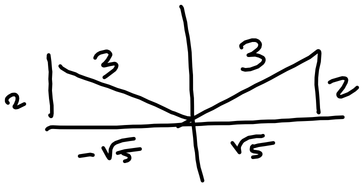
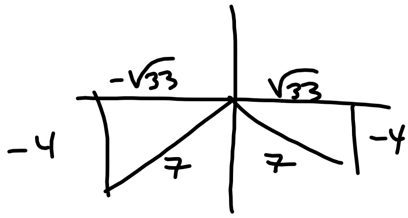
$\sin\theta = \frac{2}{3}$



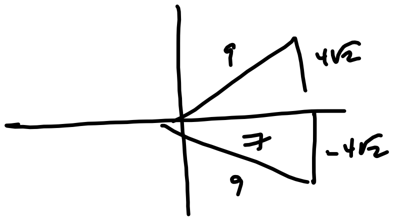
$3^2 - 2^2 = 5 \to \sqrt{5}$

$\sin\theta = -\frac{4}{7}$



$7^2 - 4^2 = 49 - 16 = 33$

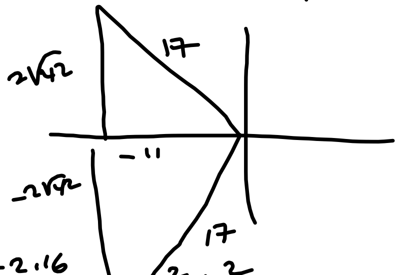
$\cos\theta = \frac{21}{27} = \frac{7}{9}$



$9^2 - 7^2 = 81 - 49 = 32 = 2 \cdot 16$

$\sqrt{32} = 4\sqrt{2}$

$\cos\theta = -\frac{11}{17}$



$17^2 - 11^2 =$

$289 - 121 = 168$

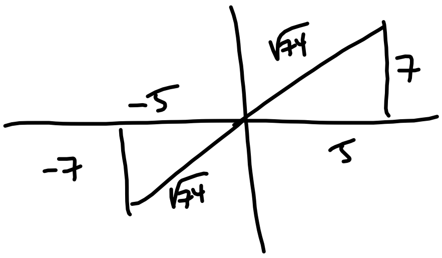
$$\begin{array}{r} 17 \\ \times 17 \\ \hline 119 \\ 170 \\ \hline 289 \end{array}$$

$\sqrt{168}$

$$\begin{array}{r|l} 2 & 168 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array}$$

