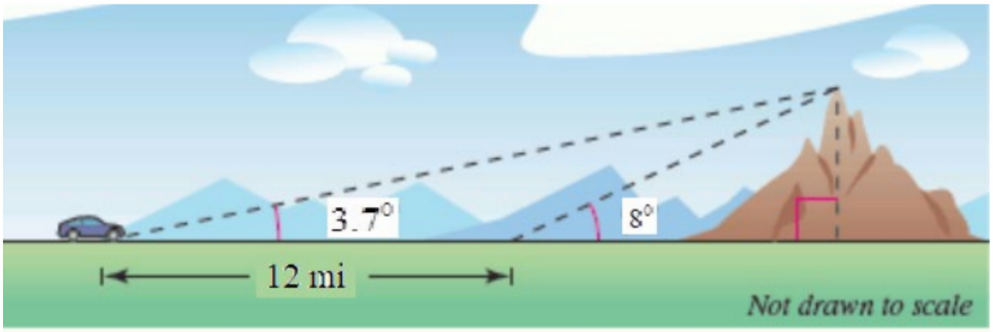
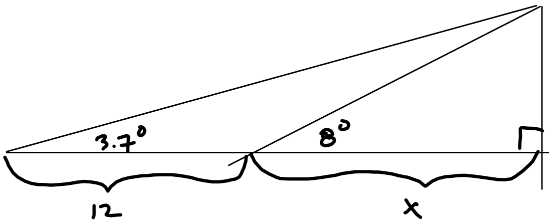
8. (5 pts) In traveling across flat land, you see a mountain directly in front of you. Its angle of elevation (to the peack) is 3.7^0. After you drive 12 miles closer to the mountain, the angle of elevation is 8^0. Approximate the height of the mountain above the plain. Give your answer in feet, to the nearest foot. See figure, below:





y = height of mountain, in miles.

x = distance from 2nd sighting to the mtn, in miles.

Then $\frac{y}{x+12} = \tan(3.7^\circ) = a$

and $\frac{y}{x} = \tan(8^\circ) = b$

$\Rightarrow \left.\begin{array}{l} a(x+12) = y \\ bx = y \end{array}\right\}$ $y = y$!

$a(x+12) = bx$ Solve for x:

$ax + 12a = bx$

$ax - bx = -12a$

$(a-b)x = -12a \Rightarrow$

$x = \frac{-12a}{a-b} = \frac{-12\tan(3.7^\circ)}{\tan(3.7^\circ) - \tan(8^\circ)}$

$10.2275864587 \approx x$. Now,

$y = \tan(8^\circ)\cdot x \approx (\tan(8^\circ))(10.2275864587)$

≈ 1.4373935379 mi.

$= (1.4373935379 \text{ mi})(5280 \frac{\text{ft}}{\text{mi}})$

$\approx 7589.43788011 \approx \boxed{y \approx 7589 \text{ ft}}$

$-\frac{12\tan(3.7)}{\tan(3.7) - \tan(8)}$

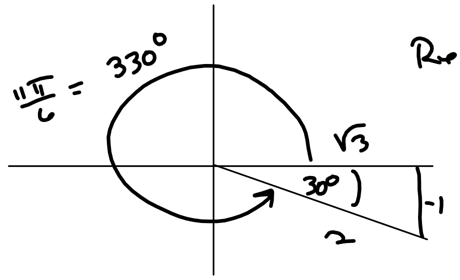
$= 10.2275864587$

$10.2275864587 \cdot \tan(8)$

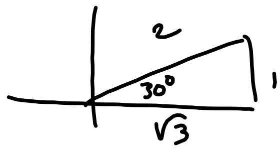
$= 1.4373935379$

Point on unit circle corresponding to $\frac{11\pi}{6}$

$$\left(\frac{11\pi}{6}\right)\left(\frac{180^\circ}{\pi}\right) = 11(30)^\circ = 330^\circ$$



Reference angle: 30°

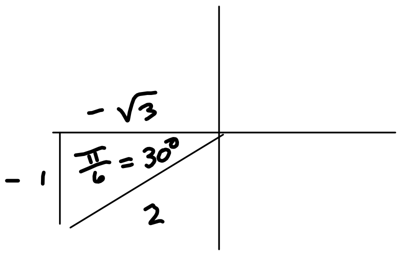


Points on unit circle: $(x, y) = (\cos\theta, \sin\theta)$

$$= \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

If possible, find the value of the 6 trigonometric functions at the real number *t*. Then sketch the angle.

