1 $r = 8\,cm$,

a $\theta = 2290^\circ \longrightarrow$

5pts $S = \text{arclength} = r\theta = 8(290^\circ)\left(\frac{\pi}{180^\circ}\right) = 8\left(\frac{129}{18}\right)\pi = 4\left(\frac{129}{9}\right)\pi$

$= \frac{4(43)}{3}\pi = \boxed{\frac{172\pi}{3}\,cm = S} \approx 180.1179788\,cm$

b 5pts $\theta = 225^\circ \longrightarrow$

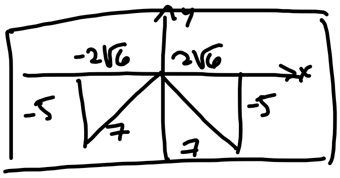
$\text{Area} = A = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(225^\circ)\left(\frac{\pi \text{ radians}}{180^\circ}\right) =$

$= 32\left(\frac{225\pi}{180}\right) = 8\left(\frac{225\pi}{45}\right) = \frac{8(45)}{9}\pi = 8(5)\pi = \boxed{40\pi\,cm^2}$ = Area of sector.

$\approx 125.6637062\,cm^2$

(2) $\sin\theta = -\frac{5}{7}$

(a) (5pts)

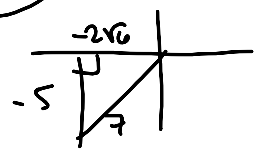


2 triangles

$7^2 - 5^2 = 49 - 25 = 24$

$\sqrt{24} = 2\sqrt{6}$

(b) (5pts) Given $\cos\theta < 0$ & $\sin\theta = -\frac{5}{7}$



$\sin\theta = \frac{-5}{7}$

$\cos\theta = \frac{-2\sqrt{6}}{7}$

$\tan\theta = \frac{5}{2\sqrt{6}}$ or $\frac{5\sqrt{6}}{12}$

$\csc\theta = -\frac{7}{5}$

$\sec\theta = -\frac{7}{2\sqrt{6}}$ or $-\frac{7\sqrt{6}}{12}$

$\cot\theta = \frac{2\sqrt{6}}{5}$

(c) (5pts) Find exact value of θ.

More than one answer

$\boxed{\pi + \arctan\left(\frac{5}{2\sqrt{6}}\right) \text{ or } \pi + \arcsin\left(\frac{5}{7}\right) \text{ or } \pi + \arccos\left(\frac{2\sqrt{6}}{7}\right)}$

plus variations on the themes.

Approximately

Radians to 3 places:

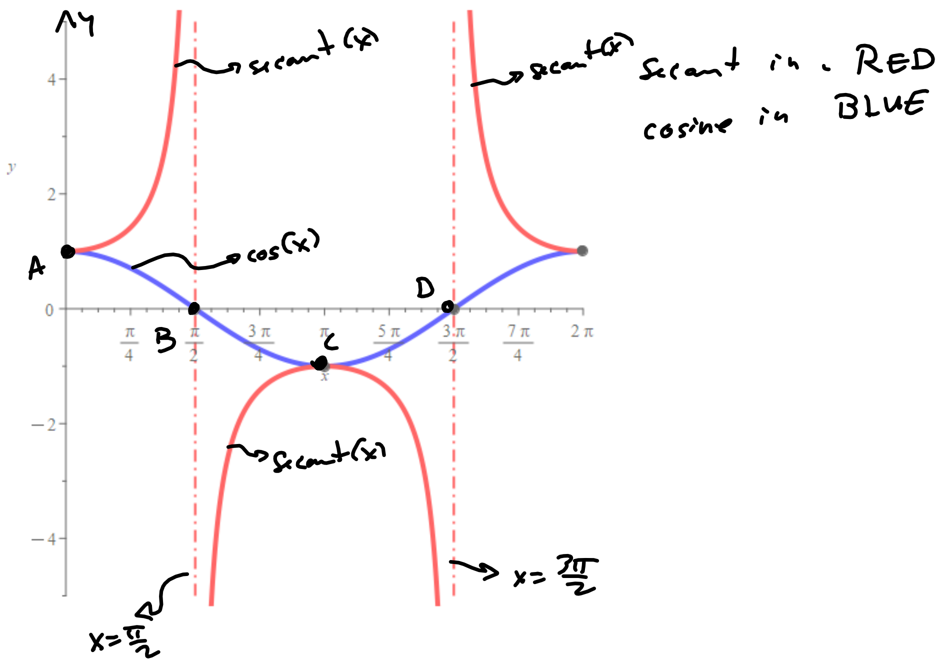
$\theta \approx 3.937195607 \approx \boxed{3.937 \approx \theta}$

