1) velocity $= 20 \frac{mi}{hr}$. Wheels' radius is 18in. How fast are the wheels spinning, in rpm, if the bike is moving @ 20 mph?

Let $\boxed{x = \text{how fast the wheels are spinning, in revolutions per minute.}}$

Following the hint, we convert x to miles per hour, set $= 20$ & solve for x.

Scratch:

$$\left(x\,\frac{rev}{min}\right)\left(\frac{2\pi\cdot 18\,in}{rev}\right)\left(\frac{1\,ft}{12\,in}\right)\left(\frac{1\,mi}{5280\,ft}\right)\left(\frac{60\,min}{1\,hr}\right) = \frac{2\pi(18)(60)}{12(5280)} = \frac{2\pi(3)(6)}{2(528)}$$

$$= \frac{(2\pi)(18)(60)}{12(5280)}x = \frac{3\pi}{88}x \overset{SET}{=} 20\,\frac{mi}{hr} \qquad = \frac{\pi(6)}{176} = \frac{3\pi}{88}$$

$$\Longrightarrow x = \frac{20(88)}{3\pi} = \boxed{\frac{1760}{3\pi}\,\frac{rev}{min} = x \approx 186.7417999\,\frac{rev}{min}}$$

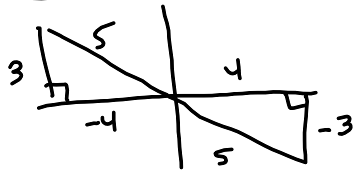
check:

$$\left(\frac{1760}{3\pi}\,\frac{revs}{min}\right)\left(\frac{60\,min}{hr}\right)\left(\frac{(18)(2\pi)\,in}{rev}\right)\left(\frac{1\,ft}{12\,in}\right)\left(\frac{1\,mi}{5280\,ft}\right)$$

$$= \frac{1760(60)(18)(2\pi)}{3\pi(12)(5280)}\,\frac{mi}{hr} = 20\,\frac{mi}{hr} \checkmark$$

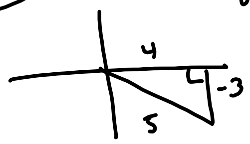
(2) $\tan\theta = -\frac{3}{4}$ and $\theta \in [0, 2\pi)$

(a) (5pts) sketch 2 triangles for $\tan\theta = -\frac{3}{4}$ on same set of axes.



(b) (5pts) In what quadrant is θ if $\sin\theta < 0$?

$\tan\theta = -\frac{3}{4}$ and $\sin\theta < 0$.



QIV

(c) (5pts) Find exact value of θ, in radians

More than one form possible.

$$\boxed{\theta = 2\pi - \arctan\left(\frac{3}{4}\right) = 2\pi - \arcsin\left(\frac{3}{5}\right) = 2\pi - \arccos\left(\frac{4}{5}\right)}$$

(d) (5pts) Round previous ans to 3 decimal places

$2\pi - \arctan\left(\frac{3}{4}\right) \approx 5.639684199 \approx \boxed{5.640 \approx \theta}$

$\approx 323.1301024^\circ$ But shouldn't be in degrees.

Close tries:

-36.8698976° is not expressed as an angle between 0 & 2π.

(e) (5pts) Find exact values of other 5 trig functions

$$\boxed{\begin{array}{ll} \sin\theta = -\frac{3}{5} & \csc\theta = -\frac{5}{3} \\ \cos\theta = \frac{4}{5} & \sec\theta = \frac{5}{4} \\ \tan\theta = -\frac{3}{4} & \cot\theta = -\frac{4}{3} \end{array}}$$

5.639684199

(f) (5pts) Find approximate values $\sin\left(\frac{\theta}{2}\right)$ & $\cos\left(\frac{\theta}{2}\right)$. Round to 4 places.

$$\sin\left(\frac{5.639684199}{2}\right) \approx 0.3162277660 \approx 0.3162$$

$$\cos\left(\frac{5.639684199}{2}\right) \quad -0.9486832980 \approx -0.9487$$

Common error!

