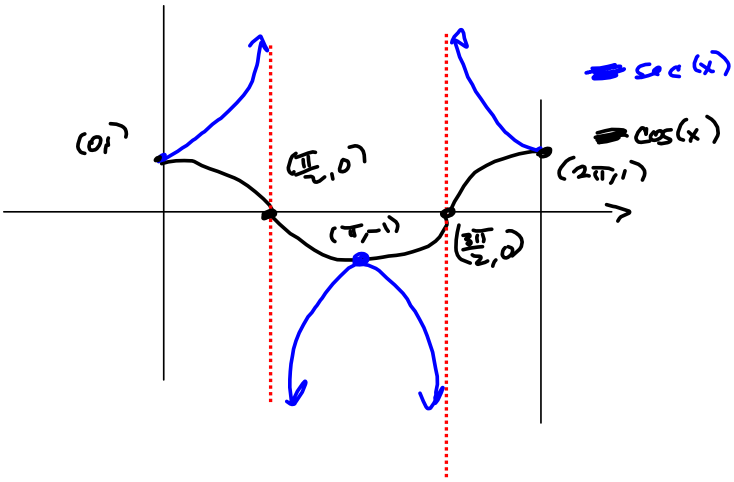
Midterm covers everything from Chapter 1 and Chapter 2.

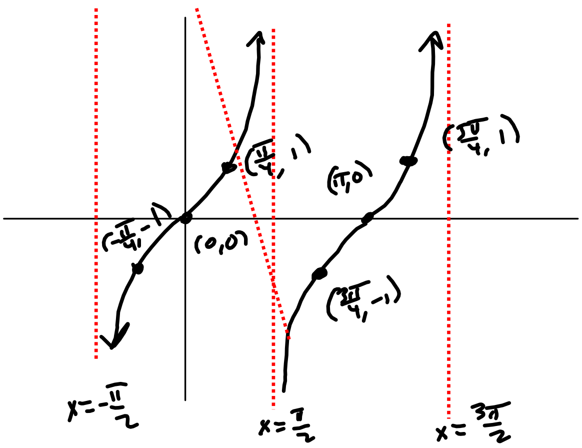
Briefly discuss Midterm, Monday or Tuesday, October 13th or 14th 8 am - 6 pm.
HOR 107. Bring pencil or pen, 1-page, 2-sided cheat sheet.

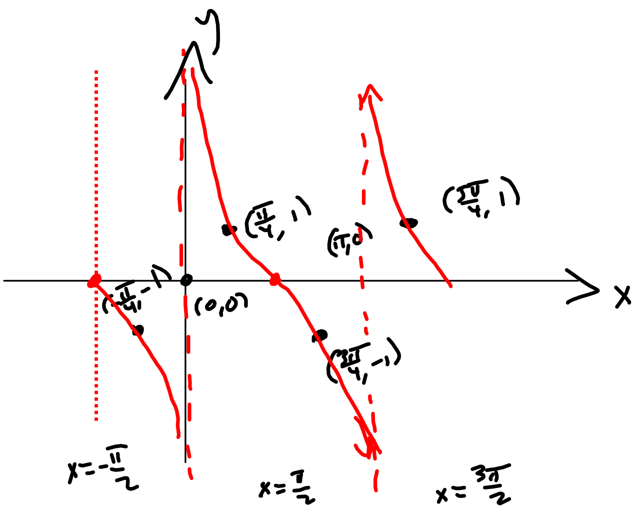
Test is 2 hours.

2-sided cheat sheet, scientific calculator, and photo ID.

