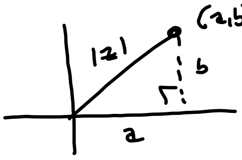
$z = a+bi \Rightarrow \bar{z} = a-bi$

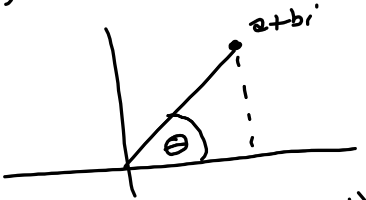
$|z| = \text{modulus} = \sqrt{a^2+b^2} = \sqrt{z\bar{z}}$



Similar structure

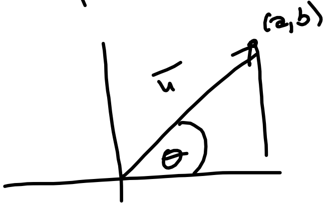
Vectors $\bar{u} = \langle a,b \rangle \Rightarrow \|\bar{u}\| = \sqrt{a^2+b^2} = \sqrt{\bar{u}\cdot\bar{u}}$

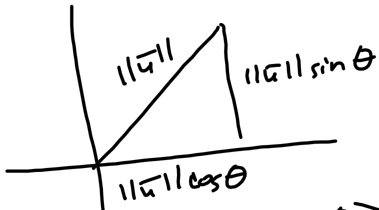
Trig Form



θ (or any angle coterminal with θ)

$a+bi = |z|(\cos\theta + i\sin\theta)$





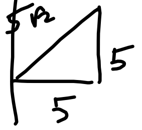
$\bar{u} = \|\bar{u}\|\langle \cos\theta, \sin\theta \rangle$

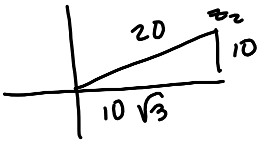
DeMoivre: $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 z_2 = r_1 r_2 \left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$

Multiply moduli & add $\underbrace{\text{arguments}}_{\text{angles.}}$





$\frac{\pi}{4}\cdot\frac{3}{3} + \frac{\pi}{6}\cdot\frac{2}{2} = \frac{5\pi}{12}$

$z_1 = 5\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$z_2 = 20\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$z_1 z_2 = 100\sqrt{2}\left(\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right)$

---

n^{th} powers

$z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$

The big brain fever is doing the n^{th} roots of z

Read 4.5 on <u>n^{th} Roots</u>

$\sqrt{-6}\,\sqrt{-2} \neq \sqrt{(-6)(-2)}$

$2 \mid 12$, $2 \mid 6$, 3, $2\sqrt{3}$

$\sqrt{a}\,\sqrt{b} = \sqrt{ab}$ only if $a \geq 0, b \geq 0$

Rather, $\sqrt{-6}\,\sqrt{-2} = (i\sqrt{6})(i\sqrt{2}) = i^2\sqrt{12} = -2\sqrt{3}$

$x^2-2x+2=0$

Quadratic Formula

$a=1, b=-2, c=2$

$b^2-4ac = 2^2-4(1)(2)$

$= 4-8 = -4 < 0 \Rightarrow$ 2 nonreal (imaginary)

$\rightsquigarrow \sqrt{-4} = 2i$

$$x = \frac{b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm 2i}{2(1)} = \frac{2 \pm 2i}{2} = \frac{2(1 \pm i)}{2} = \boxed{1 \pm i}$$

Completing the Square

$= x^2+2x+1^2-1+2 = (x+1)^2+1 \overset{\text{SET}}{=} 0$

$\frac{2}{2} = 1 \rightsquigarrow 1^2$

$\Rightarrow (x+1)^2 = -1$

$\Rightarrow x+1 = \pm\sqrt{-1} = \pm i$

$\Rightarrow \boxed{x = -1 \pm i}$

$5x^2-3x-7=0$

$5\left(x^2-\frac{3}{5}x-\frac{7}{5}\right)=0 \Rightarrow$

$x^2-\frac{3}{5}x+\left(\frac{3}{10}\right)^2-\frac{9}{100}-\frac{7}{5}=0$

$\left(x-\frac{3}{10}\right)^2-\frac{149}{100}=0 \Rightarrow$

$\left(x-\frac{3}{10}\right)^2 = \frac{149}{100}$

$x-\frac{3}{10} = \pm\frac{\sqrt{149}}{10}$

$x = \frac{3 \pm \sqrt{149}}{10}$

To Divide $5x^2\ 3x^2 + 2x + 1$ by $x-2$:

$$\begin{array}{r|rrrr} 2 & 5 & -3 & 2 & 1 \\ & & 10 & 14 & 32 \\ \hline & 5 & 7 & 16 & 33 \end{array}$$

Synthetic Division

remainder (Theorem!)