This will be wrong

$$\sin\left(\frac{5.640}{2}\right) \approx 0.3160779642 \approx \boxed{0.3161 \approx \cos\left(\frac{\theta}{2}\right)}$$

$$\cos\left(\frac{5.640}{2}\right) \approx -0.9487332188 \approx \boxed{-0.9487 \approx \cos\left(\frac{\theta}{2}\right)}$$

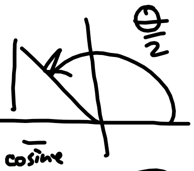
(g) (5pts) Exact answers.

$\theta \in QIV$

$\frac{3\pi}{2} < \theta < 2\pi$

$\frac{3\pi}{4} < \frac{\theta}{2} < \pi$

$\Longrightarrow \frac{\theta}{2} \in QII$, + sine



$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{5-4}{10}} = \sqrt{\frac{1}{10}} = \boxed{\frac{1}{\sqrt{10}} = \sin\frac{\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\frac{4}{5}}{2}} = -\sqrt{\frac{\frac{9}{5}}{2}} = -\sqrt{\frac{9}{10}} = \boxed{-\frac{3}{\sqrt{10}} = \cos\left(\frac{\theta}{2}\right)}$$

(h) Find ALL solns to $\tan\theta = -\frac{3}{4}$. Give exact answers.

$$\boxed{\theta = 2\pi - \arcsin\left(\frac{3}{5}\right) + n\pi}$$

OR $\arctan\left(-\frac{3}{4}\right)$ OR $\arccos\left(\frac{4}{5}\right)$

③ $f(x) = 22\sin\left(\frac{\pi x}{6} - \frac{25\pi}{6}\right) + 11 = 22\sin\left(\frac{\pi}{6}(x-25)\right) + 11$

$a = 22$

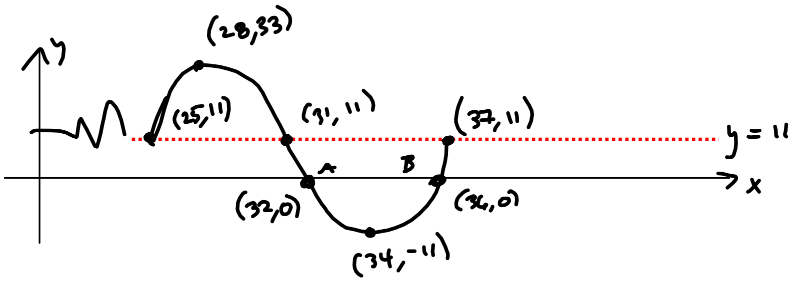
$\frac{\pi}{6}x = 2\pi$

$x = 2\pi\left(\frac{6}{\pi}\right) = 12$

$x \mapsto x + 25$

$y = 11$ mid

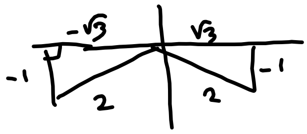
ⓐ Sketch one period



ⓑ 5pts Solve $f(x) = 0$

$22\sin\left(\frac{\pi}{6}x - \frac{25\pi}{6}\right) = -11$

$\sin\left(\frac{\pi}{6}x - \frac{25\pi}{6}\right) = -\frac{11}{22} = -\frac{1}{2}$



$\frac{6}{\pi}\left(\frac{\pi}{6}x - \frac{25\pi}{6} = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}\right)$

$x - 25 = 7$ or 11

$\boxed{x = 32 \text{ or } 36}$

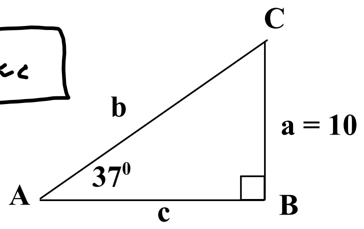
$f(0) = 22\sin\left(-\frac{25\pi}{6}\right) + 11$

$= 22\sin\left(-\frac{24\pi}{6} - \frac{\pi}{6}\right) + 11$

$= 22\sin\left(-\frac{\pi}{6}\right) + 11$

$= 22\left(-\frac{1}{2}\right) + 11$

4 (5pts) Solve the triangle. Give Exact Answers, Then round to 3 places.



