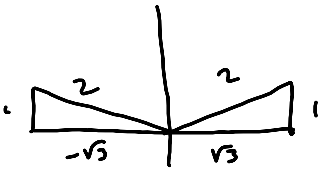
Find all sol'ns $x \in [0, 2\pi)$

$\sin(3x) = \frac{1}{2}$ Capture all $x \in [0, 2\pi] \longrightarrow$

.. .. $3x \in [0, 6\pi)$

$2\pi = \frac{12\pi}{6}$

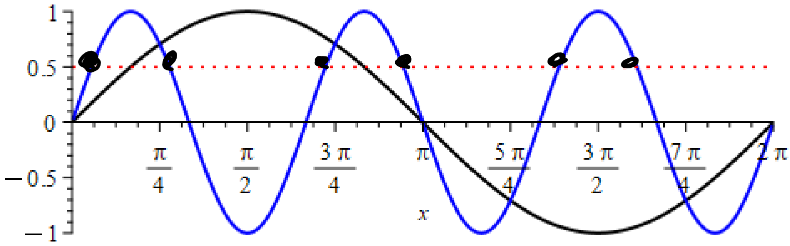



$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

$\Longrightarrow x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

$\cancel{\frac{11\pi}{6} = \frac{(36+5)\pi}{6}} > 6\pi$

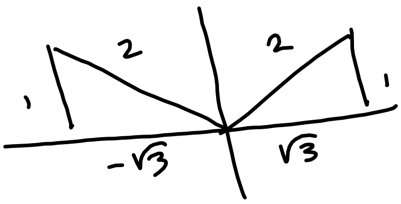
$\frac{25\pi}{6} + \frac{12\pi}{6} = \frac{37\pi}{6} > 6\pi$




FIND ALL SOLUTIONS!

Capture ALL 3x's in $[0, 2\pi)$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}$




$3x = \frac{\pi}{6} + 2n\pi \longrightarrow$

$\boxed{x = \frac{\pi}{18} + \frac{2n\pi}{3}}$

OR

$3x = \frac{5\pi}{6} + 2n\pi \longrightarrow$

$\boxed{x = \frac{5\pi}{18} + 2n\pi}$

**Section 2.4 - Sum and Difference Formulas: I only cover the SUM formulas. The rest is managing odd and even functions: Sine and Tangent are odd. Cosine is even.**

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$\sin u \cos v + \sin v \cos u$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

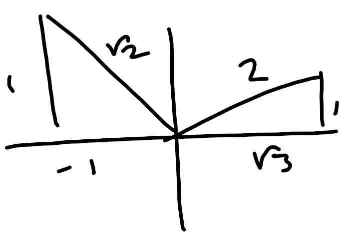
$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\sin(u-v) = \sin(u+(-v))$$
$$= \sin(u)\cos(-v) + \sin(-v)\cos(u)$$
$$= \sin u \cos v - \sin(v)\cos(u)$$

Find the exact value of $\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{2\pi}{12} + \frac{9\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)$

Scratch $\left(\underbrace{\pi + 10\pi}_{12}, \frac{(2+9)\pi}{12}\right)$

$$= \sin\frac{\pi}{6}\cos\frac{3\pi}{4} + \sin\frac{3\pi}{4}\cos\frac{\pi}{6} = \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\frac{\sqrt{3}}{2}$$




$$= \boxed{-\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}}$$ FINE

$$= \frac{-1+\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}+\sqrt{6}}{4}}$$

simplified radical form.

write the trig expression as an algebraic expression

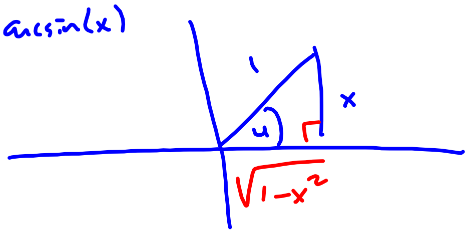
(54) $\sin(\arcsin(x) + \arccos(x))$

$= \sin(u+v)$, where $u = \arcsin(x)$, $v = \arccos(x)$

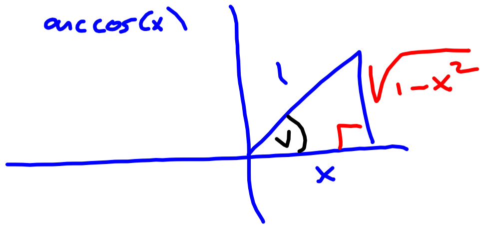
$= \sin(u)\cos(v) + \sin(v)\cos(u)$

$= \sin(\arcsin(x))\cos(\arccos(x)) + \sin(\arccos(x))\cos(\arcsin(x))$

arcsin(x)




arccos(x)




$= x(x) + \underbrace{\sqrt{1-x^2}\,\sqrt{1-x^2}}_{} = x^2 + 1 - x^2 = 1$

$(\sqrt{1-x^2})^2 = |1-x^2| = 1-x^2$, b/c $1-x^2 \geq 0$ to keep it real

Here, $1-x^2 \geq 0$
$\Rightarrow x \in [-1,1]$

However $\sqrt{(1-x^2)^2} = |1-x^2|$
& x can be anything!

