HOR 107 on Greeley Campus

8 am to 6 pm, 10/13 or 10/14

Bring scientific calculator (not graphing calculator)

Bring 1-page, 2-sided cheat sheet.

Studying from Spring, 2025 Midterm?

Everything beyond #13 is Chapter 3 stuff, which we didn't cover, yet!

Nothing on Page 3.

One of the Bonus questions is Angle-Sum question (#20) that we did cover.

Spring, 2024 midterm is all just Chapter 2. So no Chapter 3 on that one to make you panic.

§2.5 #5

$5\sin(4x) = 10\sin(2x)$ Find all sol'ns in $[0, 2\pi)$.

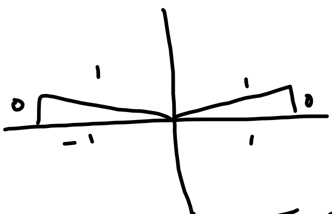
$5(2\sin(2x)\cos(2x)) = -10\sin(2x)$ $\quad 0 \le x < 2\pi$

$\sin(2x)\cos(2x) + \sin(2x) = 0$ $\quad 0 \le 2x < 4\pi$

$\sin(2x)(\cos(2x)+1) = 0$

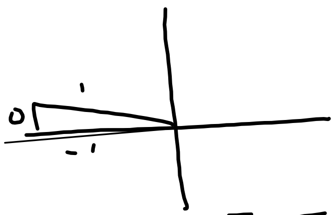
$\sin(2x) = 0$ OR $\cos(2x)+1 = 0$

$\cos(2x) = -1$



$2x = 0, \pi, 2\pi, 3\pi, \cancel{4\pi, 5\pi}$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$



$2x = \pi, 3\pi, \cancel{5\pi, 7\pi}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

S2.5 #9 Power-Reducing Formula.

$$\sin^2 x = \frac{1-\cos(2x)}{2}, \quad \cos^2 x = \frac{1+\cos(2x)}{2}$$

$$5\cos^4(x) = 5\left(\cos^2(x)\right)^2$$

$$= 5\left(\frac{1+\cos(2x)}{2}\right)^2 = 5\left(\frac{(1+\cos(2x))^2}{2^2}\right) = \frac{5}{4}\left(1+\cos(2x)\right)^2$$

$$= \frac{5}{4}\left(1 + 2(1)(\cos(2x)) + \cos^2(2x)\right)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \frac{5}{4}\left(1 + 2\cos(2x) + \left(\frac{1+\cos(4x)}{2}\right)\right) \text{ STOP!}$$

$$= \frac{5}{4} + \frac{5}{2}\cos(2x) + \frac{5}{8}\left(1+\cos(4x)\right)$$

$$= \frac{5}{4} + \frac{5}{2}\cos(2x) + \frac{5}{8} + \frac{5}{8}\cos(4x)$$

$$= \frac{15}{8} + \frac{5}{2}\cos(2x) + \frac{5}{8}\cos(4x)$$

$$\boxed{\frac{5}{8}(4\cos(2x) + \cos(4x) + 3)}$$

$\frac{1}{2}$-Angle: $\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$ $\qquad \cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$

$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$$

$$\cos(2u) = \cos(u)\cos(u) - \sin(u)\sin(u)$$

$$= \cos^2 u - \sin^2(u)$$

$$= \cos^2(u) - \left(1 - \cos^2(u)\right)$$

$$= \cos^2(u) - 1 + \cos^2(u)$$

$$= 2\cos^2(u) - 1 = \cos(2u)$$

$$\Rightarrow 2\cos^2(u) = \cos(2u) + 1$$

$$\Rightarrow \cos^2(u) = \frac{\cos(2u)+1}{2}$$

$$\cos(u) = \pm\sqrt{\frac{1+\cos(2u)}{2}}$$

Re-label:

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\vdots$$

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}$$

spring '25 midterm #5

5. (5 pts) Sketch one period of the graph of $f(x)=3\sin\left(\frac{\pi}{6}x-2\pi\right)+7$.

Clearly label all highs, lows, and intersections with the midline.