$\frac{c}{10} = \cot 37^0$

$\boxed{c = 10\cot 37^0} \approx 13.2704482162 \approx \boxed{13.270 \approx c}$

$\frac{b}{10} = \csc 37^0$

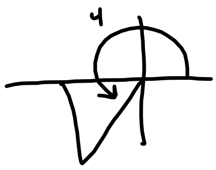
$\boxed{b = 10\csc 37^0} \approx 16.6164014112 \approx \boxed{16.616 \approx b}$

$\boxed{B = 90^0}$

$C = 180^0 - 90^0 - 37^0 =$

$= 90^0 - 37^0 = \boxed{53^0 = C}$

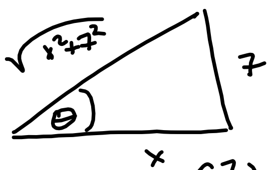
5 (5pts) arctan (tan (4))



$\arctan(\tan(4)) = \boxed{4 - \pi \approx 0.858407346 \approx 0.858}$

This one is tricky, if you don't understand domain and especially range of arctangent.

6 (5pts) Re-write $\cos(\arctan(\frac{7}{x}))$ as an algebraic expression



$\theta = \arctan(\frac{7}{x})$

$\boxed{\cos(\arctan(\frac{7}{x})) = \frac{x}{\sqrt{x^2+7^2}}}$

7 (5pts) Use a sum identity to find the exact value of

$\cos(\frac{5\pi}{12}) = \cos(\frac{2\pi}{12} + \frac{3\pi}{12}) = \cos(\frac{\pi}{6} + \frac{\pi}{4})$

$= \cos(\frac{\pi}{6})\cos(\frac{\pi}{4}) - \sin(\frac{\pi}{6})\sin(\frac{\pi}{4}) = (\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2}) - (\frac{1}{2})(\frac{\sqrt{2}}{2})$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4} = \cos(\frac{5\pi}{12})}$

(8) $z_1 = 3\left(\cos\left(\frac{\pi}{7}\right) + i\sin\left(\frac{\pi}{7}\right)\right)$, $z_2 = 2\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

(a) (5pts) $z_1 z_2 = 6\left(\cos\left(\frac{\pi}{7}+\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{7}+\frac{\pi}{4}\right)\right)$

$= 6\left(\cos\left(\frac{11\pi}{28}\right) + i\sin\left(\frac{11\pi}{28}\right)\right)$

(b) (5pts) Rectangular form for part (a). Round to 3 places.

1.981674371 + 5.663299981i $\approx \boxed{1.982 + 5.663i \approx z_1 z_2}$

(c) (5pts) 3rd roots of $z_1 = 3\left(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7}\right)$

$\frac{\theta}{n} = \frac{\theta}{3} = \frac{\pi}{7} \div 3 = \frac{\pi}{21}$

Increment $= \frac{2\pi}{n} = \frac{2\pi}{3} = \frac{14\pi}{21}$

$\frac{14\pi}{21} + \frac{1}{21} = \frac{5\pi}{21} = \frac{5\pi}{7}$

$\frac{15\pi}{21} + \frac{14\pi}{21} = \frac{29\pi}{21}$

$\frac{(29+14)}{21}\pi = \frac{43\pi}{21}$

$= \frac{42\pi}{21} + \frac{\pi}{21} = 2\pi + \frac{\pi}{21}$

$$\boxed{\begin{aligned} z_0 &= \sqrt[3]{3}\left(\cos\left(\tfrac{\pi}{21}\right) + i\sin\left(\tfrac{\pi}{21}\right)\right) \\ z_1 &= \sqrt[3]{3}\left(\cos\left(\tfrac{5\pi}{7}\right) + i\sin\left(\tfrac{5\pi}{7}\right)\right) \\ z_2 &= \sqrt[3]{3}\left(\cos\left(\tfrac{29\pi}{21}\right) + i\sin\left(\tfrac{29\pi}{21}\right)\right) \end{aligned}}$$

Check $z_3 = \sqrt[3]{3}(\cos \ldots$ is coterminal with z_0 ✓

(d) (5pts) We convert z_1 to rectangular form. Round to 3 places.

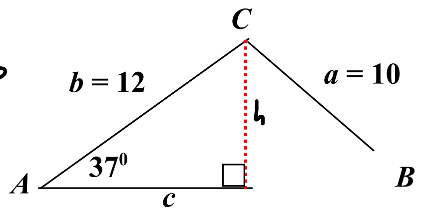
$z_1 = 3\left(\cos\left(\frac{\pi}{7}\right) + i\sin\left(\frac{\pi}{7}\right)\right) \approx$ 2.506845720 +1.647945550 i

$\approx \boxed{2.507 + 1.648i \approx z_1}$

9 $A = 37^0,\ b = 12,\ a = 10$

a Show $\exists$ 2 solns.