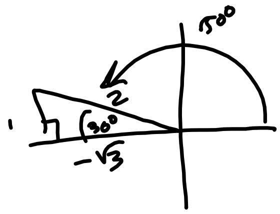
970-290-0550


sec(x)
cos(x)
(0,1)
(π/2, 0)
(π, -1)
(3π/2, 0)
(2π, 1)


(π/4, 1)
(π, 0)
(5π/4, 1)
(-π/4, -1)
(0,0)
(3π/4, -1)
x = -π/2
x = π/2
x = 3π/2

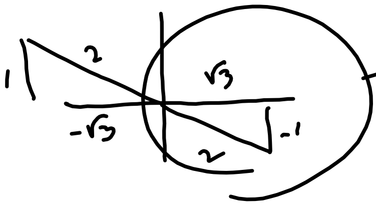

y
x
(π/4, 1)
(π, 0)
(5π/4, 1)
(-π/4, -1)
(0,0)
(3π/4, -1)
x = -π/2
x = π/2
x = 3π/2

$\arctan(\tan(\frac{5\pi}{6})) = \frac{5\pi}{6}$? No



$(\frac{5\pi}{6})(\frac{180°}{\pi}) = 150°$

$\arctan(-\frac{1}{\sqrt{3}}) = -30°$



Arctangent only sees this one & it thinks it's $-30° = -\frac{\pi}{6}$

$\arctan(\tan(\frac{5\pi}{6})) = -\frac{\pi}{6}$

We restrict sine and tangent to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ **to keep it 1-to-1, so that the** ***inverse*** **is a function.**

$(-\frac{\pi}{2}, \frac{\pi}{2})$ for tangent.

10. Consider the equation $4\cos^2(2x)-1=0$.

a. (5 pts) Find all solutions x, in radians, to the equation in the interval $[0,2\pi)$. I better see pictures!

b. (5 pts) Find all real solutions x, in radians.

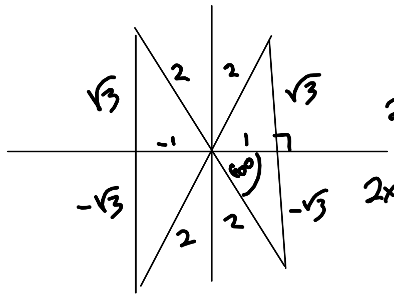
(a) Want all sol'ns $x \in [0,2\pi)$ so
" " " $2x \in [0,4\pi)$ or $2x \in [0,720^\circ)$

$4\cos^2(2x)-1=0$

$4\cos^2(2x)=1$

$\cos^2(2x)=\frac{1}{4}$

$\cos(2x)=\pm\frac{1}{2}$



$2x$ $60^\circ, 180^\circ-60^\circ, 180^\circ+60^\circ, 360^\circ-60^\circ$

$2x = 60^\circ, 120^\circ, 240^\circ, 300^\circ, 360^\circ+60^\circ, 360^\circ+120^\circ, 360^\circ+240^\circ,$
$360^\circ+300^\circ$ "

$2x = 60^\circ, 120^\circ, 240^\circ, 300^\circ, 420^\circ, 480^\circ, 600^\circ, 660^\circ$

$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{\pi\cdot 10}{3}, \frac{11\pi}{3},$

$(420^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{7\pi}{3}$

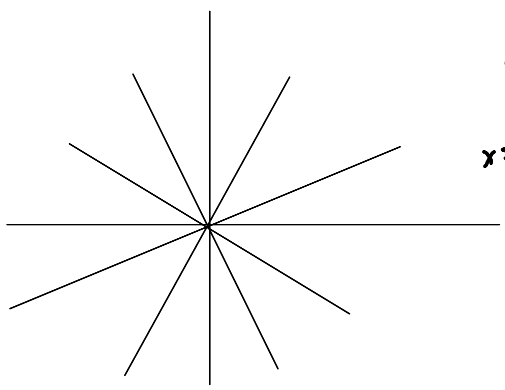
$$\boxed{x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}}$$

(b) Find All solutions

ONE WAY

$x = y + 2n\pi$, where $y = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}$, $n \in \mathbb{Z}$

$x \in \{ y + 2n\pi \mid y = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}, n \in \mathbb{Z}\}$



Short Answer: (Takes more thought)

$x = \frac{\pi}{6} + n\pi, \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi, \frac{5\pi}{6} + n\pi, n \in \mathbb{Z}$

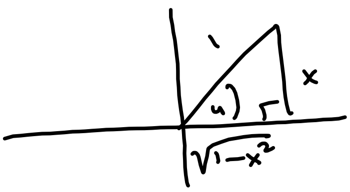
$x \in \{ y + n\pi \mid y = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, n \in \mathbb{Z}\}$

