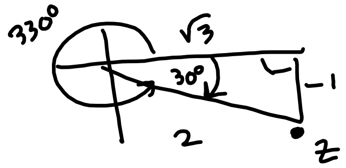
determines an ellipse. Prove it.

DeMoivre's Theorem: Add arguments and multiply moduli.

$z_1 z_2 = 35\left(\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right)$ $\qquad \frac{2\pi}{3}\cdot\frac{4}{4} + \frac{\pi}{4}\cdot\frac{3}{3} = \frac{11\pi}{12}$

$z = 81\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)$ Find z's 4th roots



$81\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \boxed{\frac{81\sqrt{3}}{2} - \frac{81}{2}i}$ In rect. form

$\theta = \frac{11\pi}{6}$

$\frac{\theta}{n} = \frac{\theta}{4} = \frac{\frac{11\pi}{6}}{4} = \frac{11\pi}{24}$

$\sqrt[4]{81} = 3$

So

$z_0 = 3\left(\cos\left(\frac{11\pi}{24}\right) + i\sin\left(\frac{11\pi}{24}\right)\right)$

Increment $= \frac{2\pi}{n} = \frac{2\pi}{4} = \frac{12\pi}{24}$

$z_1 = 3\left(\cos\left(\frac{23\pi}{24}\right) + i\sin\left(\frac{23\pi}{24}\right)\right)$

$z_2 = 3\left(\cos\left(\frac{35\pi}{24}\right) + i\sin\left(\frac{35\pi}{24}\right)\right)$

$z_3 = 3\left(\cos\left(\frac{47\pi}{24}\right) + i\sin\left(\frac{47\pi}{24}\right)\right)$

$z_4 = 3\left(\cos\left(\frac{59\pi}{24}\right) + i\sin\left(\frac{59\pi}{24}\right)\right)$ is coterminal w/ z_0 ✓

$59 = 48 + 11$

$\frac{59\pi}{24} = \frac{48\pi}{24} + \frac{11\pi}{24}$

Representation isn't unique

$$z = 81\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$$

$$z_0 = 3\left(\cos\left(-\frac{\pi}{24}\right) + i\sin\left(-\frac{\pi}{24}\right)\right)$$

$$z_1 = 3\left(\cos\left(\frac{11\pi}{24}\right) + i\sin\left(\frac{11\pi}{24}\right)\right)$$

$$z_2 = 3\left(\cos\left(\frac{23\pi}{24}\right) + i\sin\left(\frac{23\pi}{24}\right)\right)$$

$$z_3 = 3\left(\cos\left(\frac{35\pi}{24}\right) + i\sin\left(\frac{35\pi}{24}\right)\right)$$

$z_4 = 3\left(\cos\left(\frac{47\pi}{24}\right) + i\sin\left(\frac{47\pi}{24}\right)\right)$ is coterminal ~~to~~ with z_0.

$$\frac{47\pi}{24} = \frac{48\pi}{24} - \frac{\pi}{24}$$

§6.6

The parametric equations

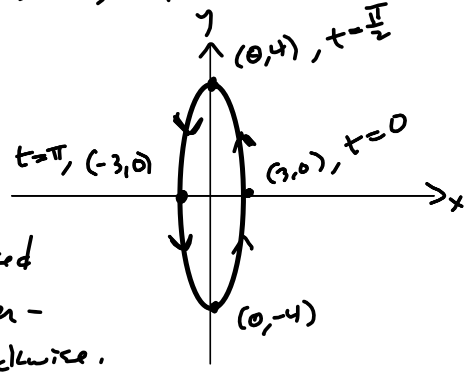
$x = 3\cos(t) \implies \frac{x}{3} = \cos(t)$

$y = 4\sin(t) \longrightarrow \frac{y}{4} = \sin(t)$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = \cos^2(t) + \sin^2(t) = 1$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

t	x	y
0	3	0
$\frac{\pi}{2}$	0	4
π	-3	0



Oriented counter-clockwise.

Plot

$x = t^2 - 2t$ for $t \geq 0$

$y = 3t$

Eliminate the parameter (to "see" it as a parabola).

$t = \frac{y}{3} \Rightarrow x = \left(\frac{y}{3}\right)^2 - 2\left(\frac{y}{3}\right)$

$= \frac{y^2}{9} - \frac{2y}{3} = \frac{1}{9}\left(y^2 - 6y\right)$

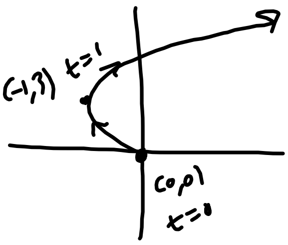
$= \frac{1}{9}\left(y^2 - 6y + 3^2 - 9\right)$

$x = \frac{1}{9}(y-3)^2 - 1$

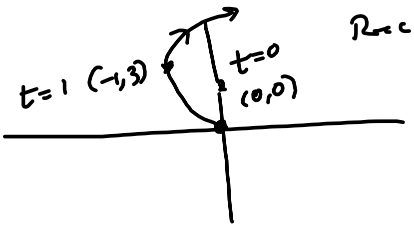
$(h,k) = (-1,3)$

t	x	y
0	0	0
1	-1	3

Rectangular

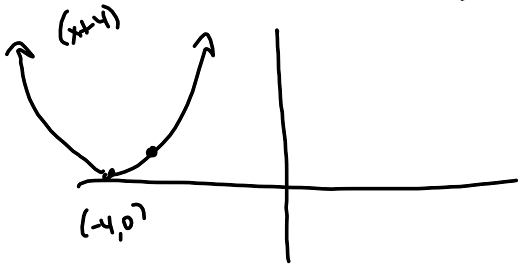


$x = \frac{1}{3}(y-k)^2 + h$

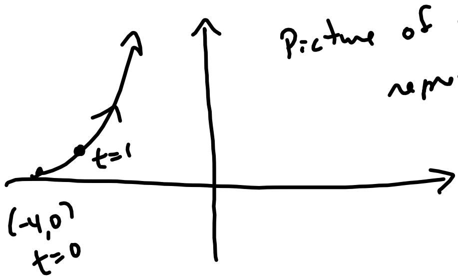


$x = \sqrt{t} - 4 \implies x+4 = \sqrt{t} \implies (x+4)^2 = t$

$y = t^3 \implies y = \left((x+4)^2\right)^3 = (x+4)^6$



t	x	y
0	-4	0
1	-3	1



Picture of parametric representation, for $t \geq 0$.