Vectors

3.1 or 3.2 Questions?

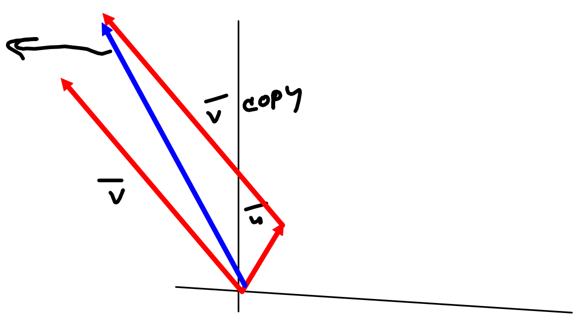
$\bar{u} = \langle 2, 3 \rangle$

$\bar{v} = \langle -5, 7 \rangle$

Then $\bar{u} + \bar{v} =$ resultant

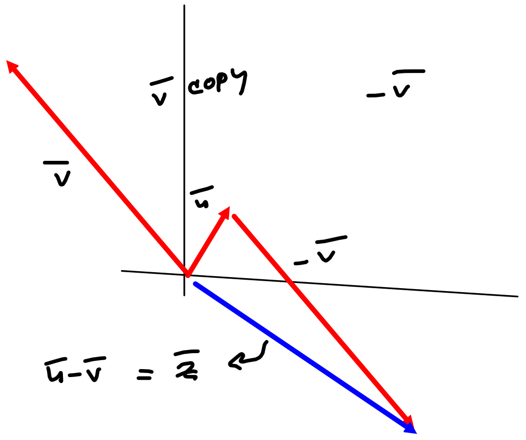
$= \langle -3, 10 \rangle$

$\bar{w} = \bar{u} + \bar{v} =$



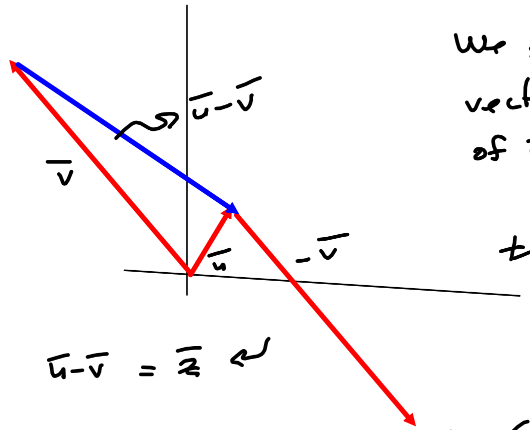
Subtraction: $\bar{u} - \bar{v} = \langle 2-(-5), 3-7 \rangle = \langle 7, -4 \rangle = \bar{z}$

$= \bar{u} + (-\bar{v})$



$\bar{u} - \bar{v} = \bar{z}$

$-\bar{v}$



We see that $\bar{u}-\bar{v}=\bar{z}$ is the vector from the tip of $\bar{v}$ to the tip of $\bar{u}$.

We need this to connect trig (Law of Cosines) to Dot Products & Projections in Section 3.4 (After Spring Break)

SCALAR MULTIPLICATION (number times vector)

$\bar{u} = \langle 2,3 \rangle \Rightarrow 5\bar{u} = 5\langle 2,3\rangle = \langle 10,15\rangle =$ vector in the same direction as $\bar{u}$, but 5 times longer.

Linear Combinations:

$5\bar{u} - 2\bar{v}$

In general, let $a_1, a_2, a_3, \dots, a_n$ be real #s and $\bar{u}_1, \bar{u}_2, \bar{u}_3, \dots, \bar{u}_n$ be vectors in the plane.

Then $\sum_{k=1}^{n} a_k \bar{u}_k = a_1\bar{u}_1 + a_2\bar{u}_2 + \dots + a_n\bar{u}_n$

is the sum of multiples of vectors, or Linear Combination of vectors, or a linear combo on the set $\{\bar{u}_1, \bar{u}_2, \dots, \bar{u}_n\}$

Canonical Basis for $\mathbb{R}^2$ (the plane)

Let $\bar{i} = \langle 1,0 \rangle$ and $\bar{j} = \langle 0,1 \rangle$.

Then any vector can be decomposed into a linear combo on $\bar{i}, \bar{j}$ as follows:

$$\bar{u} = \langle 2,3 \rangle = 2\langle 1,0 \rangle + 3\langle 0,1 \rangle$$
$$= 2\bar{i} + 3\bar{j}$$

Book used bold-faced font for $\bar{i}$ & $\bar{j}$

To say it's a basis is to say

1. Any vector can be written as a linear combo of the set SPANS.
2. It's the smallest possible set that can do this MINIMAL SPANNING SET.

Direction angle: The angle a vector makes with the positive x-axis, measured counterclockwise.

Unit Vector $\bar{u}$ has length 1.

Length : $\|\bar{u}\|$ = Length of $\bar{u}$.

If $\bar{u} = \langle u_1, u_2 \rangle$, then $\|\bar{v}\|$ = Magnitude of $\bar{v}$

$= \sqrt{u_1^2 + u_2^2}$ (Pythagorus).