$\sqrt{168} = 2\sqrt{42}$

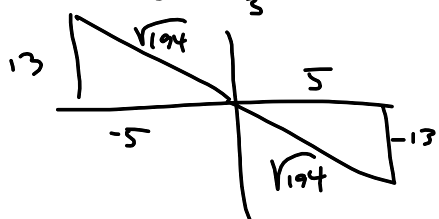
$\tan\theta = \frac{7}{5}$



$7^2 + 5^2 = 49 + 25 = 74$

$2 \,|\, 74$, 37

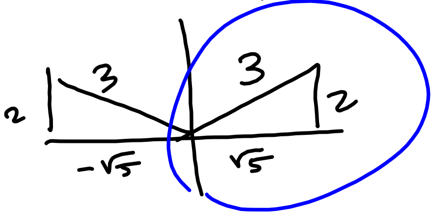
$\tan\theta = -\frac{13}{5}$



$13^2 + 5^2 = 169 + 25 = 194$

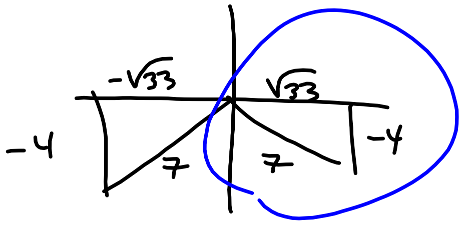
$2 \,|\, 194$, 97

$\sin\theta = \frac{2}{3}$ & $\cos\theta > 0$



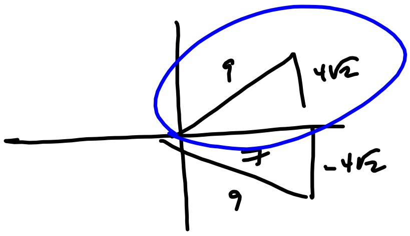
$3^2 - 2^2 = 5 \to \sqrt{5}$

$\sin\theta = -\frac{4}{7}$ & $\tan\theta < 0$



$7^2 - 4^2 = 49 - 16 = 33$

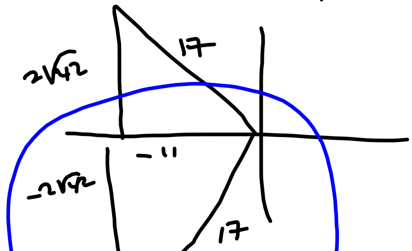
$\cos\theta = \frac{21}{27} = \frac{7}{9}$ & $\sin\theta > 0$



$9^2 - 7^2 = 81 - 49 = 32 = 2 \cdot 16$

$\sqrt{32} = 4\sqrt{2}$

$\cos\theta = -\frac{11}{17}$ & $\sin\theta < 0$



$17^2 - 11^2 =$

$289 - 121 = 168$

$$\begin{array}{r} 17 \\ \times 17 \\ \hline 119 \\ 170 \\ \hline 289 \end{array}$$

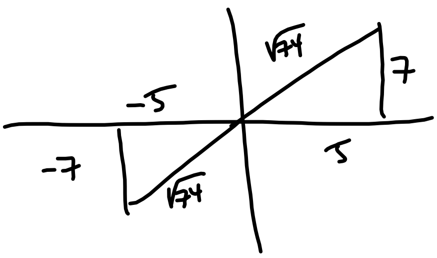
$\sqrt{168}$

$$\begin{array}{r|l} 2 & 168 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array}$$

$\sqrt{168} = 2\sqrt{42}$

etc.

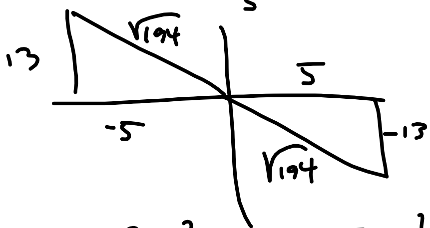
$\tan\theta = \frac{7}{5}$



$7^2 + 5^2 = 49 + 25 = 74$

$$\begin{array}{r|l} 2 & 74 \\ & 37 \end{array}$$

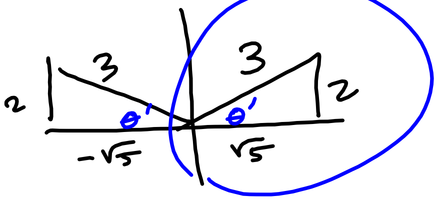
$\tan\theta = -\frac{13}{5}$



$13^2 + 5^2 = 169 + 25 = 194$

$$\begin{array}{r|l} 2 & 194 \\ & 97 \end{array}$$

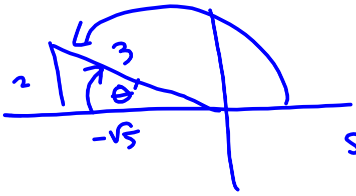
$\sin\theta = \frac{2}{3}$ & $\cos\theta > 0$



$\theta' = \arcsin\left(\frac{2}{3}\right)$

$\theta' = \arctan\left(\frac{2}{\sqrt{5}}\right)$

$\theta' = \arccos\left(\frac{\sqrt{5}}{3}\right)$



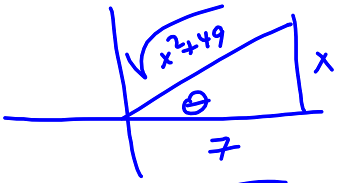
Find exact value of θ

$\sin\theta = \frac{2}{3}$ & $\tan\theta < 0$

$\theta' = \arcsin\left(\frac{2}{3}\right)$

$\theta = \pi - \arcsin\left(\frac{2}{3}\right)$

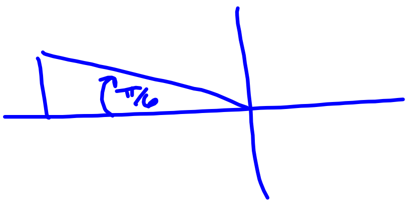
(11) $\sin\left(\arctan\left(\frac{x}{7}\right)\right) = \boxed{\sin\theta = \frac{x}{\sqrt{x^2+49}}}$

