2.3 #14

Solve the equation. (Enter your answers as a comma-separated list. Use n as an integer constant. Enter your response in radians.)

$$(\tan(6x))(\tan(x) - 1) = 0 \longrightarrow$$

Find all solutions between 0 and 2Pi, inclusive.

$\tan(6x) = 0$ OR $\tan(x) - 1 = 0$

Want all $x \in [0, 2\pi]$

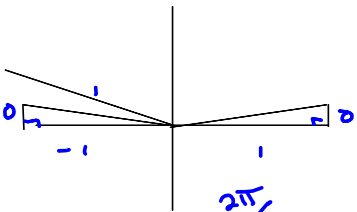
OR frequently

all $x \in [0, 2\pi) \longrightarrow$ How I usually phrase it

$\longrightarrow$ Want all $6x \in [0, 12\pi)$

$0 \le x < 2\pi \longrightarrow$

$0 \le 6x < 12\pi$



$6x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi, 8\pi, 9\pi, 10\pi, 11\pi$

$\longrightarrow$ $\boxed{x = 0, \frac{\pi}{6}, \frac{2\pi}{6} = \frac{\pi}{3}, \frac{3\pi}{6} = \frac{\pi}{2}, \frac{4\pi}{6} = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{6\pi}{6} = \pi, \frac{7\pi}{6}, \frac{8\pi}{6} = \frac{4\pi}{3}, \frac{9\pi}{6} = \frac{3\pi}{2}, \frac{10\pi}{6} = \frac{5\pi}{3}, \frac{11\pi}{6}}$

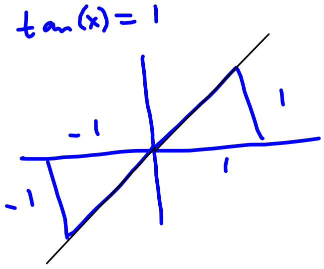
Missed $\frac{2\pi}{6} = \frac{\pi}{3}$!

OR

See next page...

$\tan(x) - 1 = 0$

$\tan(x) = 1$



$\Rightarrow \boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

Find ALL solutions!

$\frac{n\pi}{6}$, $n \in \mathbb{Z}$ by observing the pattern.

$$\left.\begin{matrix} \frac{\pi}{4} + 2n\pi \\ \frac{5\pi}{4} + 2n\pi \end{matrix}\right\} \text{collapses to } \boxed{\frac{\pi}{4} + n\pi = x, \quad n \in \mathbb{Z}}$$

2.3 #16

Solve the equation. (Enter your answers as a comma-separated list. Use n as an integer constant. Enter your response in radians.)

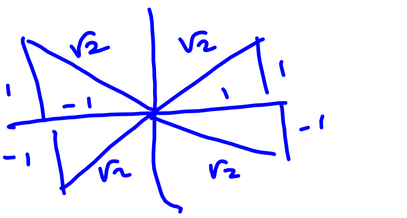
$$(2\sin^2 x - 1)(3\tan^2 x - 1) = 0$$

$2\sin^2 x - 1 = 0$

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$



On $[0, 2\pi)$, we have

$$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

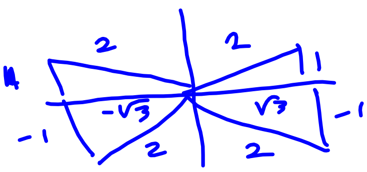
OR

$3\tan^2(x) - 1 = 0$

$3\tan^2(x) = 1$

$\tan^2(x) = \frac{1}{3}$

$\tan(x) = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

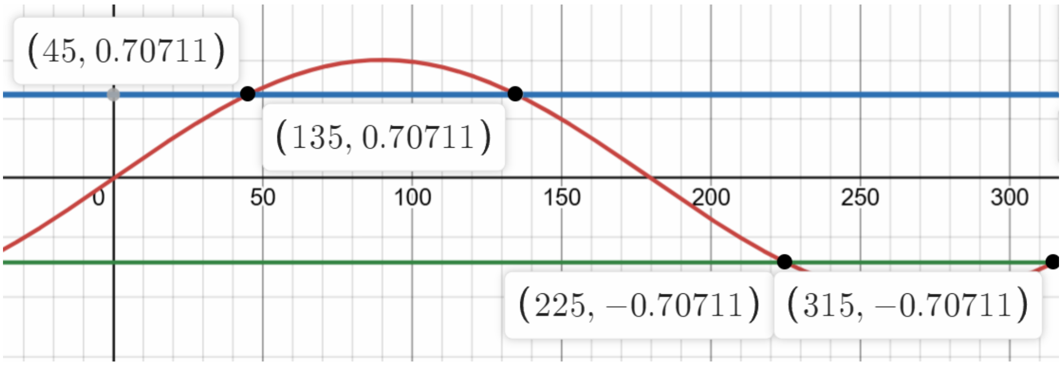


On $[0, 2\pi)$, we have

$$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

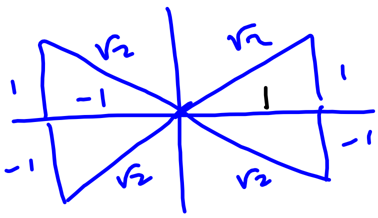
See next page for alternate approaches when the Pi radians aren't jumping out at you.

If you're not great with radians, yet, solve in degrees and then convert to radians.



Scientific Calculator in degrees mode

$\sin(x) = \pm \frac{1}{\sqrt{2}}$



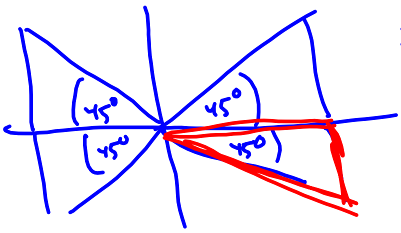
Use $\sin^{-1}$ Key

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \arcsin\left(\frac{1}{\sqrt{2}}\right) = 45^\circ = \frac{\pi}{4}$

$180^\circ - 45^\circ$ $180^\circ + 45^\circ$ $360^\circ - 45^\circ$

$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

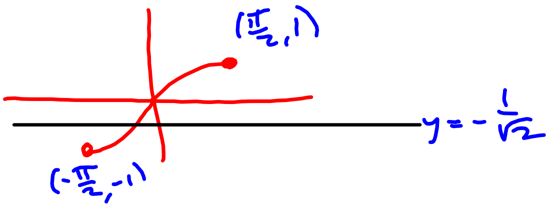
$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



All have the same reference angle.

Note: $\sin(x) = -\frac{1}{\sqrt{2}} \longrightarrow$

$\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -45^\circ \notin [0, 2\pi)$



SIN^{-1} **= virtue? (3 students immediately leave class. Didn't like the weak joke?)**