Find all solutions of

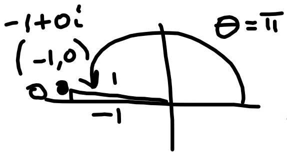
$x^4+1=0$ No real solutions, BUT

The Fundamental Theorem of Algebra says this thing has 4 (possibly nonreal) solutions!

$x^4=-1$

we need to find all the 4^{th} roots of $z=-1$.

$z=-1 \Longrightarrow \theta=\pi$



Principal 4^{th} root.

$z=1\left(\cos(\pi)+i\sin(\pi)\right)$

$\sqrt[4]{z} = \sqrt[4]{1}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$ $k=0$

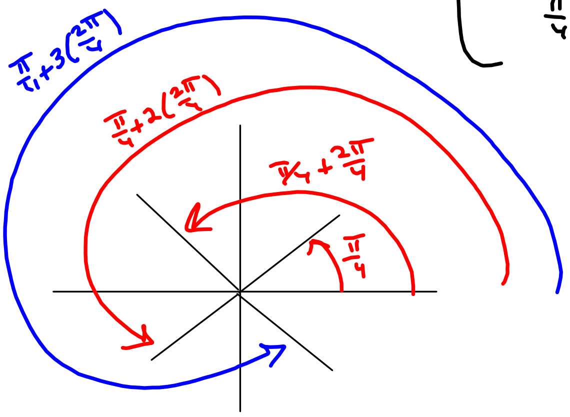
increment is $\frac{2\pi}{4}=\frac{2\pi}{n}$

$\theta_k=\frac{\theta+2k\pi}{4}$, $k=0,1,2,3$

$\frac{\pi}{4}+\frac{2\pi}{4}$ $k=1$

$\frac{\pi}{4}+2\left(\frac{2\pi}{4}\right)$ $k=2$

$\frac{\pi}{4}+3\left(\frac{2\pi}{4}\right)$ $k=3$



$\sqrt[4]{z} = \sqrt[4]{1}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$ $k=0$

increment $= \frac{2\pi}{4}$ ✓

$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$ $k=1$ $\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$

$\ldots = \frac{5\pi}{4}$ $k=2$

$\ldots = \frac{7\pi}{4}$ $k=3$

$\ldots = \frac{9\pi}{4}$ is coterminal w/ $\frac{\pi}{4}$

$1\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$ = Bonus/extra question $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ Rectangular

$1\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$ $= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$1\left(\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right)$ $= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

$1\left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right)$ $= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

$1\left(\cos\left(\frac{9\pi}{4}\right) + i\sin\left(\frac{9\pi}{4}\right)\right) = 1\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

7. (10 pts) Use the fact that $z = 1 + 2i$ is a zero of
$$f(x) = x^4 - x^3 - 3x^2 + 17x - 30$$
to obtain a linear factorization of $f(x)$, that is, split f into linear factors by finding all its zeros.

Book Way: Recall: Conjugate-Pairs Theorem:

If the coefficients of the polynomial are all real, then any nonreal (complex) zeros occur in conjugate pairs.

So, 1 + 2*i* is a zero ----> 1 - 2*i* is also a zero.

So, the Factor Theorem says (*x* - (1 + 2*i*)) and (*x* - (1 - 2*i*)) are factors.

So, $(x-(1+2i))(x-(1-2i))$ is a factor

$= (x-1-2i)(x-1+2i) = x^2 - x - 2ix - x + 1 - 2i + 2ix + 2i - (2i)^2$

$= x^2 - x - x + 1 - (-4)$

$= x^2 - 2x + 5$

$(a+bi)(a-bi)$
$= a^2 + b^2$
$= |1+2i|^2$

$|z| = |a+bi| = \sqrt{a^2+b^2} = \sqrt{z\bar{z}}$

$(x-(1+2i))(x-(1-2i))$

$= x^2 - (1-2i)x - (1+2i)x + (1+2i)(1-2i)$

$= x^2 - x + 2ix - x - 2ix + 1^2 + 2^2$

$= x^2 - 2x + 5$

$$
\begin{array}{r}
x^2 + x - 6 \\
x^2 - 2x + 5 \enclose{longdiv}{x^4 - x^3 - 3x^2 + 17x - 30} \\
-(x^4 - 2x^3 + 5x^2) \\
\hline
x^3 - 8x^2 + 17x - 30 \\
-(x^3 - 2x^2 + 5x) \\
\hline
-6x^2 + 12x - 30 \\
-(-6x^2 + 12x - 30) \\
\hline
0
\end{array}
$$

$\frac{x^4}{x^2} = x^2$

$\frac{x^3}{x^2} = x$

$\frac{-6x^2}{x^2}$

So, $\underset{\text{Divisor}}{(x^2-2x+5)}\underset{\text{Quotient}}{(x^2+x-6)}$

This is 2nd degree. We can clobber a quadratic!

