#5 Practice: using ALL of our old skills.

2 5pts Sketch in rectangular coordinates over $[0,2\pi]$

$r = 2\cos(2\theta) - 1$ ← $r=-1$ = midline

Amplitude = 2

$2\theta = 2\pi$

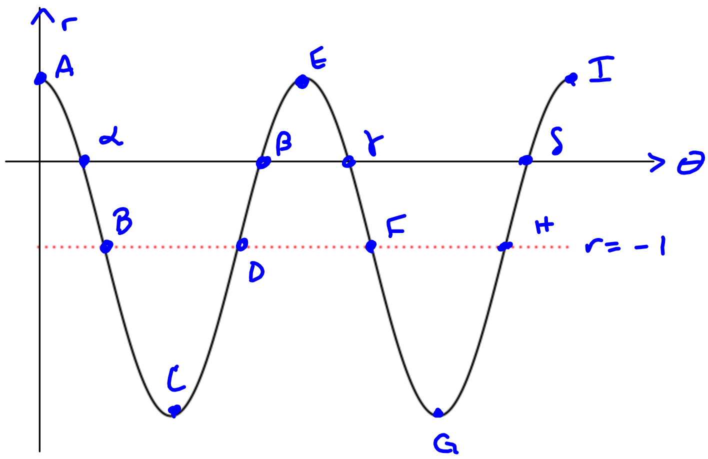
$\theta = \pi$ = Period

Increment between the key points:

$\frac{\pi}{4}$ (4 equal parts per period

Cosine starts on its high point @ $\theta = 0$

Every point is $\frac{\pi}{4}$ apart, so $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \ldots$



$A = (0, 1)$
$B = (\frac{\pi}{4}, -1)$
$C = (\frac{\pi}{2}, -3)$
$D = (\frac{3\pi}{4}, -1)$
$E = (\pi, 1)$
$F = (\frac{5\pi}{4}, -1)$
$G = (\frac{3\pi}{2}, -3)$
$H = (\frac{7\pi}{4}, -1)$
$I = (2\pi, 1)$

All we need, now, are its zeros:

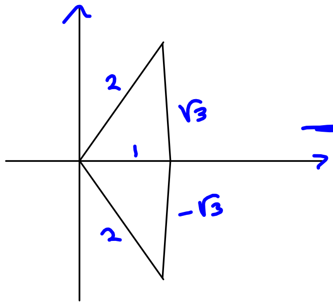
To find all zeros $\theta \in [0, 2\pi]$,
we need all $2\theta \in [0, 4\pi]$

$r = 0$

$2\cos(2\theta) - 1 = 0$

$2\cos(2\theta) = 1$

$\cos(2\theta) = \frac{1}{2}$



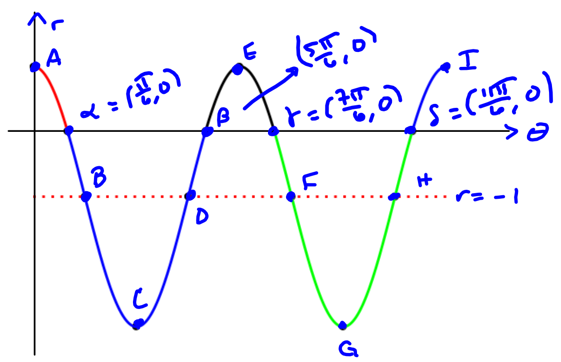
$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

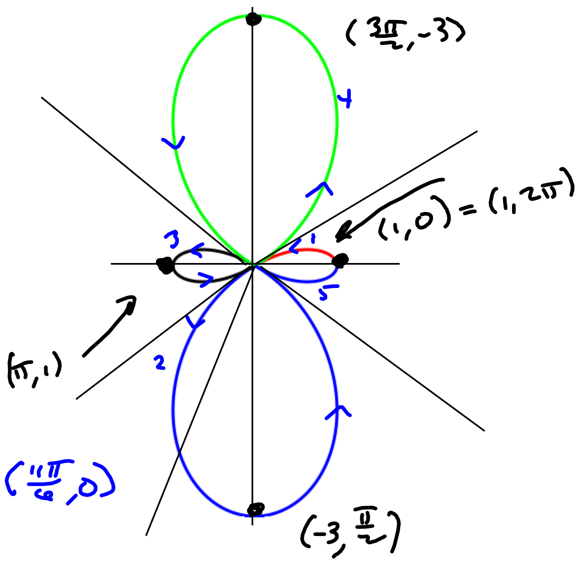
$\frac{\pi}{3} + 2\pi = \frac{\pi + 6\pi}{3} = \frac{7\pi}{3}$

$\frac{5\pi}{3} + \frac{6\pi}{3} = \frac{11\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow \alpha = (\frac{\pi}{6}, 0)$, $\boxed{\beta = (\frac{5\pi}{6}, 0),\ \gamma = (\frac{7\pi}{6}, 0),\ \delta = (\frac{11\pi}{6}, 0)}$





$\alpha = (\frac{\pi}{6}, 0)$, $\beta = (\frac{5\pi}{6}, 0)$, $\gamma = (\frac{7\pi}{6}, 0)$, $\delta = (\frac{11\pi}{6}, 0)$

$A = (0, 1)$

$B = (\frac{\pi}{4}, -1)$

$C = (\frac{\pi}{2}, -3)$

$D = (\frac{3\pi}{4}, -1)$

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$I = (2\pi, 1)$

In the polar plot, each stretch between zeros of *r* represents a loop! The tricky parts are the "loops" that lie below the θ-**axis, where the *r* values are negative, which means "reflect through the pole.**