MAT 1420-R11 Written Final Test – Spring, 2025 Print Name_____
160 Points Covers Chapters 1 – 4 and parts of Chapter 6

Show all work. Do your own work. Work down the page. The right half of the page is for scratch work. Show all scratch work. Circle final answers. Leave plenty of space.

Paper will be provided. Do not take anything from the test with you when you leave. We'll sell your car keys back to you. Don't worry.

Turn in your test sheets, your work, and your cheat sheet.

The main difference between this test and your Written Assignments is that you don't have to write out the question on a test. No annoying "Context" deductions. That doesn't relieve you of the need for a lexicon for all word problems that defines all variables in words, and the units in which the variables are measured.

Draw Pictures! Fare well!

1. A unicycle, whose wheels have an 18-inch radius, is rolling along at 20 miles per hour (mph).

   a. (5 pts) How fast is the unicycle's wheel spinning, in revolutions per minute (rpm)? Give an exact, simplified answer. Then round to 3 places. Hint: I think the lexicon is the key to this one. I would start with what I wanted, convert rpm to mph, and set it equal to 20 mph.

   b. (5 pts) What's the probability that the cyclist will wreck? Assume the cyclist has one week of experience on a unicycle. Give your answer as a percentage, rounded to 3 decimal places.

2. Answer the questions about the equation $\tan(\theta) = -\frac{3}{4}$. Assume $\theta \in [0, 2\pi)$.

   a. (5 points) Sketch two triangles on the same set of coordinate axes that satisfy $\tan(\theta) = -\frac{3}{4}$.

   b. (5 pts) Given $\sin(\theta) < 0$, in what quadrant does θ lie?

   c. (5 pts) Again, assuming $0 \le \theta < 2\pi$, find the *exact* value of θ, in radians.

   d. (5 pts) Round your previous answer to 3 decimal places.

   e. (5 pts) Find the exact value of the other 5 trigonometric functions for θ, based on previous work.

   f. (5 pts) Find approximate values of $\sin\left(\frac{\theta}{2}\right)$ and $\cos\left(\frac{\theta}{2}\right)$. Round to 4 decimal places.

   g. (5 pts) Find the exact values of $\sin\left(\frac{\theta}{2}\right)$ and $\cos\left(\frac{\theta}{2}\right)$.

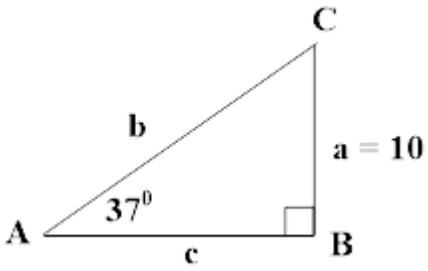
   h. (5 pts) Based on your work in part *c*, find *all* solutions to the equation $\tan(\theta) = -\frac{3}{4}$, in radians. Give *exact* answers.

3. Let $f(x) = 22\sin\left(\frac{\pi}{6}x - \frac{25\pi}{6}\right) + 11$.

   a. (5 pts) Sketch one period of the graph of $f(x)$. Clearly label all highs, lows, and intersections with the midline. I don't care where you start the one period. Just graph the one period correctly.

   b. (5 pts) Solve $f(x) = 0$. Label these points on your graph for part *a*.

4. (5 pts) Solve the triangle in the figure on the right. That means, find all lengths and angles. Give exact answers and then round to 3 decimal places.



5. (5 pts) Find the exact value of $\arctan\left(\tan(4)\right)$. Then round to 3 decimal places.

6. (5 pts) Draw the picture and use it to re-write $\cos\left(\arctan\left(\frac{7}{x}\right)\right)$ as an algebraic expression.

7. (5 pts) Use a sum identity to find the *exact* value of $\cos\left(\frac{5\pi}{12}\right)$.

8. Let $z_1 = 3\left(\cos\left(\frac{\pi}{7}\right) + i\sin\left(\frac{\pi}{7}\right)\right)$ and $z_2 = 2\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$.

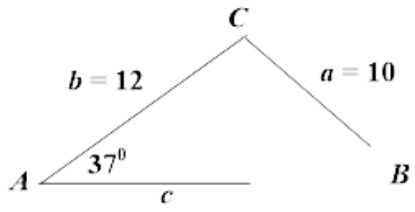
   a. (5 pts) Compute the product $z_1 z_2$. Give an exact answer.

   b. (5 pts) Round your answer to part *a* to 3 decimal places.

   c. (5 pts) Find the 3rd roots of z_1.

   d. (5 pts) Convert z_1 to rectangular form. Round to 3 decimal places.

9. Suppose $a = 10$, $b = 12$ and $A = 37^0$ in triangle ABC. (See figure.)



   a. (5 pts) Show that there are two solutions.

   b. (5 pts) Find two answers for angle *B*. Call them B_1 and B_2. Round your answers to 3 decimal places.

10. Suppose $b = 12$, $c = 10$, and $A = 37^0$ for a triangle ABC.

    a. (5 pts) Find the exact value of *a*. Express your answer in simplified radical form.

    b. (5 pts) Find side *a*. Round to 3 decimal places.

11. Let $\overline{u} = \langle 2,3 \rangle$ and $\overline{v} = \langle -3,19 \rangle$

a. (5 pts) Find $\overline{u} \bullet \overline{v}$

b. (5 pts) Find $\|\overline{u}\|$ and $\|\overline{v}\|$.

c. (5 pts) Find the angle between $\overline{u}$ and $\overline{v}$, in degrees, rounded to 1 decimal place.

d. (5 pts) Find $\text{proj}_{\overline{v}}\overline{u}$. Round answer to 1 decimal place.

e. (5 pts) Sketch the 3 vectors $\overline{u}, \overline{v}$, and $\text{proj}_{\overline{v}}\overline{u}$ on the same set of coordinate axes.

12. Let $r = 2\cos(2\theta) + 1$.

a. (5 pts) Sketch the graph of $r(\theta)$ in rectangular coordinates.

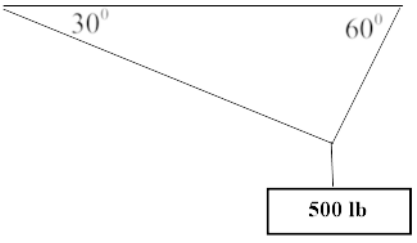
b. (5 pts) Sketch the graph of $r(\theta)$ in polar coordinates.

13. (5 pts) Identify the conic section and graph $r(\theta) = \dfrac{16}{4 - 2\cos(\theta)}$. Then sketch its graph. Be sure to find and label the vertex/vertices, focus/foci, and directrix.

Bonus Section

1. (**Bonus** 5 pts) Re-write $\tan(\arcsin(x) + \arccos(x))$ as an algebraic expression. (Hint: Use Sum identity.)

2. (**Bonus** 5 pts) A weight of 500 pounds is suspended from two cables, as shown in the figure. Find the tension in both cables. Round to the nearest pound.



3. (**Bonus** 5 pts) The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 3 inches and 15 inches, respectively. A cyclist is pedaling at a rate of 2 revolutions per second. Find the *exact* speed of the bicycle in miles per hour. Then round your answer to 3 decimal place.