Re-write $\cos(\arcsin(x) + \arctan(x)) = \cos(\theta_1 + \theta_2) = \cos(u+v)$

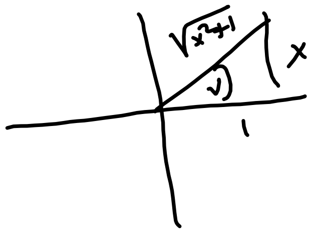
as an algebraic expression

$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v) = \frac{\sqrt{1-x^2}}{1}\,\frac{1}{\sqrt{x^2+1}} - \frac{x}{1}\cdot\frac{x}{\sqrt{x^2+1}}$$

$u = \arcsin(x)$



$v = \arctan(x)$



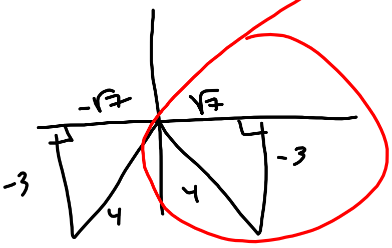
$$= \boxed{\frac{\sqrt{1-x^2} - x^2}{\sqrt{x^2+1}}}$$

CALC II Skill.

If you're studying off the old midterm, remember that #s 14 - 19 are from Chapter 3 and are not included in your midterm.

Click here for last spring's Midterm and its solutions.

9. (5 pts) Find the exact values of $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u) = -\frac{3}{4}$ and $\cos(u) > 0$.

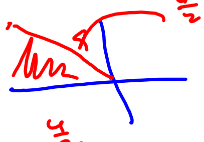


Now, we're in QIV, so

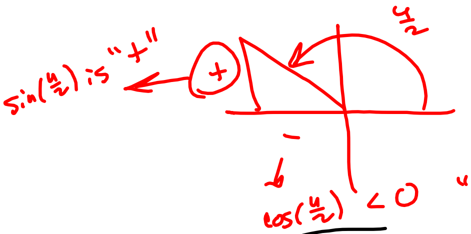
$\frac{3\pi}{2} < u < 2\pi$

$\Rightarrow \frac{3\pi}{4} < \frac{u}{2} < \pi$

$\frac{u}{2}$ lives here



$\sqrt{16-9} = \sqrt{7}$



$\sin(\frac{u}{2}) = +\sqrt{\frac{1-\cos(u)}{2}} = \sqrt{\frac{1-\frac{\sqrt{7}}{4}}{2}} = \sqrt{\frac{\frac{4-\sqrt{7}}{4}}{2}} = \boxed{\sqrt{\frac{4-\sqrt{7}}{8}} = \sin(\frac{u}{2})}$

$\cos(\frac{u}{2}) = -\sqrt{\frac{1+\cos(u)}{2}} = -\sqrt{\frac{1+\frac{\sqrt{7}}{4}}{2}} = \boxed{-\sqrt{\frac{4+\sqrt{7}}{8}}}$

$\tan(\frac{u}{2}) = \frac{\sin(\frac{u}{2})}{\cos(\frac{u}{2})} = \frac{\sqrt{\frac{4-\sqrt{7}}{8}}}{-\sqrt{\frac{4+\sqrt{7}}{8}}} = -\sqrt{\frac{\frac{4-\sqrt{7}}{8}}{\frac{4+\sqrt{7}}{8}}} = \boxed{-\sqrt{\frac{4-\sqrt{7}}{4+\sqrt{7}}} = \tan(\frac{u}{2})}$

Alternate: Have $\tan(\frac{u}{2})$ on cheat sheet.

- $\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

$= \frac{1-\frac{\sqrt{7}}{4}}{-\frac{3}{4}} = \frac{\frac{4-\sqrt{7}}{4}}{-\frac{3}{4}} = \frac{4-\sqrt{7}}{-3} = \tan(\frac{u}{2})$?!

$\frac{(4-\sqrt{7})}{-3}$

$= -0.451416229645$

$-\sqrt{\frac{(4-\sqrt{7})}{4+\sqrt{7}}}$

$= -0.451416229645$