

. [- / 1 Points]

Solve the equation. (Enter your answers as a comma-separated list. Use n as an integer constant. Enter your response in radians.)

$$8 \cos^2(x) + 4 \cos(x) - 4 = 0$$

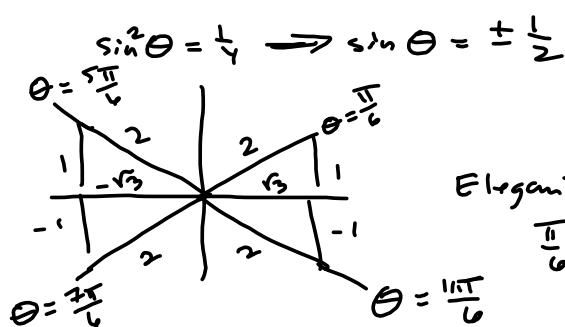
$$x = \boxed{} \quad \times \quad \boxed{2\pi n + \frac{\pi}{3}, 2\pi n + \pi, 2\pi n + \frac{5\pi}{3}}$$

As a set handwritten

$$\left\{ 2\pi n + \frac{\pi}{3}, 2\pi n + \pi, 2\pi n + \frac{5\pi}{3} \mid n \in \mathbb{Z} \right\}$$

WebAssign isn't as formal.

$$2\pi n + \frac{\pi}{3}, 2\pi n + \pi, \\ 2\pi n + \frac{5\pi}{3}, \text{ where } n \in \mathbb{Z}$$



$$\theta = 2\pi n + \frac{\pi}{3}, 2\pi n + \pi, \\ 2\pi n + \frac{5\pi}{3} \quad \forall n \in \mathbb{Z}.$$

Elegant:

$$\frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi, n \in \mathbb{Z}.$$

Brute Force:

$$\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in \mathbb{Z}$$

Solve the equation. (Enter your answers as a comma-separated list. Use n as an integer constant. Enter your response in radians.)

$$\csc^2(x) - 9 \csc(x) = 0$$

$x =$

$$\pi - 0.1113 + 2\pi n, 3.0303 + 2\pi n$$



Your answer includes 2 characters that can't be graded.

Delete your recent changes and use the pad tools to finish your answer. [More information](#)

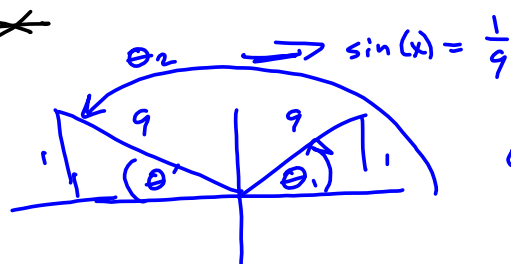
$$2\pi n + \sin^{-1}\left(\frac{1}{9}\right), 2\pi n + \pi - \sin^{-1}\left(\frac{1}{9}\right)$$

$$\csc^2(x) - 9 \csc(x) = 0$$

$$\csc(x) [\csc(x) - 9] = 0$$

$$\Rightarrow \csc(x) = 0 \quad \text{OR} \quad \csc(x) = 9$$

~~$\Rightarrow$~~



$\theta_1 = \arcsin\left(\frac{1}{9}\right)$ is irrational.

$$\theta_2 = \pi - \arcsin\left(\frac{1}{9}\right)$$

All solutions

Angle Sum Formula

$$\begin{aligned} \sin(x+y) &= \sin(x)\cos(y) + \sin(y)\cos(x) \\ \cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\ \tan(x+y) &= \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \tan(x+y) = \\ \frac{\sin(x)\cos(y) + \sin(y)\cos(x)}{\cos(x)\cos(y) - \sin(x)\sin(y)} \end{array}$$

Double-Angle.

$$\sin(2x) = \sin(x)\cos(x) + \sin(x)\cos(x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) \quad \text{used to derive } \frac{1}{2}\text{-angle}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

Not on my cheat sheet. Should be. [Click for Cheat Sheet](#)

 $\frac{1}{2}$ -angle :

$$\begin{aligned} \cos(2x) &= \cos^2(x) - \sin^2(x) \\ \cos(u) &= \cos^2\left(\frac{u}{2}\right) - \sin^2\left(\frac{u}{2}\right) = \cos^2\left(\frac{u}{2}\right) - (1 - \cos^2\left(\frac{u}{2}\right)) \\ &= 2\cos^2\left(\frac{u}{2}\right) - 1 = \cos(2x) \end{aligned}$$

$$\begin{aligned} \Rightarrow 2\cos^2\left(\frac{u}{2}\right) &= 1 + \cos(u) \\ \cos^2\left(\frac{u}{2}\right) &= \frac{1 + \cos(u)}{2} \end{aligned}$$

$$\Rightarrow \cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 + \cos(u)}{2}}$$

