Section 2.1 #15

Use the trigonometric substitution to write the algebraic equation as a trigonometric equation of θ, where $-\pi/2 < \theta < \pi/2$.

$5\sqrt{3} = \sqrt{100 - x^2}$, $x = 10 \cos \theta$

$5\sqrt{3}$ = $10\sin(\theta)$ ✓ $10\sqrt{\sin^2(\theta)}$ $= 10\,|\sin(\theta)|$ $\longrightarrow$

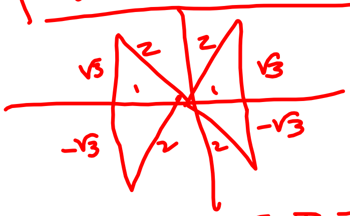
Find sin θ and cos θ. (Enter your answers as a comma-separated list.)

sin θ = $\frac{\sqrt{3}}{2}$ ✗ $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$

cos θ = $\frac{1}{2}$ ✓ $\frac{1}{2}$

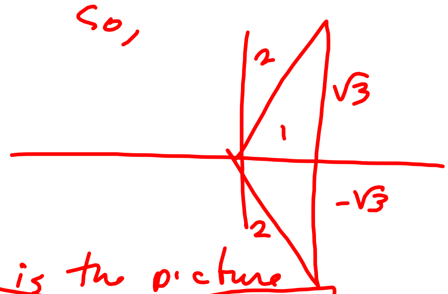
$10\sin\theta = \pm 5\sqrt{3}$ $\longrightarrow$

$\boxed{\sin\theta = \pm\frac{\sqrt{3}}{2}}$



Domain: $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

So,



is the picture and $\boxed{\cos\theta = \frac{1}{2}}$ for both pictures.

Reciprocals

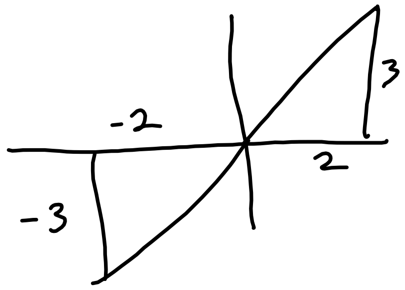
$\sin\theta = \frac{1}{\csc\theta}$

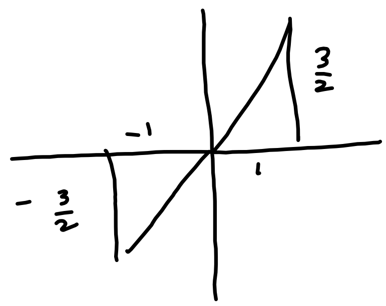
$\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

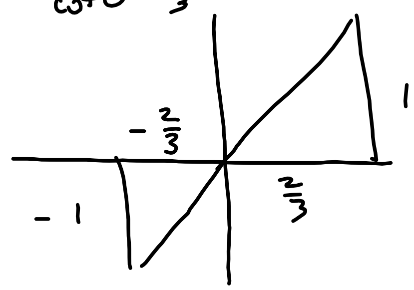
Me: $\cot\theta = \frac{2}{3}$

$\Rightarrow \tan\theta = \frac{3}{2}$





$\cot\theta = \frac{2}{3}$



Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\cot^2\theta + 1 = \csc^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

Cofunction Identities

$$\boxed{\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta}$$

$$\boxed{\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta}$$

$$\boxed{\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta} \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

$$\boxed{\sec\left(\frac{\pi}{2}-\theta\right)} = \text{cosecant of theta} \boxed{= \csc\theta}$$

§2.2 Stuff

Common tricks:

Turn everything into sines and cosines $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$,

"Rationalizing" trick

$$a^2-b^2 = (a-b)(a+b) \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

E We show that $\dfrac{6\sec\theta - 6}{1-\cos\theta} = 6\sec\theta$

$$\frac{6(\sec\theta-1)}{(1-\cos\theta)}\left(\frac{1+\cos\theta}{1+\cos\theta}\right) = \frac{6(\sec\theta-1)(\cos\theta+1)}{1-\cos^2\theta}$$

$$= \frac{6(\sec\theta\cos\theta + \sec\theta - \cos\theta - 1)}{\sin^2\theta} = \frac{6\left(1 + \frac{1}{\cos\theta} - \cos\theta - 1\right)}{\sin^2\theta}$$

$$= \frac{6\left(\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}\right)}{\sin^2\theta} = \frac{6\left(\frac{1-\cos^2\theta}{\cos\theta}\right)}{\frac{\sin^2\theta}{1}} = 6\left(\frac{\sin^2\theta}{\cos\theta}\right)\left(\frac{1}{\sin^2\theta}\right)$$

$$= 6\left(\frac{1}{\cos\theta}\right) = 6\sec\theta.$$

$$\left(\frac{1}{\sqrt{3}+2}\right)\left(\frac{\sqrt{3}-2}{\sqrt{3}-2}\right) = \frac{\sqrt{3}-2}{(\sqrt{3})^2-2^2} = \frac{\sqrt{3}-2}{3-4} = \frac{\sqrt{3}-2}{-1}$$

$$= -2+\sqrt{3}.$$

$$\left(\frac{2+i}{3-i}\right)\left(\frac{3+i}{3+i}\right) = \frac{6+2i+3i+i^2}{3^2-i^2}$$

$$= \frac{6+5i-1}{9-(-1)} = \frac{5+5i}{10} = \frac{1+i}{2} = \frac{1}{2}+\frac{1}{2}i.$$

Final Word on 2.2: For the "Graphing Utility" questions, I recommend Desmos.