(d) Find ALL solns to $\sin\theta = -\frac{5}{7}$

$$\boxed{\theta = \pi + \arcsin\left(\frac{5}{7}\right) + 2n\pi,\ \underbrace{-\arcsin\left(\frac{5}{7}\right)}_{-\arccos\left(\frac{2\sqrt{6}}{7}\right) \text{ or } -\arctan\left(\frac{5}{2\sqrt{6}}\right)} + 2n\pi,\ n \in \mathbb{Z}}$$

or $\arctan\left(\frac{5}{2\sqrt{6}}\right)$ or $\arccos\left(\frac{2\sqrt{6}}{7}\right)$

3. (5 pts) Graph one period of cosine and secant on the same coordinate axes.



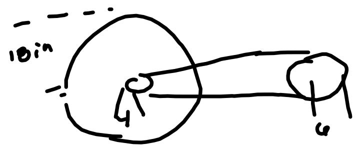
4 10 pts

Front Sprocket – 6 in

Rear " – 4 in

Rear wheel – 18-inch Radius

Pedaling at $2\frac{\text{rev}}{\text{sec}}$



$$\left(2\,\frac{\text{revs front}}{\text{sec}}\right)\left(\frac{6\text{ revs rear}}{4\text{ revs front}}\right)\left(\frac{2\pi\text{ radians}}{1\text{ rev}}\right)\left(18\text{ in radius rear}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)$$

$$= \frac{24(18)\pi}{4(12)} = \frac{36}{4}\pi = \boxed{9\pi\,\frac{ft}{s}} \approx 28.27433389\,\frac{ft}{s}$$

$$\boxed{\approx 28.274\,\frac{ft}{s}}$$

$\left(19.278\,\frac{mi}{hr}\right)$

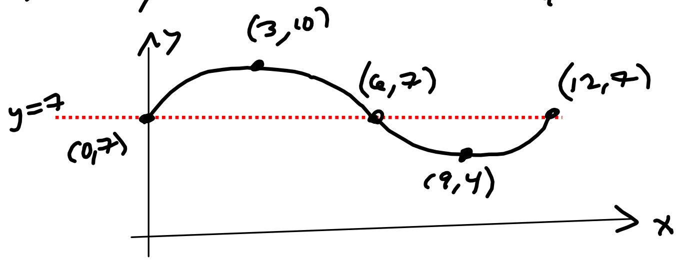
5 5pts One period of $f(x)=3\sin\left(\frac{\pi}{6}x-2\pi\right)+7$

$= 3\sin\left(\frac{\pi}{6}x\right)+7$, since sine has period 2π.

Check: $3\sin\left(\frac{\pi}{6}(x-12)\right)+7$

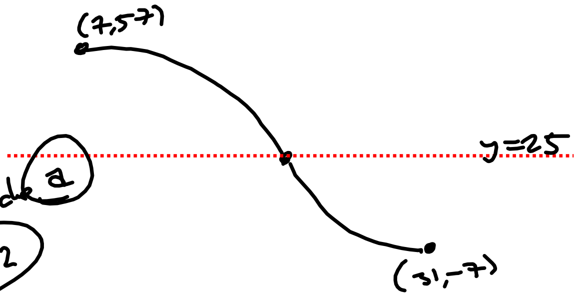
$\frac{\pi}{6}x=2\pi$ when $x=12$, so yeah. It's delayed by one full period, exactly

$\frac{12}{4}=3$



6 5pts High: (7,57)

Low: (31,−7)



$\frac{57-(-7)}{2}$

$=\frac{64}{2}=32$ = Amplitude a

$a=32$

$\frac{57+-7}{2}=\frac{50}{2}=\boxed{25=d}$

$31-7=24=\frac{1}{2}$ period,

$48=1$ period.