$h = 12\sin(37^0) \approx 7.22 < a < b \Rightarrow$ 2 solns



b 5pts $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\sin B = \frac{b\sin A}{a} = \frac{12\sin 37^0}{10} \approx 0.7221780281$

$\Rightarrow B_1 \approx 46.23459646^0 \approx \boxed{B_1 \approx 46.235^0}$

$B_2 = 180^0 - B_1 \approx 133.7654035^\circ \approx \boxed{133.765^0 \approx B_2}$

10 $b = 10,\ c = 12,\ A = 37^0$

a Find a. Give exact simplified radical form.

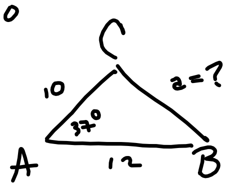
$a^2 = b^2 + c^2 - 2bc\cos(A)$

$= 10^2 + 12^2 - 2(10)(12)\cos(A)$

$= 244 - 240\cos(A) \Rightarrow$

$\boxed{a = 2\sqrt{61 - 40\cos(37^0)}}$

2 | 244   2 | 240
2 | 122   2 | 120
     61   60



b 5pts Round part a to 3 places

$a \approx 7.233773400 \approx \boxed{7.234 \approx a}$

(11) $\bar{u} = \langle 2,3\rangle$ & $\bar{v} = \langle -3, 19\rangle$

(a) 5pts $\bar{u}\cdot\bar{v} = -6 + 57 = \boxed{51 = \bar{u}\cdot\bar{v}}$

(b) 5pts $\|\bar{u}\| = \sqrt{2^2+3^2} = \boxed{\sqrt{13} = \|\bar{u}\|}$

$\|\bar{v}\| = \sqrt{3^2+19^2} = \sqrt{9+361} = \boxed{\sqrt{370} = \|\bar{v}\|}$

(c) 5pts Let θ = angle between $\bar{u}$ & $\bar{v}$. Then

$\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \dfrac{51}{\sqrt{13}\sqrt{370}}$

$\theta = \arccos\left(\dfrac{51}{\sqrt{13}\sqrt{370}}\right) \approx$ 0.74460448053 $\approx 42.6626941409^{\circ}$

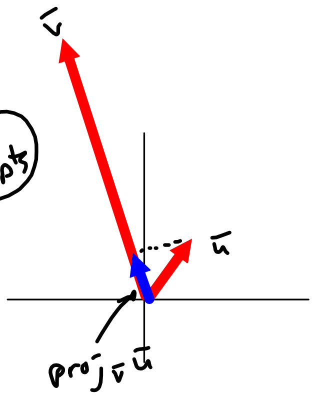
$\approx \boxed{42.7^{\circ} \approx \theta}$

(d) 5pts $\text{proj}_{\bar{v}}\bar{u} = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{v} = \boxed{\dfrac{51}{370}\langle -3, 19\rangle} = \left\langle \dfrac{-150}{370}, \dfrac{969}{370}\right\rangle$

