6.6 - Parametric Equations

6.7 - Polar Coordinates

6.8 - Graphing in Polar Coordinates

6.9 - Conic Sections in Polar Coordinates (The "p" isn't the same as the "p" for parabolas in Section 6.2.

Need to organize the Homework Notes and Videos.

Stuff from our 4.5 is in 4.4 stuff on harryzaims.com.

Same for 4.4 stuff included in 4.3 stuff on harryzaims.com.

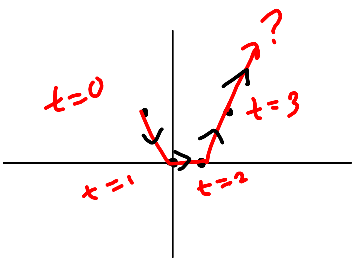
Jordan asked. Yasmin answered.

If $x=f(t)$ and $y=g(t)$ are continuous, then the set of points $(f(t), g(t))$ describes a plane curve

$x=t-1$
$y=t^2-3t+2$

t	x	y	
0	-1	2	
1	0	0	
2	1	0	
3	2	2	

$2^2-3(2)+2=4-6+2$



The arrows indicate the direction of increasing t. This is the ORIENTATION of the plane curve.

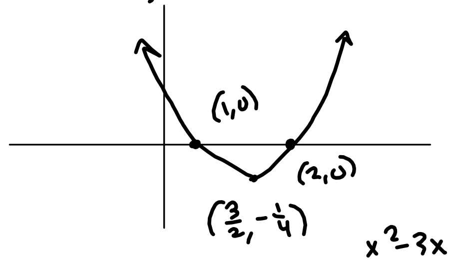
Eliminate the Parameter:

$x=t-1 \longrightarrow t=x+1 \longrightarrow$

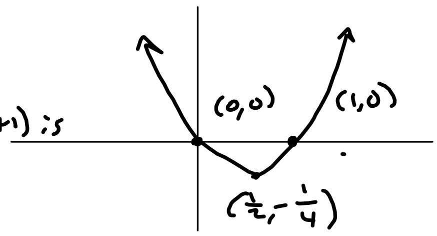
$y=(x+1)^2-3(x+1)+2$

M1 Observe this is a left shift by 1 unit of

$f(x)=x^2-3x+2=(x-1)(x-2)$



$\longrightarrow f(x+1)$ is



x^2-3x+2

$=x^2-3x+\left(\frac{3}{2}\right)^2-\frac{9}{4}+\frac{2}{1}\cdot\frac{4}{4}$

$=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}$

You lose the orientation when you eliminate the parameter.

An Ellipse!

$x = 2\cos\theta$
$y = 3\sin\theta$ is obviously an ellipse

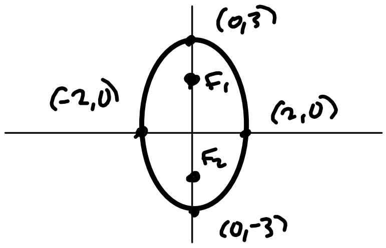
Eliminate the Parameter:

$\frac{x}{2} = \cos\theta$ & $\sin^2\theta + \cos^2\theta = 1$!

$\frac{y}{3} = \sin\theta$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$



$F_1 = (0, \sqrt{5})$
$F_2 = (0, -\sqrt{5})$

$c^2 = 3^2 - 2^2$
$= 9 - 4 = 5$

Hyperbola

$x = 5\sec\theta + 1 \Longrightarrow \dfrac{x-1}{5} = \sec\theta$

$y = 3\tan\theta - 7 \Longrightarrow \dfrac{y+7}{3} = \tan\theta$

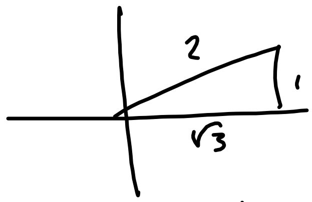
NOTE $\tan^2\theta + 1 = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$$\frac{(x-1)^2}{5^2} - \frac{(y+7)^2}{3^2} = 1$$ Lost the orientation

θ	x	y
0	6	-7
$\frac{\pi}{6}$	$\frac{10}{\sqrt{3}} + 1$	$\frac{3}{\sqrt{3}} - 7$

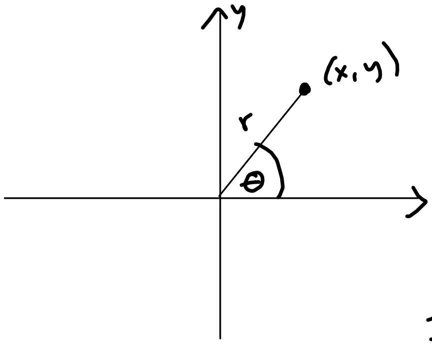
etc.