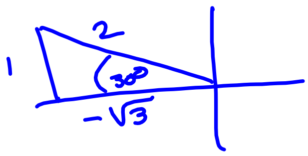


$\sqrt{x^2+7^2}$

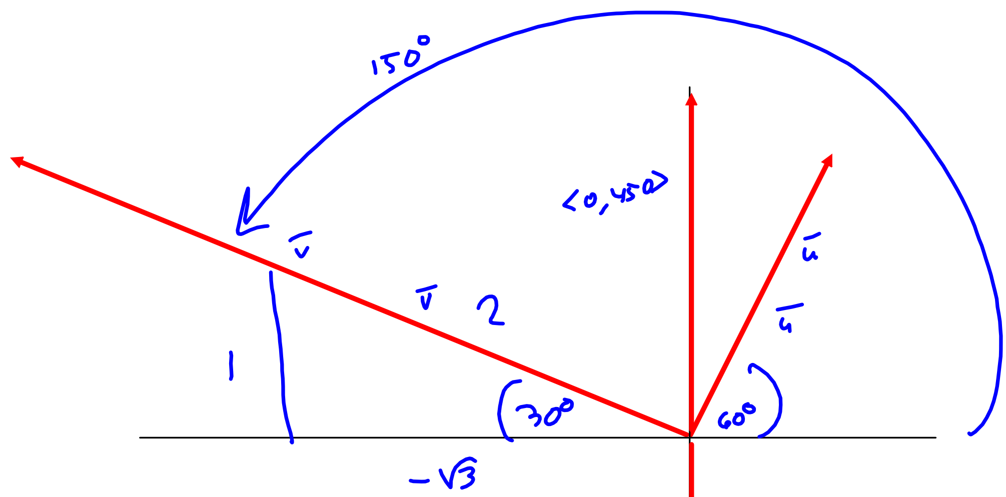
(8) $\arctan\left(\tan\left(\frac{5\pi}{6}\right)\right)$ $\theta' = \frac{\pi}{6}$

Suggest using degrees





$\arctan\left(\tan\left(\frac{5\pi}{6}\right)\right) = \arctan\left(-\frac{1}{\sqrt{3}}\right)$



$\bar{u}+\bar{v} = \langle 0, 450\rangle$

$\bar{u} = \|\bar{u}\| \langle \cos 60°, \sin 60°\rangle$

$= x\langle \frac{1}{2}, \frac{\sqrt{3}}{2}\rangle$, where $\boxed{x = \|\bar{u}\|}$

$\bar{v} = \|\bar{v}\| \langle \cos 150°, \sin 150°\rangle$

$= y\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}\rangle$, where $y = \|\bar{v}\|$

Want $\|\bar{u}\|$ & $\|\bar{v}\|$

$=$ x & y, respectively

$\bar{u}+\bar{v} = \langle \frac{1}{2}x, \frac{\sqrt{3}}{2}x\rangle + \langle -\frac{\sqrt{3}}{2}y, \frac{1}{2}y\rangle = \langle 0, 450\rangle$

$\frac{1}{2}x - \frac{\sqrt{3}}{2}y = 0 \xrightarrow{\times 2} x - \sqrt{3}y = 0 \longrightarrow x = \sqrt{3}y$

$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 450 \xrightarrow{\times 2} \sqrt{3}x + y = 900$

$\longrightarrow \sqrt{3}(\sqrt{3}y) + y = 900$

$\longrightarrow 3y + y = 4y = 900$

$y = \frac{900}{4} = \frac{450}{2} = 225 \text{ lbs} = \|\bar{v}\|$

$x = \sqrt{3}y = \sqrt{3}(225) = 225\sqrt{3} \text{ lbs} = \|\bar{u}\|$

$4\sin^2(3x) - 1 = 0 \implies$

$4\sin^2(3x) = 1$

$\sin^2(3x) = \frac{1}{4}$

$\sin(3x) = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

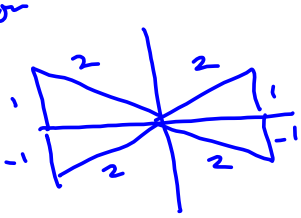
Find all $x \in [0, 2\pi)$ that solve the eq'n.