$\approx \langle -0.4135135135, 2.618918919\rangle$

$\approx \boxed{\langle -.4, 2.6\rangle \approx \text{proj}_{\bar{v}}(\bar{u})}$

(e) 5pts



Sketch $\bar{u}, \bar{v}, \text{proj}_{\bar{v}}\bar{u}$

(12) (5 pts) $r(\theta) = 2\cos(2\theta) + 1 = a\sin(b(\ -c)) + d$

$a = 2 = \text{amp}$

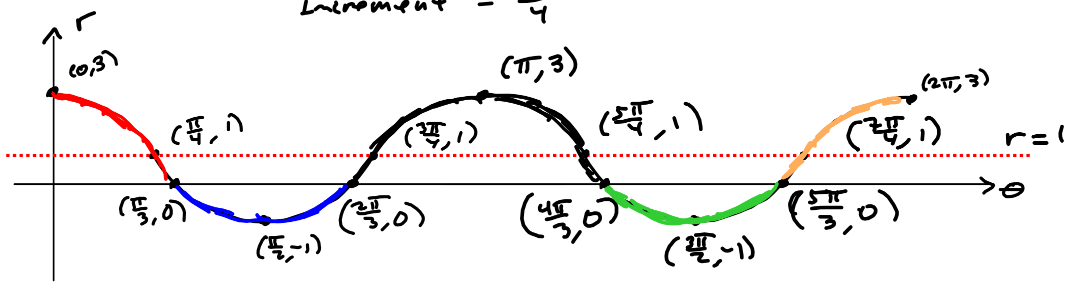
$b(\theta - c) = 2(\theta - 0)$

horizontal shift $= 0$

$2\theta = 2\pi$ when

$\theta = \pi$ = period

Increment $= \frac{\pi}{4}$



Instructions don't say. Teacher always does one period...

5 pts bonus if they graph two periods, perfectly.

x-ints:

Solve $r(\theta) = 0$

over 2π, find all

$\theta \in [0, 2\pi)$

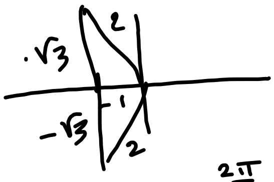
$\Rightarrow 2\theta \in [0, 4\pi)$

$r(\theta) = 0$

$\Rightarrow 2\cos(2\theta) + 1 = 0$

$2\cos(2\theta) = -1$

$\cos(2\theta) = -\frac{1}{2}$



$2\pi = 6\frac{\pi}{3}$ = inc.

$\Rightarrow 2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

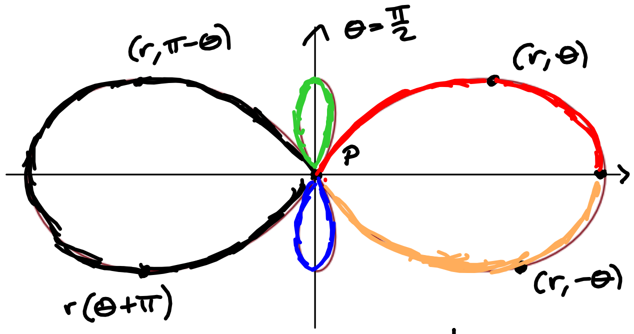
$0 \le \theta \le \frac{\pi}{3}$

$\frac{\pi}{3} < \theta \le \frac{2\pi}{3}$

$\frac{2\pi}{3} < \theta \le \frac{4\pi}{3}$

$\frac{4\pi}{3} < \theta < \frac{5\pi}{3}$

$\frac{5\pi}{3} \le \theta < 2\pi$



P = Pole is intersected @

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$r(-\theta) = r(\theta) \Rightarrow$ symmetric about polar axis.

$r(\theta+\pi) = 2\cos(2(\theta+\pi)) + 1 = 2\cos(2\pi + 2\theta) + 1 = 2\cos(2\theta) + 1$

$\Rightarrow$ symmetric thru the pole

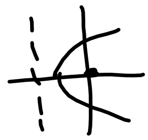
$r(\pi-\theta) = 2\cos(2(\pi-\theta)) + 1 = 2\cos(2\pi - 2\theta) + 1$

$= 2\cos(-2\theta) + 1 = 2\cos(2\theta) + 1 = r$

$\Rightarrow$ symmetric about $\theta = \frac{\pi}{2}$.

13 5pts Graph $r(\theta) = \frac{16}{4-2\cos(\theta)} = \frac{4\cdot 4}{4(1-\frac{1}{2}\cos\theta)}$

$= \frac{4}{1-\frac{1}{2}\cos\theta} = \frac{ep}{1-e\cos\theta}$.