$\dfrac{1}{\cos(\frac{\pi}{6})} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}} = \sec(\frac{\pi}{6})$

$5\left(\frac{2}{\sqrt{3}}\right) + 1 = \frac{10}{\sqrt{3}} + 1 = \frac{10\sqrt{3}}{3} + \frac{3}{3}$

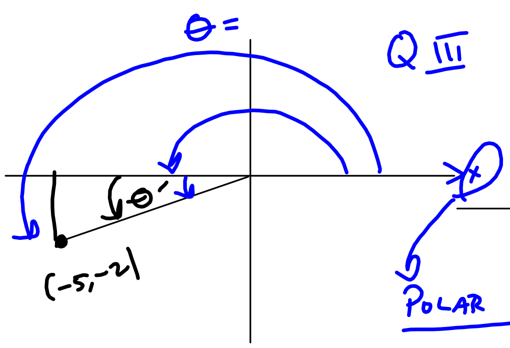
6.7 Stuff



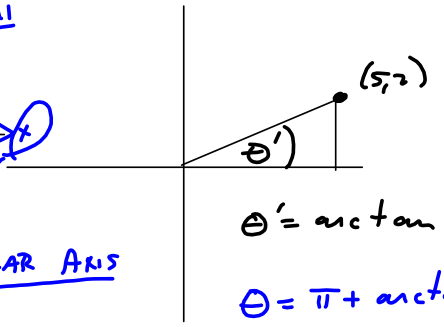
$r = \sqrt{x^2+y^2}$ = length from origin, which is now called THE POLE.

In QI, $\theta > 0$, $\theta = \arctan\left(\frac{y}{x}\right)$

It's trickier in other Quadrants

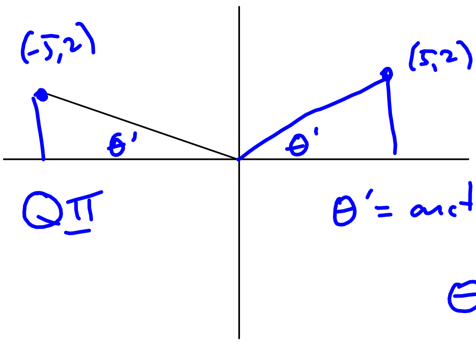


Q III



$\theta' = \arctan\left(\frac{2}{5}\right)$

$\theta = \pi + \arctan\left(\frac{2}{5}\right)$

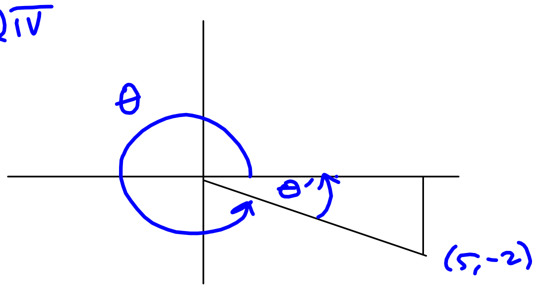


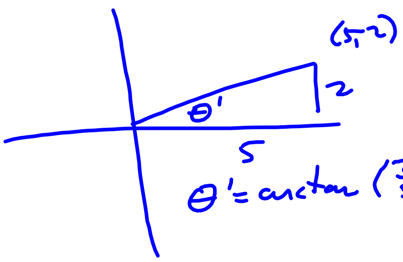
Q II

$\theta' = \arctan\left(\frac{2}{5}\right)$

$\theta = \pi - \arctan\left(\frac{2}{5}\right)$

Q IV



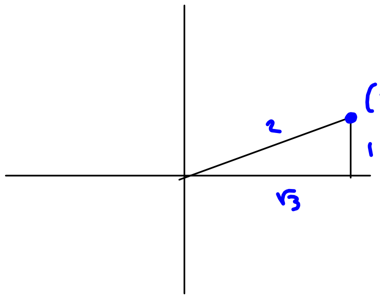


$\theta' = \arctan\left(\frac{2}{5}\right)$

$\theta = 2\pi - \arctan\left(\frac{2}{5}\right)$

In Q IV, θ is coterminal with $2\pi - \theta'$

This representation is <u>not</u> unique.

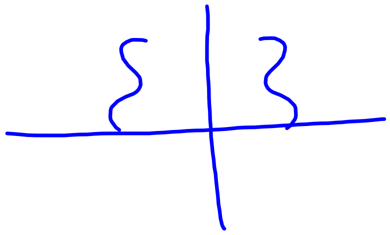


$(\sqrt{3}, 1) = (2, \frac{\pi}{6}) = (2, 30^\circ) = (-2, \frac{7\pi}{6})$!

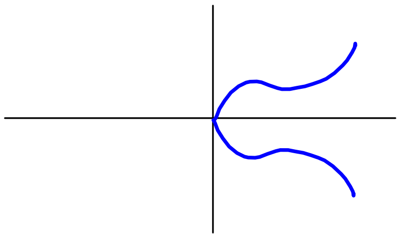
If a polar function is of the form $r = f(\sin\theta)$, it's symmetric about the line $\theta = \frac{\pi}{2}$

$1 + \sin\theta$

$3 - \sin\theta$



If a polar function is of the form $r = g(\cos\theta)$, it's symmetric about the line $\theta = 0$, which is polar axis



We'll discuss symmetry in more detail, next Monday.