

MIDTERM:

RM 107 Horizon Hall 8 am - 6 pm Monday 3/31 or Tuesday 4/1

Bring scientific calculator,

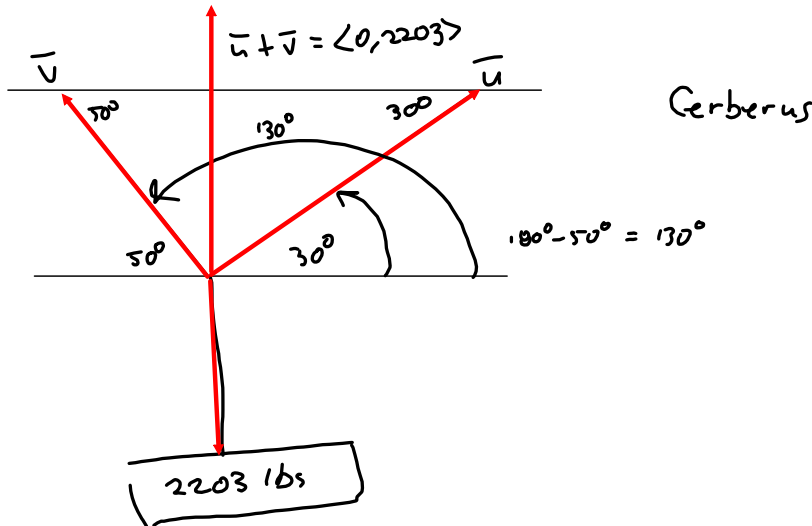
pen or pencil,

1-page cheat sheet (2 sides is ok)

Photo ID

A weight of 2203 lbs is suspended from 2 hooks as shown in the picture.

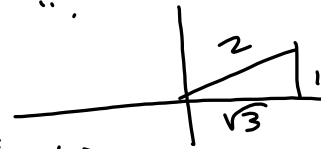
What's the tension in each cable?



Let $x = \|\vec{u}\| =$ tension in cable on right, in lbs.

$y = \|\vec{v}\| =$ left, " " "

Then $\vec{u} + \vec{v} = \langle 0, 2203 \rangle$



$$\begin{aligned} \vec{u} &= \|\vec{u}\| \langle \cos(30^\circ), \sin(30^\circ) \rangle = x \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle \\ &= \langle \frac{\sqrt{3}}{2}x, \frac{1}{2}x \rangle \\ &= \langle \frac{\sqrt{3}}{2}x, \frac{1}{2}x \rangle \end{aligned}$$

$$\vec{v} = \dots = y \langle \cos 130^\circ, \sin 130^\circ \rangle$$

$$= \langle ay, by \rangle, \text{ where } a = \cos 130^\circ, b = \sin 130^\circ$$

$$\text{So } \vec{u} + \vec{v} = \langle \frac{\sqrt{3}}{2}x, \frac{1}{2}x \rangle + \langle ay, by \rangle = \langle 0, 2203 \rangle$$

$$\frac{\sqrt{3}}{2}x + ay = 0 \rightarrow \frac{\sqrt{3}}{2}x = -ay \rightarrow \boxed{x = \frac{-2a}{\sqrt{3}}y}$$

$$\frac{1}{2}x + by = 2203 \rightarrow \frac{1}{2}(\frac{-2a}{\sqrt{3}}y) + by =$$

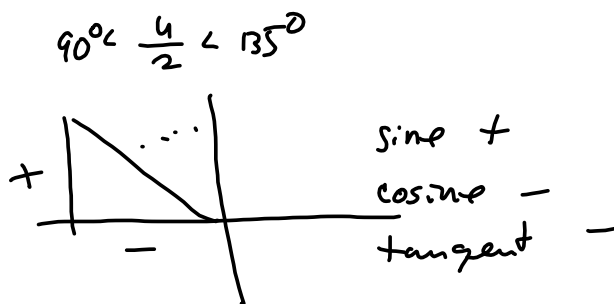
$$= (\frac{-a}{\sqrt{3}} + b)y = 2203 \rightarrow$$

$$y = \frac{2203}{\frac{-a}{\sqrt{3}} + b} = \frac{2203}{\frac{-\cos 130^\circ}{\sqrt{3}} + \sin 130^\circ} \approx 1937.28568718 \text{ lbs} \approx y$$

$$\approx 1937 \text{ lbs} \approx \|\vec{u}\|$$

$$x = \frac{-2a}{\sqrt{3}}y = \frac{-2 \cos(130^\circ)}{\sqrt{3}}y \approx 1437.9061292 \approx 1438 \text{ lbs} \approx \|\vec{v}\|$$

$$\approx \frac{-2 \cos(130^\circ)}{\sqrt{3}}(1437.9061292) \approx 1937.28568718 \approx 1937 \text{ lbs} \approx \|\vec{u}\|$$



Midterm: Old Midterm is a good thing to study [Link](#)

Find all $x \in [0, 2\pi)$ such that the following equation is solved.