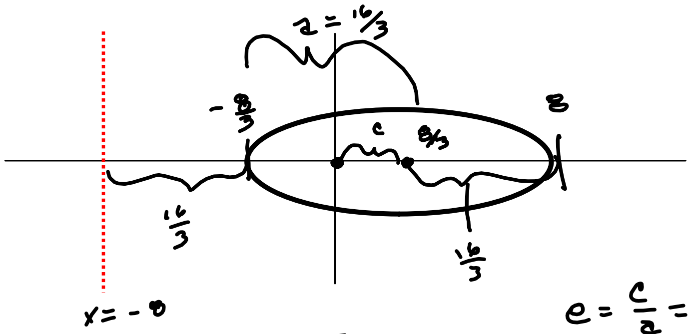


Ellipse. Directrix to left of pole

$e = \frac{1}{2} = \frac{c}{a} = \frac{3}{16}c \longrightarrow c = \frac{16}{3}\cdot\frac{1}{2} = \frac{8}{3}$

$r(0) = \frac{16}{4-2} = \frac{16}{2} = 8$

$r(\pi) = \frac{16}{4+2} = \frac{16}{6} = \frac{8}{3}$



$\frac{8+(-\frac{8}{3})}{2} = \frac{\frac{24-8}{3}}{2} = \frac{16}{6} = \frac{8}{3}$

center $= (\frac{8}{3}, 0)$ in both polar & rect.

$a = \frac{16}{3}$

$e = \frac{c}{a} = \frac{1}{2} = \frac{\frac{8}{3}}{\frac{16}{3}} = \frac{1}{2}$ ✓

$b = \sqrt{a^2 - c^2}$

$= \sqrt{\left(\frac{16}{3}\right)^2 - \left(\frac{8}{3}\right)^2}$

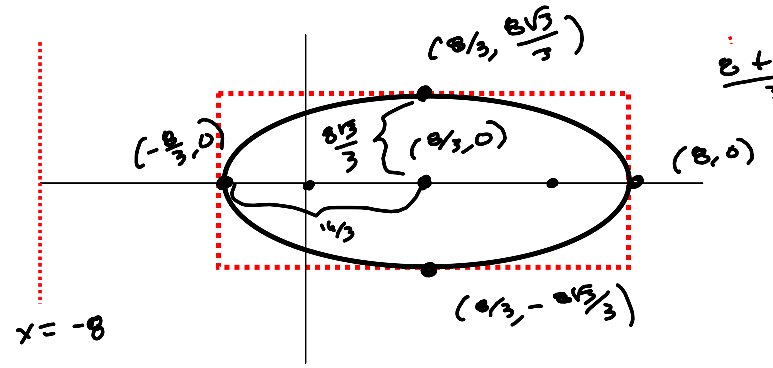
$= \sqrt{\frac{8^2\cdot 2^2 - 8^2}{3^2}} = \sqrt{\frac{8^2(2^2-1)}{3^2}} = \boxed{\frac{8\sqrt{3}}{3} = b}$

$ep = \frac{1}{2}p = 4 \longrightarrow$

$p = 8$ = distance from pole to directrix.

$x = -8$

$\frac{8+24}{3} = \frac{32}{3} = 2a$



$$r = \sqrt{x^2+y^2} = \sqrt{3^2+y^2} = \sqrt{9 + r^2\sin^2\theta}$$

$$r^2 = 9 + r^2\sin^2\theta$$

$$r^2 - r^2\sin^2\theta = r^2(1-\sin^2\theta) = 9$$

$$r^2 = \frac{9}{\cos^2\theta} = 9\sec^2\theta$$

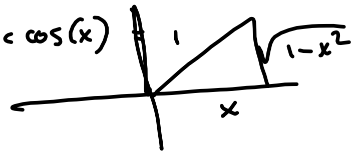
$$r = \pm 3|\sec\theta|$$

B1 5pts $\tan(\arcsin(x) + \arccos(x)) = \tan(\theta_1 + \theta_2)$

$$= \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2} = \frac{\tan(\arcsin(x)) + \tan(\arccos(x))}{1 - \tan(\arcsin(x))\tan(\arccos(x))}$$

$\arcsin(x)$

$\arccos(x)$



$$\frac{\frac{x}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{x}}{1 - \frac{x}{\sqrt{1-x^2}} \cdot \frac{\sqrt{1-x^2}}{x}}$$?!