The REAL #54

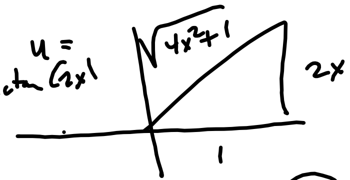
$$\sin(\arctan(2x) - \arccos(x)) = \sin(u - v)$$

$$= \sin(u)\cos(-v) + \sin(-v)\cos(u)$$

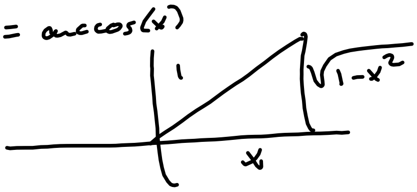
$$= \sin(u)\cos(v) - \sin(v)\cos(u)$$

$$= \sin(\arctan(2x))\cos(\arccos(x)) - \sin(\arccos(x))\cos(\arctan(2x))$$

$u = \arctan(2x)$




$v = \arccos(x)$




$(2x)^2 = 4x^2 = 2^2x^2$

Don't write $2x^2$

$$= \frac{2x}{\sqrt{4x^2+1}} \cdot x - \frac{\sqrt{1-x^2}}{1} \cdot \frac{1}{\sqrt{4x^2+1}}$$

$$= \frac{2x^2 - \sqrt{1-x^2}}{\sqrt{4x^2+1}}$$

**VitalSource is great eBook compared to WebAssign's. It's \$50 or \$60 for the semester (more like 180 days).**

We have a better proof of the Cofunction Identities:

$\tan(\frac{\pi}{2}-x) = \cot(x)$ ?

Proof:

$\tan(u+v) = \frac{\tan u + \tan v}{1-\tan u \tan v} = \frac{\tan(\frac{\pi}{2}) + \tan(-x)}{1-\tan(\frac{\pi}{2})\tan(-x)}$ $\nexists$

= DOH! What's wrong with this?

But I can do it with sine & cosine & then use

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\sin(\frac{\pi}{2}-\theta) = \cos\theta$

Proof:

$\sin(\frac{\pi}{2}-\theta) = \sin\frac{\pi}{2}\cos\theta + \sin\theta\cos\frac{\pi}{2}$

$= 1\cdot\cos\theta + \sin\theta\cdot 0$

$= \cos\theta$

$\cos(\frac{\pi}{2}-\theta) = \cos\frac{\pi}{2}\cos(-\theta) - \sin(\frac{\pi}{2})\sin(-\theta)$

$= 0\cdot\cos\theta - (1)(-\sin\theta) = \sin\theta$

$\longrightarrow \tan(\frac{\pi}{2}-\theta) = \frac{\sin(\frac{\pi}{2}-\theta)}{\cos(\frac{\pi}{2}-\theta)} = \frac{\cos\theta}{\sin\theta} = \cot\theta$ ∎

**Section 2.5**

**Multiple-Angle and Product-to-Sum Formulas**

**Double-Angle Formulas -**

$\sin(2\theta) = 2\sin\theta\cos\theta = \sin(\theta+\theta) = \sin\theta\cos\theta + \sin\theta\cos\theta$
$= 2\sin\theta\cos\theta$

$\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta$
$= \cos^2\theta - \sin^2\theta$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta} = \dfrac{2\tan\theta}{1-\tan^2\theta}$

**Power-Reducing Formulas**

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \cos(2\theta) + 1 = 2\cos^2\theta \Rightarrow$
$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

**Half-Angle Formulas**

$\theta = \frac{1}{2}u, \quad 2\theta = u$

$\cos^2\left(\frac{u}{2}\right) = \dfrac{1+\cos(u)}{2} \Rightarrow$

$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\dfrac{1+\cos(u)}{2}}$

$+$: $\cos\left(\frac{u}{2}\right) > 0$
$-$: $\cos\left(\frac{u}{2}\right) < 0$

You need to determine what quadrant $\frac{u}{2}$ is in to determine "+" or "−"

**Product-to-Sum and Sum-to-Product Formulas**

**Heuristic Learning: Learning what you need to solve a problem. Pretty much the easiest way to motivate the readings.**