Depressed Polynomial:

$x^2 + x - 6 = (x+3)(x-2) = 0 \implies x \in \{-3, 2\}$

Zeros: $\boxed{x = 1 \pm 2i, -3, 2}$

x^2+x-6

$a=1, b=1, c=-6$

$b^2-4ac = 1^2-4(1)(-6) = 1+24 = 25$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-1 \pm \sqrt{25}}{2(1)} = \dfrac{-1 \pm 5}{2}$ → $\dfrac{4}{2} = \boxed{2 = x}$, $\dfrac{-6}{2} = \boxed{-3 = x}$

Also $\boxed{x = 1 \pm 2i}$

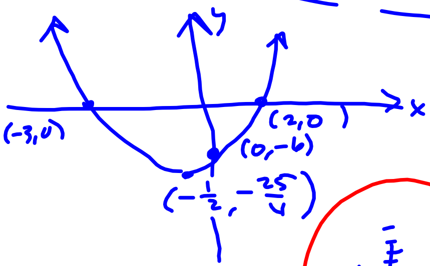
$x^2+x-6 = x^2+x+\left(\frac{1}{2}\right)^2 - \frac{1}{4} - \frac{6}{1}\cdot\frac{4}{4}$

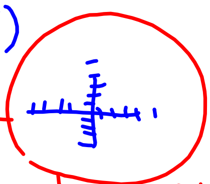
$1 \rightsquigarrow \frac{1}{2} \rightsquigarrow \left(\frac{1}{2}\right)^2$

$= \left(x+\frac{1}{2}\right)^2 - \frac{25}{4}$

$(h,k) = \left(-\frac{1}{2}, -\frac{25}{4}\right)$

from $a(x-h)^2+k$





Busy Work BLEAH!

(Waste of time).

$\left(x+\frac{1}{2}\right)^2 - \frac{25}{4} = 0$

$\left(x+\frac{1}{2}\right)^2 = \frac{25}{4}$

$x+\frac{1}{2} = \pm\sqrt{\frac{25}{4}} = \pm\frac{5}{2}$

$x = -\frac{1}{2} \pm \frac{5}{2}$ → $\frac{4}{2} = 2$, $-\frac{6}{2} = -3$

Using a more direct approach, using synthetic division, and splitting off the factors, one by one.

7. (10 pts) Use the fact that $z = 1 + 2i$ is a zero of

$$f(x) = x^4 - x^3 - 3x^2 + 17x - 30$$

to obtain a linear factorization of $f(x)$, that is, split f into linear factors by finding all its zeros.

$1+2i$ is a ~~zero~~ zero $\Longrightarrow$ $(x-(1+2i))$ is a factor. Divide by $x-(1+2i)$

$-$

$$\cancel{\begin{array}{r|rrrrr} 1+2i & 1 & -1 & 3 & 17 & -30 \\ & & 1+2i & -4+2i & -5 & \\ \hline & 1 & 2i & -1+2i & 12 & \end{array}}$$

Miscopy, fool!

$-(1-2i)$

$-(1-2i)(1+2i) = -(1^2+2^2) = -5$

$$\begin{array}{r|rrrrr} 1+2i & 1 & -1 & -3 & 17 & -30 \\ & & 1+2i & -4+2i & -11+16i & \\ \hline 1-2i & 1 & 2i & -7+2i & 6+16i & \\ \hline & 1 & 1 & -6 & & \end{array}$$

integer $(1-2i)$

$$(1+2i)(-7+2i) = -7+2i-14i-4$$
$$= \underbrace{-11-12i}_{\text{Needs to be}}$$

$\overline{1+2i} = 1-2i$

$$\begin{array}{l|ccccc}
1+2i & 1 & -1 & -3 & 17 & -30 \\
 & & 1+2i & -4+2i & -11-12i & 30 \\
\hline
1-2i & 1 & 2i & -7+2i & 6-12i & 0 \\
 & & 1-2i & 1-2i & -6+12i & \\
\hline
 & 1 & 1 & -6 & 0 &
\end{array}$$

$\boxed{(x-(1+2i))(x-(1-2i))(x+3)(x-2)}$ is fully factored, linear factorization

$x^2+x-6 = (x+3)(x-2)$

$\boxed{x = 1\pm 2i, -3, 2}$

$2i(1+2i) = 2i + (2i)^2 = 2i-4$

$(-7+2i)(1+2i) = -7-14i+2i-4$

$= -11-12i$

$(1+2i)(6-12i) = 6(1+2i)(1-2i) = 6(1^2+2^2)$

$= 6(5) = 30$!