Section 2.1

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

$$\sin\left(\frac{\pi}{2} - u\right) = \sin\left(-\left(u - \frac{\pi}{2}\right)\right)$$

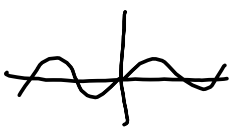
Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

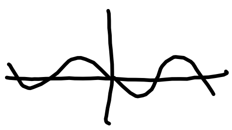
$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$$

Even

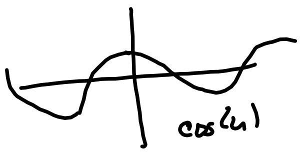
$\sin(u)$



$\sin(-u)$

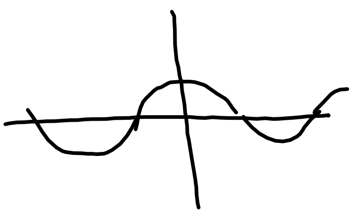


$\sin\left(-\left(u - \frac{\pi}{2}\right)\right)$

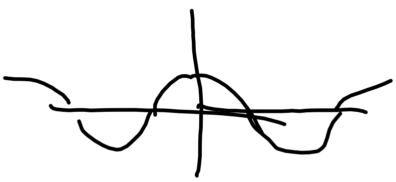


$\sin(u)$

$\sin\left(u + \frac{\pi}{2}\right)$



$\sin(-u + \pi/2)$



Pythagorean identities are sometimes used in radical form such as

$\sqrt{\sin^2(u)} = \sqrt{1-\cos^2(u)}$

$$\sin u = \pm\sqrt{1-\cos^2 u}$$

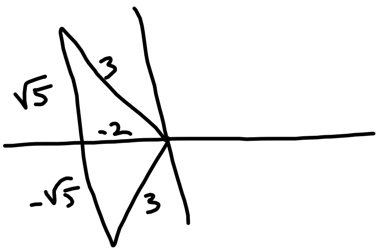
or $|\sin(u)| = \sqrt{1-\cos^2(u)}$

$$\tan u = \pm\sqrt{\sec^2 u - 1}$$

Example 1 Using Identities to Evaluate a Function

Use the conditions $\sec u = -\frac{3}{2}$ and $\tan u > 0$ to find the values of all six trigonometric functions.

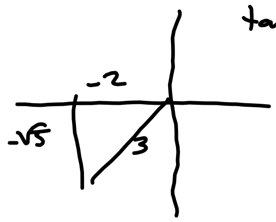
No. $\cos(u) = -\frac{2}{3}$



AND

$\tan(u) > 0$

$\cos(u) = -\frac{2}{3}$ AND $\tan(u) > 0$



$\sin(u) = -\frac{\sqrt{5}}{3}$ $\qquad$ $\csc(u) = -\frac{3}{\sqrt{5}}$

$\cos(u) = -\frac{2}{3}$ $\qquad$ $\sec(u) = -\frac{3}{2}$

$\tan(u) = \frac{\sqrt{5}}{2}$ $\qquad$ $\cot(u) = \frac{2}{\sqrt{5}}$

Factor each expression.

a. $1 - \cos^2\theta$ $= 1^2 - \cos^2\theta = a^2 - b^2 = (a-b)(a+b)$
$= (1-\cos\theta)(1+\cos\theta)$

How to factor any quadratic equation.

b. $2\csc^2\theta - 7\csc\theta + 6$

Let $u = \csc\theta$ SLEDGEHAMMER
$2u^2 - 7u + 6 \overset{\text{SET}}{=} 0$ FACTORING.
$a=2, b=-7, c=6$
$b^2 - 4ac = 7^2 - 4(2)(6) = 49 - 48 = 1$
$u = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{7 \pm \sqrt{1}}{2(2)} = \dfrac{7 \pm 1}{4}$

$\frac{8}{4} = 2$
$\frac{6}{4} = \frac{3}{2}$

$2(u-2)(u-\frac{3}{2})$
$= 2(\csc\theta - 2)(\csc\theta - \frac{3}{2})$
$= (\csc\theta - 2)(2\csc\theta - 3)$

Factor $\csc^2 x - \cot x - 3$

$(\cot^2(x) + 1) - \cot(x) - 3$
$= \cot^2(x) - \cot(x) - 2$
$= u^2 - u - 2$
$= (u-2)(u+1)$
$= (\cot(x) - 2)(\cot(x) + 1$

Verify the identity by converting the left side into sines and cosines. (Simplify at each step.)