When in doubt, you should be in Radians Mode. The wrench at the top right has those settings.

$\sin^2(83^\circ) + \sin^2(7^\circ) =$

$\sin^2(83^\circ) + \cos^2(83^\circ) = 1$

$\cos^2(90^\circ - 83^\circ) + \sin^2 7^\circ$

$= \cos^2(7^\circ) + \sin^2(7^\circ) = 1$

2.3 Stuff

I discovered a hole at the end of the 2.3 Notes and Videos.

I added notes, but there isn't a corresponding video.

So #s 46 - 50 that have no videos.

Section 2.3 Solving Trigonometric Equations

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$2\sin(\theta) = 1$

$(2\sin\theta - 1) = 0$

$(2\sin\theta - 1)(2\sin\theta + 1)$

SOLVE $= 4\sin^2\theta - 1 = 0$

$(2\sin\theta + 1)(2\sin\theta - 1) = 0 \longrightarrow$

$2\sin\theta + 1 = 0$ OR $2\sin\theta - 1 = 0 \longrightarrow$

$\sin\theta = -\frac{1}{2}$ $\quad$ $\sin\theta = \frac{1}{2}$

$\boxed{\text{On } [0, 2\pi) \text{ OR } [0, 360^\circ)}$ $\quad$ $0 \le \theta < 2\pi$ OR $0 \le \theta < 360^\circ$

$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$ Degrees

$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ Radians

---

$\mathbb{N} = \{1, 2, 3, \ldots\}$

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{R} = (-\infty, \infty)$

$\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0\}$

$\forall$ = "for all, for each, or for every, or for any"

$\nexists$ = DNE = Does not exist

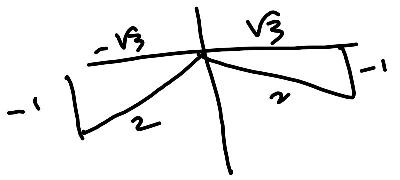
$\exists$ = "exists or there is or there exists"

$\longrightarrow$ = "implies."

Find all solutions in $[0,2\pi)$ **of**

$2\sin(2\theta)+1=0$

$\sin(2\theta) = -\frac{1}{2}$



$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

webAssign

To find all θ satisfying $\sin(2\theta) = -\frac{1}{2}$, $0 \le \theta < 2\pi$,

me:

Find all $\theta \in [0,2\pi)$

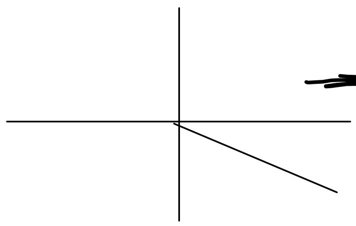
$2\theta \in [0,4\pi)$

$0 \le \theta < 2\pi \longrightarrow$

$0 \le 2\theta < 4\pi$

$2\theta = \frac{7\pi}{6}, \frac{7\pi}{6}+2\pi, \frac{11\pi}{6}, \frac{11\pi}{6}+2\pi$

$\longrightarrow \theta = \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{11\pi}{12}, \frac{23\pi}{12}$



All solutions in $[0, 2\pi)$

$4\sin^2(2\theta) - 1 = 0$

$\Rightarrow (2\sin(2\theta)+1)(2\sin(2\theta)-1) = 0$

$\Rightarrow \sin(2\theta) = -1$ OR $\sin(2\theta) = 1$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\sqrt{\frac{1}{4}}$

$= \pm\frac{1}{2}$

Find all $2\theta \in [0, 4\pi)$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ DONE

Find all $\theta \ni 4\sin^2(2\theta) - 1 = 0$

STEP 1 Find all $2\theta \in [0, 4\pi)$

That captures all sol'ns $\theta \in [0, 2\pi)$

Then add $2n\pi$ $\forall n \in \mathbb{Z}$ to capture the rest.

we have $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

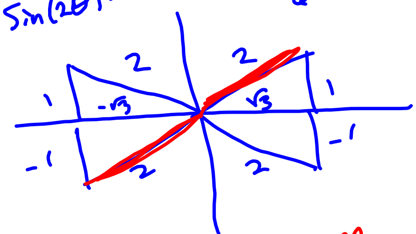
$\frac{\pi}{12} + 2n\pi, \frac{5\pi}{12} + 2n\pi, \frac{7\pi}{12} + 2n\pi, \ldots, \frac{23\pi}{12} + 2n\pi$

You can shorten it up by observing

that $\frac{\pi}{12} + \pi = \frac{13\pi}{12}$

$\frac{\pi}{6} + \pi = \frac{13\pi}{6}$

$\sin(2\theta) = \pm\frac{1}{2}$



Solve for all the 2θ's. Then divide by 2

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\theta = \frac{\pi}{6} + n\pi, \quad 2\theta = \frac{5\pi}{6} + n\pi$

$\boxed{\theta = \frac{\pi}{12} + \frac{n\pi}{2}, \quad \frac{5\pi}{12} + \frac{n\pi}{2}}$