That means, find all lengths and angles. Exact answers required.



8. (5 pts) Find the exact value of $\arctan\left(\tan\left(\frac{5\pi}{6}\right)\right)$

9. (5 pts) Find the exact values of $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u)=-\frac{3}{4}$ and $\cos(u)>0$.

10. Consider the equation $4\cos^2(2x)-1=0$.

    a. (5 pts) Find all solutions x, in radians, to the equation in the interval $[0,2\pi)$. I better see pictures!

    b. (5 pts) Find all real solutions x, in radians.

11. (5 pts) Draw the picture and use it to re-write $\sin\left(\arctan\left(\frac{x}{7}\right)\right)$ as an algebraic expression.

12. Find the *exact* value of $\sin\left(\frac{7\pi}{12}\right)$ in two ways:

    a. (5 pts) Use a Sum identity.

    b. (5 pts) Use a Half-Angle identity.

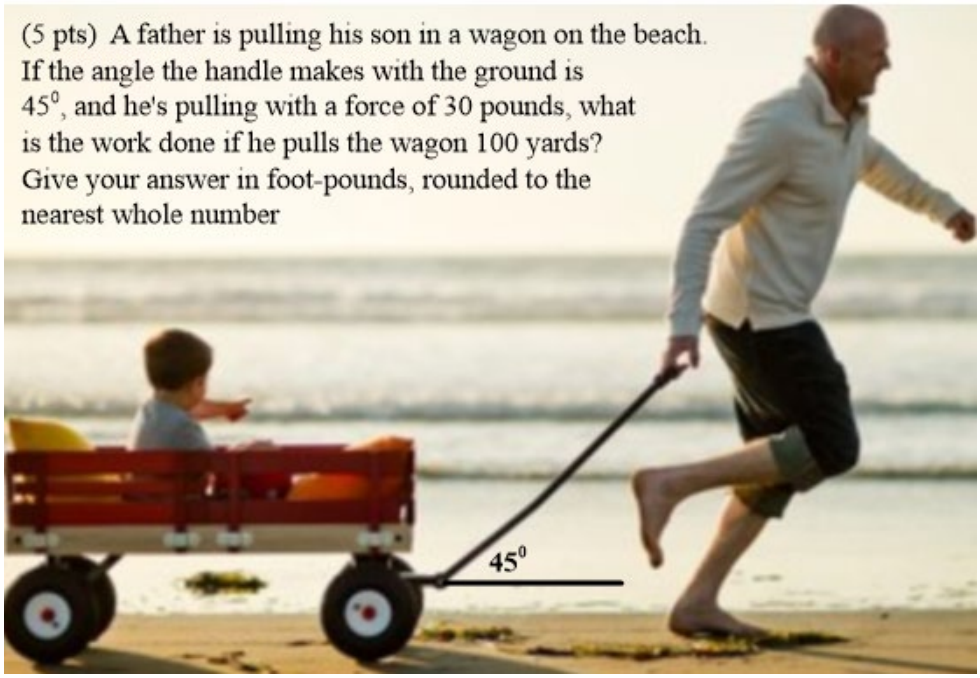
13. (5 pts) Find the *exact* values of $\sin(2u)$, $\cos(2u)$ and $\tan(2u)$, given that $\sin(u)=\frac{4}{11}$ and $\cos(u)<0$.

14. Suppose that there's a triangle ABC with the measure of angle A of 37^0, and sides $a = 12$ and $b = 16$.

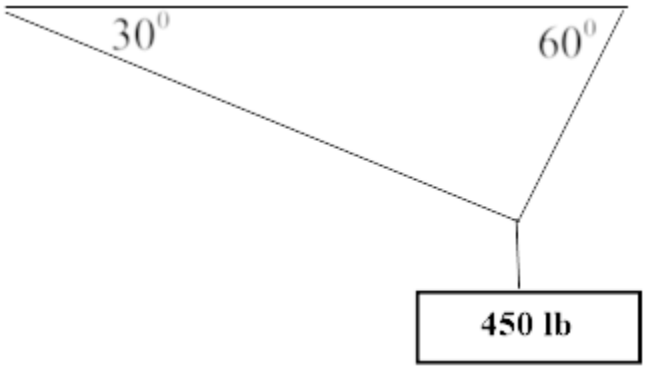
    a. (5 pts) Show that there are two possibilities for the measure of angle B.

    b. (5 pts) Find both solutions B, in degrees. Round your final answers to 5 decimal places.

15. (5 pts) Suppose that there's a triangle ABC with the measure of angle A of 37^0, and sides $b = 12$ and $c = 16$. Use the Law of Cosines to find the length of side a.

16. 

17. (**Bonus 10 pts**) A 450-lb weight is suspended from two cables as shown in the diagram. Find the tension in each cable. Round your final answers to 1 decimal place.



18. Let $\overline{u} = \langle 6, 8 \rangle$ and $\overline{v} = \langle -7, -1 \rangle$

    a. (5 pts) Find $\overline{u} + \overline{v}$.

    b. (**Bonus 5 pts**) Find $\overline{u} \bullet \overline{v}$

    c. (5 pts) Find $\|\overline{u}\|$ and $\|\overline{v}\|$.

    d. (**Bonus 5 pts**) Find the angle between $\overline{u}$ and $\overline{v}$, in degrees, rounded to 1 decimal place.

e. (**Bonus 5 pts**) Find $\text{proj}_{\overline{v}}\overline{u}$. Round answer to 1 decimal place.

f. (**Bonus 5 pts**) Find 2 vectors, $\overline{w}_1$ and $\overline{w}_2$, such that $\overline{w}_1$ is parallel to $\overline{v}$ and $\overline{w}_2$ is orthogonal to $\overline{w}_1$. Round answer to 1 decimal place.

19. (**Bonus 5 pts**) A triangle has vertices $A(3,2), B(10,5),$ and $C(5,9)$. Use vectors to find the measure of angle C. Round final answers to 4 places. Use degrees mode in your calculator for this one.

20. (**Bonus 5 pts**) Re-write $\cos\left(\arcsin(x)+\arctan(x)\right)$ as an algebraic expression. (Hint: Use Sum identity.)