$$2\sin(4x) + 2\sin(2x) = 0$$

$$\Rightarrow 2(2\sin(2x)\cos(2x)) + 2\sin(2x) =$$

$$= 4\sin(2x)\cos(2x) + 2\sin(2x) = 0 \Rightarrow$$

$$2\sin(2x)[2\cos(2x) + 1] = 0$$

Want all $x \in [0, 2\pi) \rightarrow$ Find all $2x \in [0, 4\pi)$

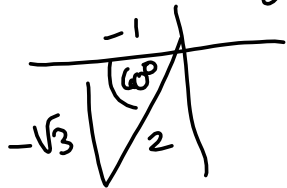
$$\angle \sin(2x) = 0$$

$$\sin(2x) = 0$$



$$2x = 0, \pi, 2\pi, 3\pi$$

$$\rightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$



$$2\cos(2x) + 1 = 0$$

$$\rightarrow 2\cos(2x) = -1$$

$$\cos(2x) = -\frac{1}{2}$$



$$180^\circ - 60^\circ = 120^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{2\pi}{3}$$

$$60^\circ = \frac{\pi}{3}$$

$$30^\circ = \frac{\pi}{6}$$

$$45^\circ = \frac{\pi}{4}$$

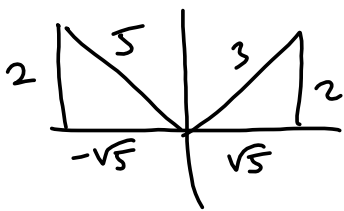
$$120^\circ = \frac{2\pi}{3}$$

$$2x = \frac{2\pi}{3}$$

$$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

$$\rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

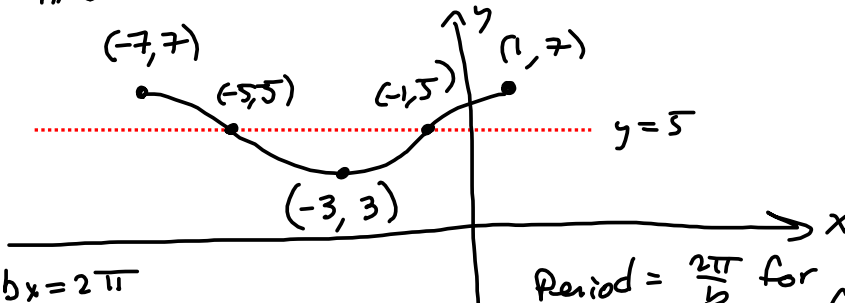
$$\sin \theta = \frac{2}{3}$$



$$3^2 - 2^2 = 9 - 4 = 5 \rightsquigarrow \sqrt{5}$$

$$f(x) = 2 \cos\left(\frac{\pi}{4}x + \frac{7\pi}{4}\right) + 5$$

Amplitude \swarrow up 5!
 $\frac{\pi}{4}x + \frac{7\pi}{4} = \frac{\pi}{4}(x+7)$ \swarrow left + 7!



$$bx = 2\pi$$

$$\frac{\pi}{4}x = 2\pi \rightarrow$$

$$x = \frac{2\pi}{\frac{\pi}{4}} = 2\pi \cdot \frac{4}{\pi} = 8$$

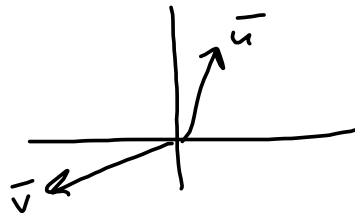
Period = $\frac{2\pi}{b}$ for $\cos(bx)$ & $\cos(bx+c)$

There will be a Law of Sines 3.1

Law of Cosines 3.2

Vectors 3.3

Dot Product 3.4



$$\vec{u} = \langle 1, 5 \rangle$$

$$\vec{v} = \langle -4, -2 \rangle$$

$$\vec{u} + \vec{v} = \langle -5, 3 \rangle$$

$$\|\vec{u}\| = \sqrt{1^2 + 5^2} = \sqrt{26} = \|\vec{u}\|$$

$$\|\vec{u} + \vec{v}\| = \sqrt{5^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34} = \|\vec{u} + \vec{v}\|$$

$$\|\vec{v}\| = \sqrt{6^2 + 2^2} = \sqrt{40}$$

Angle Between \vec{u} & \vec{v}

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \frac{\langle 1, 5 \rangle \cdot \langle -4, -2 \rangle}{\sqrt{26} (2\sqrt{10})} = \frac{-6 - 10}{2\sqrt{260}} = \frac{-16}{2\sqrt{260}} = \frac{-8}{\sqrt{260}}$$

$$= \frac{-16}{2(2\sqrt{65})} = \frac{-16}{4\sqrt{65}} = \frac{-8}{\sqrt{65}} = \cos \theta$$

$$\theta = \arccos\left(\frac{-8}{\sqrt{65}}\right)$$

$$\begin{array}{r} 2 \overline{) 260} \\ 2 \overline{) 130} \\ \cdot 5 \overline{) 65} \\ 13 \end{array}$$

To nearest 4 digits

$$172.874983651^\circ \approx 172.8750^\circ$$