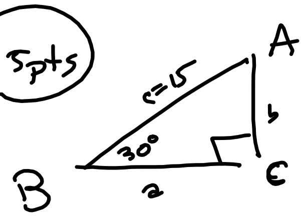
Want $bx=2\pi$ when $x=48$

$\rightarrow \frac{2\pi}{48}=\boxed{\frac{\pi}{24}=b}$

Start at top: $x=\boxed{7=c}$

$$\rightarrow \boxed{f(x)=32\sin\left(\frac{\pi}{24}(x-7)\right)+25}$$

7 5pts



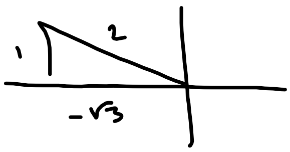
Solve

$\Rightarrow b = 15\sin 30^\circ = \boxed{\frac{15}{2} = b}$

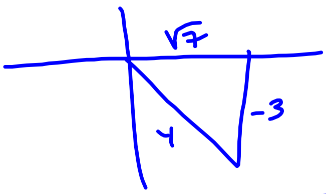
$a = 15\cos 30^\circ = \boxed{\frac{15\sqrt{3}}{2} = a}$

$\boxed{A = 60^\circ}$

8 $\arctan\left(\tan\left(\frac{5\pi}{6}\right)\right) = \arctan\left(-\frac{1}{\sqrt{3}}\right) = \boxed{-30^\circ \text{ OR } -\frac{\pi}{6}}$



9 (5pts) $\sin(u) = -\frac{3}{4}$ & $\cos(u) > 0$



$\frac{3\pi}{2} < u < 2\pi$

$\frac{3\pi}{4} < u < \pi$ Q II

sin

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1-\cos(u)}{2}} = \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{2}} = \sqrt{\frac{4-\sqrt{7}}{8}} = \frac{1}{8}\sqrt{32-8\sqrt{7}} = \sin\left(\frac{u}{2}\right)$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1+\cos(u)}{2}} = -\frac{1}{8}\sqrt{32+8\sqrt{7}} = \cos\left(\frac{u}{2}\right)$$

$$\tan\left(\frac{u}{2}\right) = -\sqrt{\frac{32-8\sqrt{7}}{32+8\sqrt{7}}} \quad \text{OR} \quad \frac{1-\cos(u)}{\sin(u)} = \frac{1+\frac{1}{8}\sqrt{32-8\sqrt{7}}}{-\frac{1}{8}\sqrt{32+8\sqrt{7}}}$$

$$= \frac{8+\sqrt{32-8\sqrt{7}}}{\sqrt{32+8\sqrt{7}}} = \tan\left(\frac{u}{2}\right)$$

10) ~~20pts~~ Solve $4\cos^2(2x)-1=0$

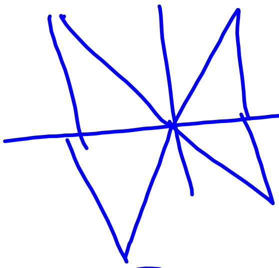
a) 5pts on $[0,2\pi) \longrightarrow$

$2x \in [0,4\pi)$

$4\cos^2(2x)=1$

$\cos^2(2x)=\frac{1}{4}$

$\cos(2x)=\pm\frac{1}{2}$



$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3} \ldots \longrightarrow$

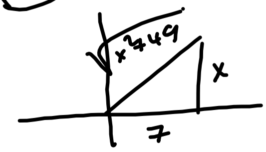
$$\boxed{x=\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}}$$

b) 5pts All sol'ns:

$2x=\frac{\pi}{3}+n\pi, \frac{2\pi}{3}+n\pi$

$$\Rightarrow \boxed{x=\frac{\pi}{6}+\frac{n\pi}{2}, \frac{\pi}{3}+\frac{n\pi}{2}, n\in\mathbb{Z}}$$