$\pm$ depends on what quadrant $\frac{u}{2}$ is in.

$$\cos(2x) = 1 - \sin^2(x) - \sin^2(x) = 1 - 2\sin^2(x)$$

$$\cos(u) = 1 - 2\sin^2\left(\frac{u}{2}\right) \rightarrow$$

$$-2\sin^2\left(\frac{u}{2}\right) = \cos(u) - 1$$

$$\sin^2\left(\frac{u}{2}\right) = \frac{1 - \cos(u)}{2}$$

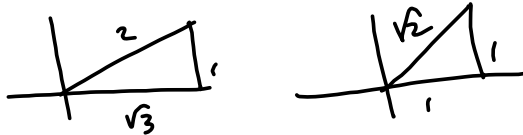
$$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos(u)}{2}}$$

Find exact value:

 $\sin\left(\frac{5\pi}{12}\right)$ in 2 ways:

$$\text{Angle Sum: } \sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{6} \cos\frac{\pi}{4} + \sin\frac{\pi}{4} \cos\frac{\pi}{6} = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}} \quad \text{FINE}$$



$$\text{Rationalize: } \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4} \quad \text{optional usually.}$$

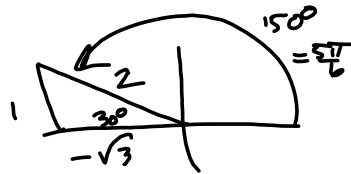
 $\frac{1}{2}$ - angle

$$\frac{5\pi}{12} = \frac{4}{2} \Rightarrow u = \frac{5\pi}{6}$$

$$\frac{5\pi}{12} \in (0, \frac{\pi}{2}) \text{, i.e. } \in \text{QI}$$

$$\sin \frac{5\pi}{12} > 0 \quad "+"$$

$$\sin\left(\frac{4}{2}\right) = \pm \sqrt{\frac{1 - \cos(4)}{2}} = + \sqrt{\frac{1 - \cos\left(\frac{5\pi}{6}\right)}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$$



$$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

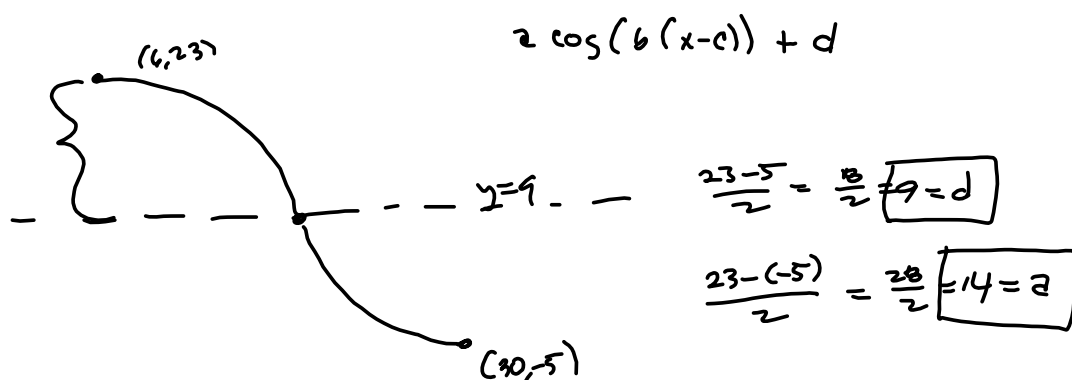
$$\frac{(\sqrt{2} + \sqrt{6})}{4}$$

$$= 0.965925826289$$

$$\frac{\sqrt{(2 + \sqrt{3})}}{2}$$

$$= 0.965925826289$$

(5 pts) Write the cosine function that achieves its maximum height of $y = 23$ centimeters at time $t = 6$ seconds and its minimum height of $y = -5$ centimeters at $t = 30$ seconds.



$$\text{Period} = 2(30-6) = 2(24) = 48$$

$$\text{want } bx = 2\pi \text{ when } x = 48$$

$$48b = 2\pi$$

$$b = \frac{2\pi}{48} = \frac{\pi}{24}$$

Start @ the high point $t = 6$:

$$\boxed{14 \cos\left(\frac{\pi}{24}(x-6)\right) + 9}$$