$$\frac{\cot^2(t)}{6\csc(t)} = \frac{1-\sin^2(t)}{6\sin(t)}$$

m1

$$\frac{\frac{\cos^2 t}{\sin^2 t}}{6\csc t} = \frac{\frac{\cos^2 t}{\sin^2 t}}{\frac{6}{\sin(t)}} = \frac{\cos^2 t}{\sin^2 t}\cdot\frac{\sin t}{6} = \frac{\cos^2 t}{6\sin t} = \frac{1-\sin^2 t}{6\sin t} \checkmark$$

m2

$$\frac{\csc^2 t - 1}{6\csc t} = \frac{\csc^2 t}{6\csc t} - \frac{1}{6\csc t} = \frac{\csc t}{6} - \frac{1}{6\csc t}$$

No joy

$$= \frac{\frac{1}{\sin^2 t} - 1}{6\left(\frac{1}{\sin t}\right)} = \frac{\frac{1}{\sin^2 t} - \frac{\sin^2 t}{\sin^2 t}}{\frac{6}{\sin t}} =$$

$$\frac{\sin t}{6}\left(\frac{1}{\sin^2 t} - \frac{\sin^2 t)}{\sin^2 t}\right) = \text{Yuck}$$

Verify the identity. (Simplify at each step.)

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{|\sin\theta|}$$

$$= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

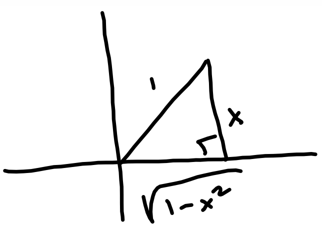
$$= \frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{\sin^2\theta}} = \frac{|1+\cos\theta|}{|\sin\theta|} = \frac{1+\cos\theta}{|\sin\theta|}$$

2.2 #19

Verify the identity.

$$7\cos\theta = 7\cos(\sin^{-1}x) = \sqrt{49-49x^2}$$

$$= 7\sqrt{1-x^2}$$



These problems are for trig substitution in Calculus II. Draw pictures. The rest just falls into place.

20. [– / 2 Points] DETAILS MY I

Verify the identity. (Simplify at each step.)

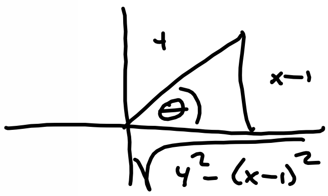
$$\tan\theta = \tan\left(\sin^{-1}\left(\frac{x-1}{4}\right)\right) = \frac{x-1}{\sqrt{16-(x-1)^2}}$$

Let $\theta = \sin^{-1}\frac{x-1}{4} \Rightarrow \sin\theta = \frac{x-1}{4}$. Thus,

$$\tan\left(\sin^{-1}\frac{x-1}{4}\right) = \tan\left(\boxed{\theta}\right)$$

$$= \frac{\dfrac{\boxed{}}{4}}{\dfrac{\sqrt{16-(x-1)^2}}{4}}$$

$$= \frac{x-1}{\sqrt{16-(x-1)^2}}.$$



$$\tan\theta = \frac{x-1}{\sqrt{16-(x-1)^2}}$$

2.1 Part II #1

$3\sin^2\theta - 4\sin\theta - 4$

$3u^2 - 4u - 4 \overset{SET}{=} 0$

$a=3, b=-4, c=-4$

$b^2-4ac = 4^2-4(3)(-4) = 16+48 = 64$

$u = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{64}}{2(3)} = \frac{4 \pm 8}{6}$ → $\frac{12}{6} = 2$, $-\frac{4}{6} = -\frac{2}{3}$

$\Rightarrow 3(u-2)(u-(-\frac{2}{3})) = (u-2)(3)(u+\frac{2}{3})$

$= (u-2)(3u+2)$

$= (\sin\theta - 2)(3\sin\theta + 2)$

FACTOR THEOREM.

If $f(x)$ is a polynomial and $f(c)=0$, then $x-c$ is a factor of $f(x) = (x-c)g(x)$, where $g(x)$ is a polynomial of degree $n-1$ (assuming $f(x)$ has degree n.

Intermediate Algebra

Factoring to find zeros.

$f(x) = x^2-1 = 0$

$\Rightarrow f(x) = (x-1)(x+1) = 0$

$\Rightarrow x = 1, -1$

Sledgehammer for Factoring.

Using zeros to help you factor.

$f(x) = x^2-1$ has zeros ± 1

$\Rightarrow f(x) = (x-1)(x+1)$