(11) (5pts) $\sin\theta = \sin\left(\arctan\left(\frac{x}{7}\right)\right) = \boxed{\frac{x}{\sqrt{x^2+49}}}$



(12) $\sin\left(\frac{7\pi}{12}\right)$

(a) (5pts) $\frac{7\pi}{12} = \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

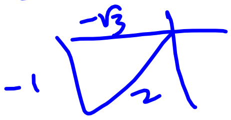
$\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$

$\boxed{= \frac{\sqrt{3}+1}{2\sqrt{2}} \text{ OR } \frac{\sqrt{6}+\sqrt{2}}{4}}$

0.9659258263

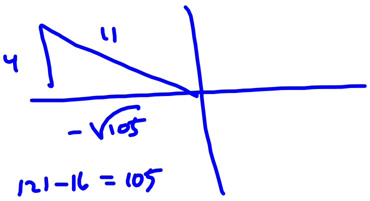
(b) (5pts) $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\frac{7\pi}{6}}{2}\right)$ $u = \frac{7\pi}{6}$



$\cos(u) = -\frac{\sqrt{3}}{2}$

$\sin\left(\frac{7\pi}{12}\right) = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$ OR $\sqrt{\frac{2+\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2} = \sin\frac{7\pi}{12}}$

13 $\sin(u) = \frac{4}{11}$, $\cos(u) < 0$



$121 - 16 = 105$

$\sin(2u) = 2\sin u\cos u = 2\left(\frac{4}{11}\right)\left(-\frac{\sqrt{105}}{11}\right) = \boxed{-\frac{8\sqrt{105}}{121} = \sin(2u)}$

$\cos(2u) = 2\cos^2(u) - 1 = 2\left(\frac{\sqrt{105}}{11}\right)^2 - 1$

$= \frac{2(105)}{121} - \frac{121}{121} = \frac{210-121}{121} = \boxed{\frac{89}{121} = \cos(2u)}$

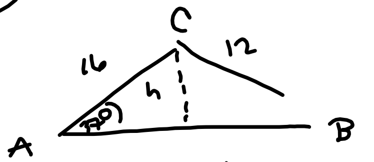
$\boxed{\tan(2u) = -\frac{8\sqrt{105}}{89}}$

OR $\tan(2u) = \frac{2\tan(u)}{1-\tan^2(u)} = \frac{2\left(-\frac{4}{\sqrt{105}}\right)}{1-\left(\frac{16}{105}\right)} = \frac{\frac{-8}{\sqrt{105}}}{\frac{105-16}{105}} = \frac{-8}{\sqrt{105}}\left(\frac{105}{89}\right)$

$= -\frac{8\sqrt{105}}{89}$ ✓

(14) $\triangle ABC$, $A = 37^\circ$, $a = 12$, $b = 16$

(a) 5 pts $\exists$ 2 sol'ns.



$h = 16\sin 37^\circ \approx 9.629040374 < a = 12 < b = 16 \Longrightarrow$

2 sol'ns.

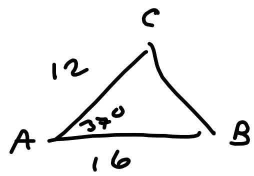
(b) $\dfrac{\sin B}{b} = \dfrac{\sin A}{a} \Longrightarrow \sin B = \dfrac{b \sin A}{a} = \dfrac{16 \sin 37^\circ}{12} \approx 0.8024200307$

$\Longrightarrow B_1 = \arcsin(\sin(B)) \approx \arcsin(0.8024200307) \approx 53.36182366^\circ \approx B_1$

$\boxed{B_1 \approx 53.36182^\circ} \Longrightarrow$

$B_2 \approx 180^\circ - B_1 \approx 126.6381763^\circ \approx \boxed{126.63817^\circ \approx B_2}$

(15) 5 pts $\triangle ABC$, $A = 37^\circ$, $b = 12$, $c = 16$



Law of cosines to find a:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= 12^2 + 16^2 - 2(12)(16)\cos(37^\circ)$$

$$= 144 + 256 - 384\cos(37^\circ)$$

$$= 400 - 384\cos(37^\circ)$$

$$\approx 93.3239642 \Longrightarrow$$

$$a \approx \sqrt{93.3239642} \approx \boxed{9.660432920 \approx a}$$

$$\begin{array}{r} 24 \\ \times 16 \\ \hline 144 \\ 240 \\ \hline 384 \end{array}$$