$\bar{u} = \langle 2,3 \rangle \Longrightarrow \|\bar{u}\| = \sqrt{2^2+3^2} = \boxed{\sqrt{13} = \|\bar{u}\|}$

since points on the unit circle can be represented as $(x,y) = (\cos\theta, \sin\theta)$ & a position vector for (x,y) is $\langle x, y \rangle = \langle \cos\theta, \sin\theta \rangle$, where θ is the direction angle for $\langle x,y \rangle$

Find the vector with direction angle $\theta = 30^\circ$ and magnitude $\|\bar{u}\| = 5$.

Hint: All position vectors on the unit circle are unit vectors!

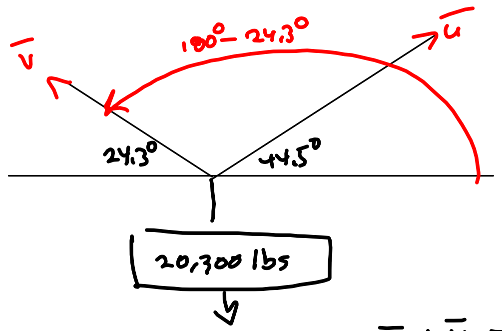
$\left(\sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = 1 \; ! \right)$

$\langle \cos 30^\circ, \sin 30^\circ \rangle$ is unit vector in the direction I want, so

$5 \langle \cos 30^\circ, \sin 30^\circ \rangle$ has length 5 & is pointed in the right direction.

$\bar{u} = 5 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle = \langle \frac{5\sqrt{3}}{2}, \frac{5}{2} \rangle$

#1 on §3.3 Applications



Find the tension in each cable.

Physical Setting: Nothing's moving. All forces are in perfect balance.

Let
$a = \cos 44.5^\circ$
$b = \sin 44.5^\circ$

We know that $\bar{u} + \bar{v} = \langle 0, 20{,}300 \rangle$

$\bar{u} = \|\bar{u}\| \langle \cos 44.5^\circ, \sin 44.5^\circ \rangle = \|\bar{u}\| \langle a, b \rangle$

$180^\circ - 24.3^\circ = 155.7^\circ$

$\bar{v} = \|\bar{v}\| \langle \cos(155.7^\circ), \sin(155.7^\circ) \rangle = \|\bar{v}\| \langle c, d \rangle$

$c = \cos(155.7^\circ)$
$d = \sin(155.7^\circ)$

Then Horizontal Forces are balanced
$a\|\bar{u}\| + c\|\bar{v}\| = 0$ and
Vertical Forces
$b\|\bar{u}\| + d\|\bar{v}\| = 20300$

$ax + cy = 0$
$bx + cy = 20300$
where $x = \|\bar{u}\|$
$y = \|\bar{v}\|$

$\Rightarrow a\|\bar{u}\| = -c\|\bar{v}\| \Rightarrow$

$\|\bar{u}\| = \dfrac{-c\|\bar{v}\|}{a} \Rightarrow$

$a\|\bar{u}\| + d\|\bar{v}\| = 20\,300 \Rightarrow$

$b\left(\dfrac{-c\|\bar{v}\|}{a}\right) + d\|\bar{v}\| = 20300 \Rightarrow$

$\dfrac{-bc}{a}\|\bar{v}\| + d\|\bar{v}\| = \left(d - \dfrac{bc}{a}\right)\|\bar{v}\| = 20300 \Rightarrow$

$\|\bar{v}\| = \dfrac{20300}{d - \frac{bc}{a}} = \cancel{\dfrac{20\,300}{\sin(155.7^\circ) - \frac{\sin(44.5^\circ)\cos(155.7^\circ)}{\cos(44.5^\circ)}}}$

Desmos clobbers TI-3

$= \dfrac{20300}{\sin(155.7^\circ) - \tan(44.5^\circ)\cos(155.7^\circ)} \approx \boxed{15529.9951572 \approx \|\bar{v}\|}$

$\approx \boxed{15530 \text{ lbs}}$

$\|\bar{u}\| =$

You can build a unit vector in the direction of any vector you want.

Find a unit vector in the direction of

$\bar{v} = \langle 3, 7 \rangle$

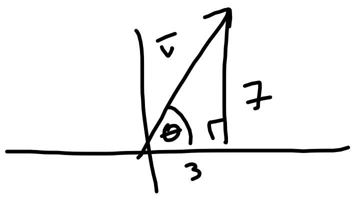
$$\|\bar{v}\| = \sqrt{3^2+7^2} = \sqrt{58}$$

Unit vector we want is

$$\frac{1}{\sqrt{58}}\bar{v} = \frac{1}{\|\bar{v}\|}\bar{v} = \boxed{\frac{1}{\sqrt{58}}\langle 3, 7 \rangle}$$

A funky way to write $\bar{v}$ is

$\|\bar{v}\| \langle \cos\theta, \sin\theta \rangle$, where θ = direction angle



$$\theta = \arctan\left(\frac{7}{3}\right)$$

$$\bar{v} = \sqrt{58}\ \left\langle \cos\left(\arctan\left(\frac{7}{3}\right)\right), \sin\left(\arctan\left(\frac{7}{3}\right)\right)\right\rangle$$