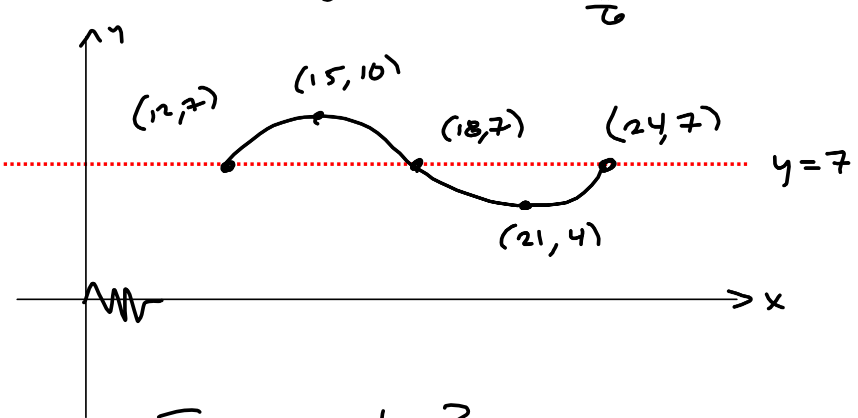
$a\sin(b(x-c))+d \rightarrow y=d$ is midline

a: Amp? b: relates to period (sine has period 2π) c: START

$f(x)=3\sin\left(\frac{\pi}{6}x-2\pi\right)+7$

$=3\sin\left(\frac{\pi}{6}(x-12)\right)+7$

$\frac{2\pi}{\frac{\pi}{6}}=12$ $\frac{2\pi}{\frac{\pi}{6}}=2\pi\cdot\frac{6}{\pi}=12$ ✓



$\frac{\pi}{6}x=2\pi$ when?

$x=\frac{2\pi}{\frac{\pi}{6}}=12$ same as the phase (horizontal) shift!

10. Consider the equation $4\cos^2(2x) - 1 = 0$.

a. (5 pts) Find all solutions x, in radians, to the equation in the interval $[0, 2\pi)$. I better see pictures!

b. (5 pts) Find all real solutions x, in radians.

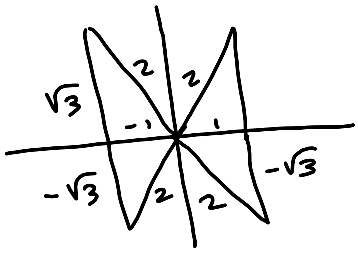
$x \in [0, 2\pi) \Longrightarrow$

$2x \in [0, 4\pi)$

$4\cos^2(2x) = 1$

$\cos^2(2x) = \frac{1}{4}$

$\cos(2x) = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$



$\cos(2x) = \frac{1}{2}$

arccos$(\cos(2x)) = \arccos\left(\frac{1}{2}\right) = 60^\circ$

$= \frac{\pi}{3}$ = reference angle.

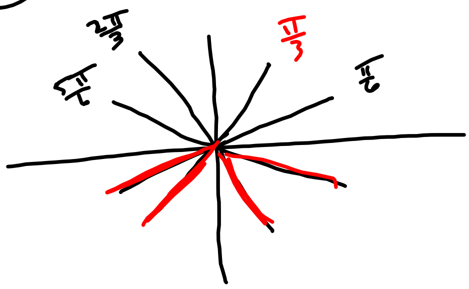
$\cos^{-1}$ key.

$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi + \frac{\pi}{3}, 2\pi + \frac{2\pi}{3}, 2\pi + \frac{4\pi}{3}, 2\pi + \frac{5\pi}{3}$

$= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

$\Rightarrow \boxed{x = \frac{\pi}{6}, \frac{2\pi}{6} = \frac{\pi}{3}, \frac{4\pi}{6} = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{8\pi}{6} = \frac{4\pi}{3}, \frac{10\pi}{6} = \frac{5\pi}{3}, \frac{11\pi}{6}}$

(b) Find ALL of 'em!



All of the above $+ 2n\pi$. (write out!)

More elegant.

$\frac{\pi}{6} + n\pi, \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi, \frac{5\pi}{6} + n\pi$

$\forall n \in \mathbb{Z}$

Using Sets:

$\Rightarrow x = \frac{\pi}{6}, \frac{2\pi}{6} = \frac{\pi}{3}, \frac{4\pi}{6} = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{8\pi}{6} = \frac{4\pi}{3}, \frac{10\pi}{6} = \frac{5\pi}{3}, \frac{11\pi}{6}$

(a) $x \in \left\{\frac{\pi}{6}, \frac{2\pi}{6} = \frac{\pi}{3}, \frac{4\pi}{6} = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{8\pi}{6} = \frac{4\pi}{3}, \frac{10\pi}{6} = \frac{5\pi}{3}, \frac{11\pi}{6}\right\} = A$

(b) $x \in \{y + 2n\pi \mid y \in A \text{ and } n \in \mathbb{Z}\}$