**Grade Reports coming soon. I'm just bogged down with a lot of crap, right now.**

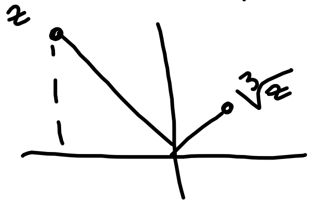
**Remaining schedule is pretty lax, if you look at it.**

**The Chapter 6 material is pretty challenging. Keep working as hard as possible. Give yourself time to reflect and review at the end.**

**No Chapter 5. Just the last part of Chapter 6: parametric equations, equations in polar coordinates, finishing with Conic Sections in Polar coordinates.**

$n^{th}$ Powers & $n^{th}$ roots.

Recall, multiplying by $1(\cos\theta + i\sin\theta)$ is a rotation by $\theta$ radians.




$z = 5\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$

$3^{rd}$ root:

$\sqrt[3]{5}\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)$ = one of the 3 $3^{rd}$ roots of $z$

$$\frac{\frac{2\pi}{3}}{3} = \frac{2\pi}{9}$$

$x^3 = z \implies$

$x^3 - z = 0 \implies$

3 (complex) zeros!

By Fundamental Theorem of Algebra.

Here's how it works.

$\sqrt[3]{z}$ = Principal cube root of $z$ is

$|z|^{\frac{1}{3}}\left(\cos\left(\frac{\theta}{3}\right) + i\sin\left(\frac{\theta}{3}\right)\right)$

$$\left(\sqrt[3]{5}\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)\right)^3$$

$$= \left(\sqrt[3]{5}\right)^3\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)^3$$

$$= (\sqrt[3]{5})^3 \left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)^3$$

$$= 5\left(\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)\right)\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)$$

$$= 5\left(\left(\cos\left(\frac{4\pi}{9}\right) + i\sin\left(\frac{4\pi}{9}\right)\right)\left(\cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)\right)\right)$$

$$= 5\left(\cos\left(\frac{6\pi}{9}\right) + i\sin\left(\frac{6\pi}{9}\right)\right)$$

$$= (\sqrt[3]{5})^3\left(\cos\left(3\cdot\frac{2\pi}{9}\right) + i\sin\left(3\cdot\frac{2\pi}{9}\right)\right) = z\ !$$

$$= 5\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$$

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How do we find the other 2 cube roots?

1. Find $\sqrt[3]{z} = \sqrt[3]{|z|}\left(\cos\left(\frac{\theta}{3}\right) + i\sin\left(\frac{\theta}{3}\right)\right) = z_1$
2. Increment is $\frac{2\pi}{3} = \frac{2\pi}{n} = \frac{2\pi}{\text{index of the root}}$

$$\theta_2 = \frac{\theta}{3} + \frac{2\pi}{3}$$

$$\theta_3 = \frac{\theta}{3} + 2\left(\frac{2\pi}{3}\right)$$

$$\boxed{z_2 = \sqrt[3]{5}\left(\cos\left(\frac{2\pi}{9} + \frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{9} + \frac{2\pi}{3}\right)\right)}$$

$$= \sqrt[3]{5}\left(\cos\left(\frac{8\pi}{9}\right) + i\sin\left(\frac{8\pi}{9}\right)\right)$$

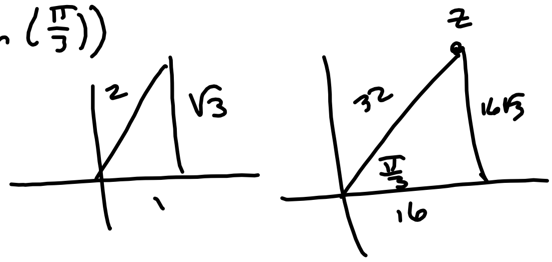
$$\frac{8\pi}{9} + \frac{2\pi}{3} = \frac{8\pi}{9} + \frac{6\pi}{9}$$

$$\boxed{z_3 = \sqrt[3]{5}\left(\cos\left(\frac{14\pi}{9}\right) + i\sin\left(\frac{14\pi}{9}\right)\right)}$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n}+k\frac{2\pi}{n}\right) + i\sin\left(\frac{\theta}{n}+k\left(\frac{2\pi}{n}\right)\right)\right), \quad k=0,1,2,\dots,n-1$$

Principle $n^{\text{th}}$ root

$5^{\text{th}}$ roots of $z = 32\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)$




$\frac{\pi}{3} \div 5 = \frac{\pi}{15}$

Increment $= \frac{2\pi}{5} = \frac{6\pi}{15}$

$\frac{\pi}{15} + \frac{6\pi}{15} = \frac{7\pi}{15}$

$\frac{7\pi}{15} + \frac{6\pi}{15} = \frac{13\pi}{15}$

$\dots \quad \frac{19\pi}{15}$

$\dots \quad \frac{25\pi}{15}$

$z_1 = 2\left(\cos\left(\frac{\pi}{15}\right)+i\sin\left(\frac{\pi}{15}\right)\right)$

$z_2 = 2\left(\cos\left(\frac{7\pi}{15}\right)+i\sin\left(\frac{7\pi}{15}\right)\right)$

$z_3 = 2\left(\cos\left(\frac{13\pi}{15}\right)+i\sin\left(\frac{13\pi}{15}\right)\right)$

$z_4 = 2\left(\cos\left(\frac{19\pi}{15}\right)+i\sin\left(\frac{19\pi}{15}\right)\right)$

$z_5 = 2\left(\cos\left(\frac{25\pi}{15}\right)+i\sin\left(\frac{25\pi}{15}\right)\right)$

$z_6 = 2\left(\cos\left(\frac{31\pi}{15}\right)+i\sin\left(\frac{31\pi}{15}\right)\right) = z_1$ ! Went too far!

( $\frac{31\pi}{15} = 2\pi + \frac{\pi}{15}$ is coterminal with $\theta_1$ )

$$(x-(2+i))(x-(2-i))$$
$$= x^2 - x(2-i) - (2+i)x + (2+i)(2-i)$$
$$= x^2-2x+ix-2x-ix+(2^2+1^2)$$
$$= x^2-4x+5$$

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$$(x-2+i)(x-2-i)$$
$$= x^2-2x-\cancel{ix}-2x+4+2i+\cancel{ix}-2i-i^2$$
$$= x^2-4x+5$$

whichever's easier for you.