EXACT answer for a:

$$a = \sqrt{400 - 384\cos(37^\circ)}$$

16 5pts work. $\bar{F} = 30\langle \cos 45^\circ, \sin 45^\circ \rangle$ (Pounds)

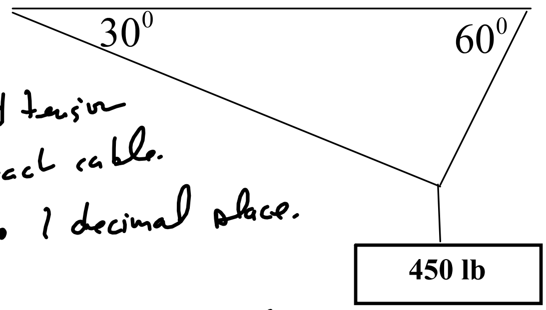
$\bar{v} = 100$ yards $= 300$ ft $= \langle 300, 0 \rangle$ (Feet)

Work $= \bar{F} \cdot \bar{v} = 30\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle \cdot \langle 300, 0 \rangle$

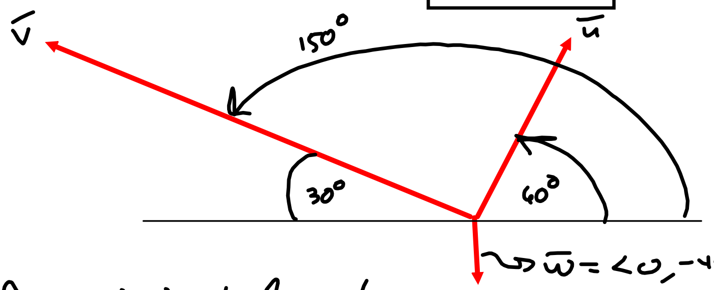
$= \langle 15\sqrt{2}, 15\sqrt{2} \rangle \cdot \langle 300, 0 \rangle = 4500\sqrt{2}$ ft-lbs

$\approx$ 6363.961029 $\approx \boxed{6364 \text{ ft-lbs}}$

17 10pts



Find tension in each cable. Round to 1 decimal place.



$\leadsto \vec{w} = \langle 0, -450 \rangle$

Want forces to be balanced

so $\vec{u} + \vec{v} = -\vec{w} = \langle 0, 450 \rangle$

$\vec{u} = \|\vec{u}\| \langle \cos 60^\circ, \sin 60^\circ \rangle = x \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$, where $x \equiv \|\vec{u}\|$

$\vec{v} = \|\vec{v}\| \langle \cos 150^\circ, \sin 150^\circ \rangle = y \langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$, where $y = \|\vec{v}\|$

We're trying to find x & y:

$\langle \frac{1}{2}x, \frac{\sqrt{3}}{2}x \rangle + \langle -\frac{\sqrt{3}}{2}y, \frac{1}{2}y \rangle = \langle 0, 450 \rangle$

$\frac{1}{2}x - \frac{\sqrt{3}}{2}y = 0 \Rightarrow x - \sqrt{3}\,y = 0 \Rightarrow x = \sqrt{3}\,y$

$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 450 \Rightarrow \sqrt{3}x + y = 900$

$\Rightarrow \sqrt{3}\,x + y = \sqrt{3}(\sqrt{3}\,y) + y = 4y = 900 \Rightarrow \boxed{y = 225 \text{ lbs}}$

$\Rightarrow x = 225\sqrt{3}$ lbs $\approx 389.7114318 \approx \boxed{389.7 \text{ lbs}}$

18 $\bar{u} = \langle 6,8\rangle,\ \bar{v} = \langle -7,-1\rangle$

a 5pts $\boxed{\bar{u}+\bar{v} = \langle -1,7\rangle}$

b 5pts Bonus $\boxed{\bar{u}\cdot\bar{v} = -42-8 = -50}$

c $\boxed{\|\bar{u}\| = \sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100} = 10}$