This says

$$5x^3-3x^2+2x+1 = (x-2)(5x^2+7x+16)+33$$

OR $$\frac{5x^2-3x^2+2x+1}{x-2} = \underbrace{5x^2+7x+16}_{} + \frac{33}{x-2}$$

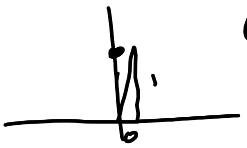
Recall: Slant / Oblique Asymptote

$$5(2)^3-3(2)^2+2(2)+1 = 40-12+4+1 = 33$$

Trig form:

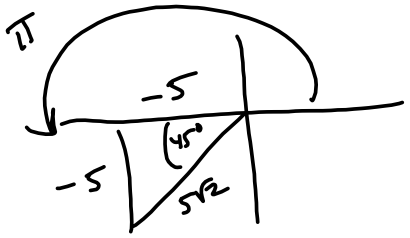
$z = 3i$

$r = |z| = 3$



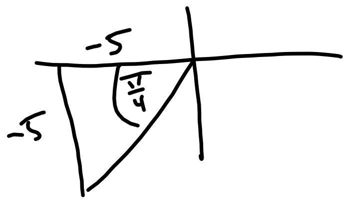
$\theta = \frac{\pi}{2} = 90°$

so $z = 3(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$



$\theta = \pi + \arctan\left(\frac{5}{5}\right) = \pi + \arctan\left(\frac{\pi}{4}\right)$

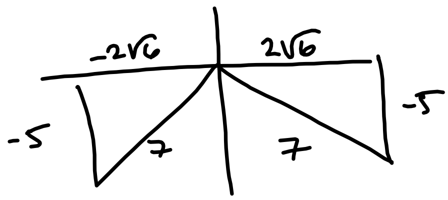
$= \pi + \frac{\pi}{4} = \frac{5\pi}{4}$



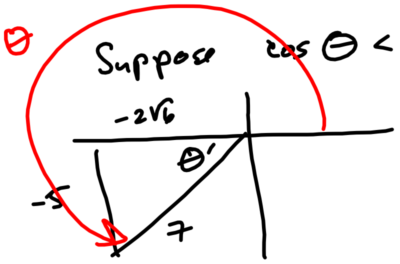
$\sin\theta = -\frac{5}{7}$

$7^2 - 5^2 = 24$

$\sqrt{24} = 2\sqrt{6}$



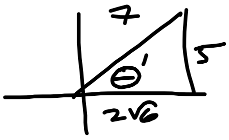
Suppose $\cos\theta < 0$



Exact value of θ:

$\theta = \pi + \arcsin\left(\frac{5}{7}\right)$

$\pi - \arcsin\left(-\frac{5}{7}\right)$



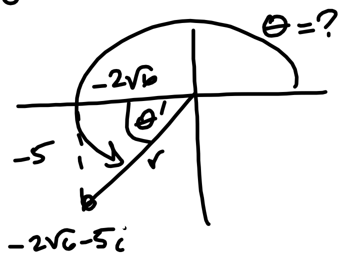
$\theta' = \arctan\left(\frac{5}{2\sqrt{6}}\right)$

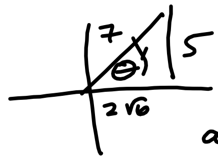
$= \arcsin\left(\frac{5}{7}\right)$

$= \arccos\left(\frac{2\sqrt{6}}{7}\right)$

New! For chapter 4!

Write $-2\sqrt{6}-5i$ in trig form





$\arcsin\left(\frac{5}{7}\right)$

$$r^2 = 5^2 + (2\sqrt{6})^2 = 25 + 24 = 49$$

$$\Rightarrow r = 7 = |z|$$

$$\theta = \pi + \arcsin\left(\frac{5}{7}\right)$$

$$z = 7\left(\cos\left(\pi + \arcsin\left(\frac{5}{7}\right)\right) + i\sin\left(\pi + \arcsin\left(\frac{5}{7}\right)\right)\right)$$

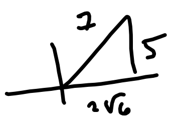
You could simplify somewhat, but not unless asked for it.

Otherwise, just crank it out to the specified precision.

$$\cos\left(\pi + \arcsin\left(\frac{5}{7}\right)\right)$$

$$= \cos(\pi)\cos\left(\arcsin\left(\frac{5}{7}\right)\right) - \sin(\pi)\sin\left(\arcsin\left(\frac{5}{7}\right)\right)$$

$$= -1\left(2\sqrt{6}/7\right) - 0 = -\frac{2\sqrt{6}}{7}$$



$$\sin\left(\pi + \arcsin\left(\frac{5}{7}\right)\right) = \underline{\sin\pi\cos\left(\arcsin\left(\frac{5}{7}\right)\right)} + \sin\left(\arcsin\left(\frac{5}{7}\right)\right)\cos\pi$$

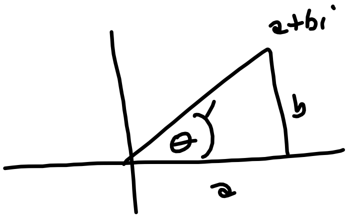
$$\sin(u+v) = \sin u\cos v + \sin v\cos u$$

$$+ \frac{5}{7}(-1) = -\frac{5}{7}$$

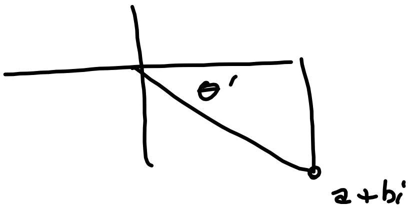
$$\Rightarrow z = |z|(\cos\theta + i\sin\theta)$$

$$= 7\left(-\frac{5}{7} - \frac{2\sqrt{6}}{7}\right) = -5 - 2\sqrt{6}$$

$a+bi$



$$\theta = \arctan\left(\frac{b}{a}\right)$$



$$\theta = 2\pi - \arctan\left(\frac{|b|}{|a|}\right)$$