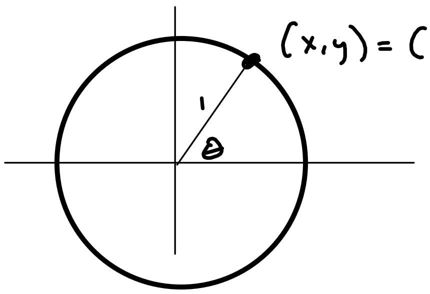
$$t = -\left(\frac{5\pi}{6}\right)\left(\frac{180^\circ}{\pi}\right) = -150^\circ \qquad t = -\frac{5\pi}{6}$$



$\sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$ $\qquad \csc\left(-\frac{5\pi}{6}\right) = -2$

$\cos\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ $\qquad \sec\left(-\frac{5\pi}{6}\right) = -\frac{2}{\sqrt{3}}$

$\tan\left(-\frac{5\pi}{6}\right) = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$ $\qquad \cot\left(-\frac{5\pi}{6}\right) = \sqrt{3}$



$(x,y) = (\cos\theta, \sin\theta)$

Pythagorus Says $x^2 + y^2 = r^2$

$(\cos\theta)^2 + (\sin\theta)^2 = 1^2$

Pythagorean Identities.

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}$$

$$\boxed{\tan^2\theta = \sec^2\theta - 1}$$

$$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{1-\sin^2\theta}{\sin^2\theta} = \dots = \csc^2\theta - 1$$

$$= \cot^2\theta$$

Use trig identities to transform the LHS into the RHS.

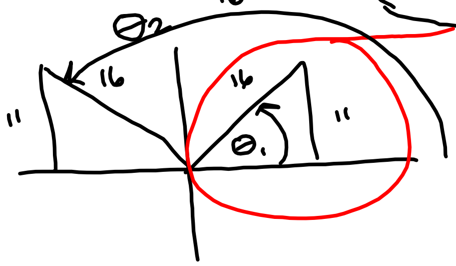
$\tan\theta + \cot\theta = \csc\theta \sec\theta.$

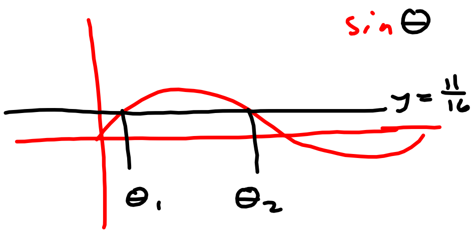
$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

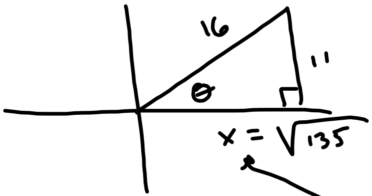
$$= \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} = \csc\theta\sec\theta$$

13. (5 pts) Sketch a right triangle corresponding to $\sin(\theta) = \dfrac{3}{13}$, where θ is an acute angle. Then find the exact values of the other five trigonometric functions of θ.

$\sin\theta = \dfrac{11}{16}$ and θ is acute $(0^\circ < \theta < 90^\circ)$







$x = \sqrt{135} = 3\sqrt{15}$

Pythagorus says

$x^2 + 11^2 = 16^2$

$x^2 = 16^2 - 11^2$

$= 256 - 121 = 135 = x^2$

$3\,|\,135$
$3\,|\,45$
$3\,|\,15$
5

$\sqrt{135} = 3\sqrt{15}$

$\sin\theta = \dfrac{11}{16}$ $\qquad \csc\theta = \dfrac{16}{11}$

$\cos\theta = \dfrac{3\sqrt{15}}{16}$ $\qquad \sec\theta = \dfrac{16}{3\sqrt{15}}$

$\tan\theta = \dfrac{11}{3\sqrt{15}}$ $\qquad \cot\theta = \dfrac{3\sqrt{15}}{11}$

Degrees, Minutes and Seconds

$\cos(27^\circ 15' 28'')$ to 4 places.

$\cos\left(\left(27 + \frac{15}{60} + \frac{28}{3600}\right)^\circ\right) \approx 0.888954977904$

$\approx \boxed{0.8890}$