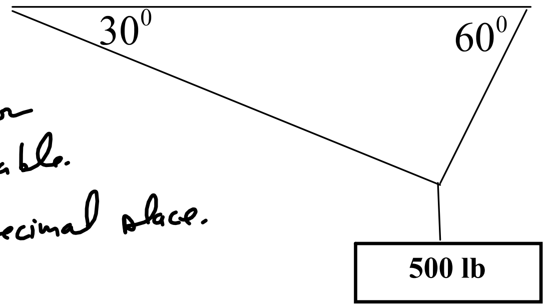
Undefined!
Division by zero!

$$= \frac{\sin(\theta_1+\theta_2)}{\cos(\theta_1+\theta_2)} = \frac{\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1}{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2}$$

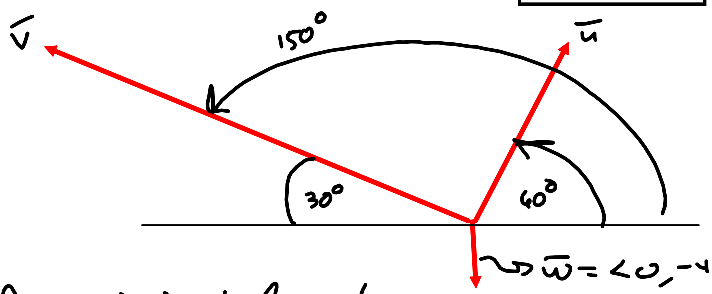
$$= \left(\frac{\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1}{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2}\right)\left(\frac{\cos\theta_1\cos\theta_2}{\cos\theta_1\cos\theta_2}\right)$$

$$= \frac{\frac{\sin\theta_1\cos\theta_2}{\cos\theta_1\cos\theta_2} + \frac{\sin\theta_2\cos\theta_1}{\cos\theta_1\cos\theta_2}}{\frac{\cos\theta_1\cos\theta_2}{\cos\theta_1\cos\theta_2} - \frac{\sin\theta_1\sin\theta_2}{\cos\theta_1\cos\theta_2}} = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2}$$

B2 5pts



Find tension in each cable.
Round to 1 decimal place.



$\rightsquigarrow \bar{w} = \langle 0, -450 \rangle$

Want forces to be balanced

So $\bar{u} + \bar{v} = -\bar{w} = \langle 0, 450 \rangle$

$\bar{u} = \|\bar{u}\| \langle \cos 60^\circ, \sin 60^\circ \rangle = x \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$, where $x \equiv \|\bar{u}\|$

$\bar{v} = \|\bar{v}\| \langle \cos 150^\circ, \sin 150^\circ \rangle = y \langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$, where $y = \|\bar{v}\|$

We're trying to find x & y:

$\langle \frac{1}{2}x, \frac{\sqrt{3}}{2}x \rangle + \langle -\frac{\sqrt{3}}{2}y, \frac{1}{2}y \rangle = \langle 0, 500 \rangle$

$\frac{1}{2}x - \frac{\sqrt{3}}{2}y = 0 \Rightarrow x - \sqrt{3}\,y = 0 \Rightarrow x = \sqrt{3}\,y$

$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 500 \Rightarrow \sqrt{3}x + y = 1000$

$\Rightarrow \sqrt{3}\,x + y = \sqrt{3}(\sqrt{3}\,y) + y = 4y = 1000 \Rightarrow \boxed{y = 250 \text{ lbs}}$

$\Rightarrow x = 250\sqrt{3}$ lbs $\approx$ 433.0127020 lbs $\approx \boxed{433 \text{ lbs}}$

B3 5pts 15-inch radius rear wheel, 3 inches rear sprocket, 5-inch front sprocket, pedaling @ $2\frac{\text{rev}}{\text{sec}}$.

Let x = the speed of the bicycle, in miles per hour. Then

$$x = \left(2\,\frac{\text{rev front}}{1\text{ sec}}\right)\left(\frac{5\text{ revs rear}}{3\text{ revs front}}\right)\left(\frac{(2\pi)(15\text{ in})}{1\text{ rev rear}}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)\left(\frac{60\,\frac{\text{mi}}{\text{hr}}}{88\,\frac{\text{ft}}{\text{sec}}}\right) =$$

$$= \frac{\cancel{(2)}(5)\cancel{(2\pi)}\overset{5}{\cancel{(15)}}\overset{5}{\cancel{(60)}}}{\cancel{3}\underset{6}{\cancel{(12)}}\underset{22}{\cancel{(88)}}} = \frac{125\pi}{22}\,\frac{\text{mi}}{\text{hr}} \approx 17.84995826\,\frac{\text{mi}}{\text{hr}} \approx \boxed{17.850\,\frac{\text{mi}}{\text{hr}} \approx x}$$