5pts $\boxed{\|\bar{v}\| = \sqrt{7^2+1^2} = \sqrt{50} = 5\sqrt{2}}$

d 5pts θ = angle between $\bar{u}$ & $\bar{v}$ $\longrightarrow$

Bonus $\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \dfrac{-50}{10\cdot 5\sqrt{2}} = -\dfrac{1}{\sqrt{2}} \longrightarrow$

$\theta = \arccos\left(-\frac{1}{\sqrt{2}}\right) = \boxed{135^\circ \text{ or } \frac{3\pi}{4} = \theta}$ EXACT

e 5pts $\text{proj}_{\bar{v}}\bar{u} = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{v} = \dfrac{-50}{50}\bar{v} = -\bar{v} = -\langle -7,-1\rangle = \langle 7,1\rangle$

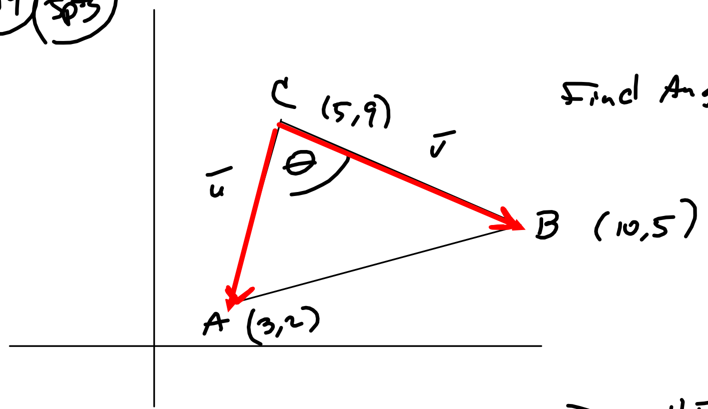
$\bar{w}_1 = -\bar{v} = \boxed{\langle 7,1\rangle = \text{proj}_{\bar{v}}\bar{u}}$

f 5pts $\bar{w}_1 = -\bar{v}$

$\bar{w}_2 = \bar{u} - \bar{w}_1 = \bar{u} + \bar{v} = \boxed{\langle -1,7\rangle = \bar{w}_2}$

Check $\bar{w}_1\cdot\bar{w}_2 = \langle 7,1\rangle\cdot\langle -1,7\rangle = -7+7 = 0$ ✓

19 5pts



Find Angle C

$\bar{u} = \langle 3-5, 2-9 \rangle = \langle -2, -7 \rangle$, $\|\bar{u}\| = \sqrt{4+49} = \sqrt{53}$

$\bar{v} = \langle 10-5, 5-9 \rangle = \langle 5, -4 \rangle$, $\|\bar{v}\| = \sqrt{25+16} = \sqrt{41} =$

$\cos C = \dfrac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \dfrac{-10+28}{\sqrt{53}\sqrt{41}} = \dfrac{18}{\sqrt{53 \cdot 41}} \approx$ 0.3861380881

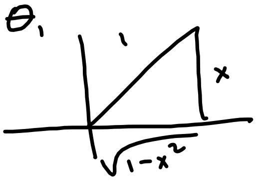
$\Rightarrow C = \arccos(\cos C) \approx \arccos(0.3861380881) \approx 67.28558766^\circ$

$\boxed{C \approx 67.2856^\circ}$ & $\theta = C$

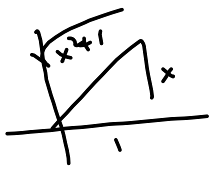
20 5pts

$\cos(\arcsin(x) + \arctan(x)) = \cos(\theta_1 + \theta_2)$

θ_1



θ_2



$\cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$

$= \left(\dfrac{\sqrt{1-x^2}}{1}\right)\left(\dfrac{1}{\sqrt{x^2+1}}\right) - \left(\dfrac{x}{1}\right)\left(\dfrac{x}{\sqrt{x^2+1}}\right)$

$= \boxed{\dfrac{\sqrt{1-x^2} - x^2}{\sqrt{x^2+1}} = \cos(\theta_1 + \theta_2)}$