10a

Harder (3x) version

$\rightarrow$ Need all $3x \in [0, 6\pi)$ $\quad 2\pi = \frac{12\pi}{6}$

$\sin(3x) = \pm\frac{1}{2}$



$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}$

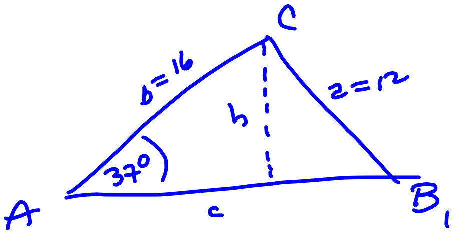
$$\boxed{x \in \left\{\frac{\pi}{18}, \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}\right\} = A}$$

Sol'n sets are nice

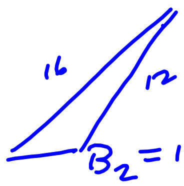
b All sol'ns:

$$\boxed{\{x + 2n\pi \mid x \in A \text{ and } n \in \mathbb{Z}\}}$$

$A = 37^\circ, \ a = 12, b = 16$



2 sol'ns



$B_2 = 180^\circ - B_1$

$\frac{h}{16} = \sin 37^\circ$

$h = 16 \sin 37^\circ \approx 9.63$

$h < a < b \Rightarrow$ 2 sol'ns.

$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\Rightarrow \sin B = \frac{b \sin A}{a} = \frac{16 \sin 37^\circ}{12} \approx 0.8024200307$

$\Rightarrow B_1 \approx \arcsin(0.8024200307) \approx \boxed{53.36182366^\circ \approx B_1}$

$\Rightarrow B_2 = 180^\circ - B_1 \approx \boxed{126.63817^\circ \approx B_2}$

$\bar{u} = \langle 6, 8 \rangle \quad \langle -7, -1 \rangle = \bar{v}$

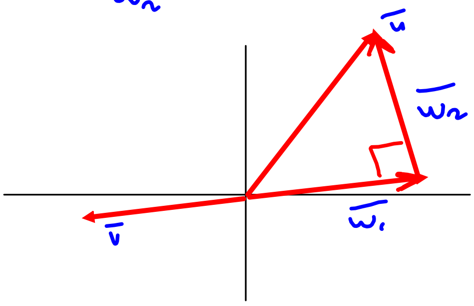
$\|\bar{u}\| = 10 \ , \ \|\bar{v}\| = 5\sqrt{2} = \sqrt{50}$

$\bar{u} \cdot \bar{v} = -42 - 8 = -50$

$\text{proj}_{\bar{v}} \bar{u} = \dfrac{\bar{u} \cdot \bar{v}}{\|\bar{v}\|^2} \bar{v} = \dfrac{-50}{50} \bar{v} = -1 \, \bar{v} = -1 \langle -7, -1 \rangle$

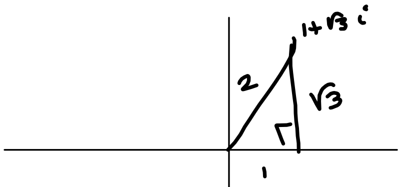
$= \boxed{\langle 7, 1 \rangle = \overline{w_1}}$

$\overline{w_2} = \bar{u} - \overline{w_1} = \langle 6, 8 \rangle - \langle 7, 1 \rangle = \boxed{\langle -1, 7 \rangle = \overline{w_2}}$



$\overline{w_2} \cdot \overline{w_1} = -7 + 7 = 0$ ✓

§4.5 #10 Find square roots of $z = 1 + \sqrt{3}\, i$



$z = 2(\cos 60^\circ + i \sin 60^\circ)$

$\frac{2\pi}{2}$ = increment (2^{nd} root) $= \pi$

$\sqrt{1^2 + \sqrt{3}^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\sqrt{z} = \sqrt{2}(\cos 30^\circ + i \sin 30^\circ)$

$= \boxed{\sqrt{2}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)}$

other 2^{nd} root: $\frac{\pi}{6}$ + increment $= \frac{\pi}{6} + \pi = \frac{\pi}{6} + \frac{6\pi}{6}$

$= \frac{7\pi}{6}$

$\boxed{\sqrt{2}\left(\cos\left(\frac{7\pi}{6}\right) + i \sin\left(\frac{7\pi}{6}\right)\right)}$