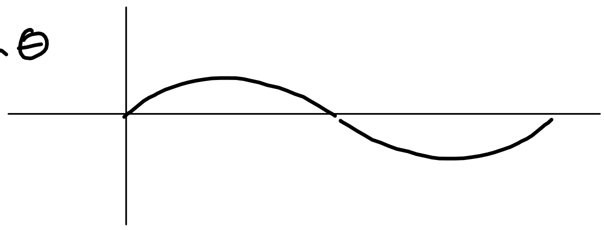
§2.1

Cofunction Identities

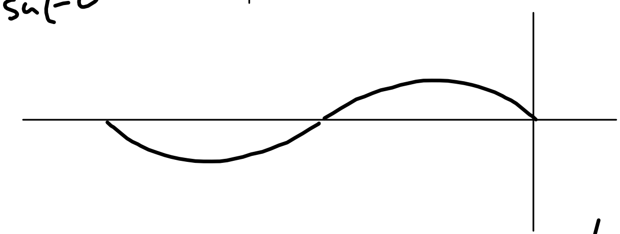
$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

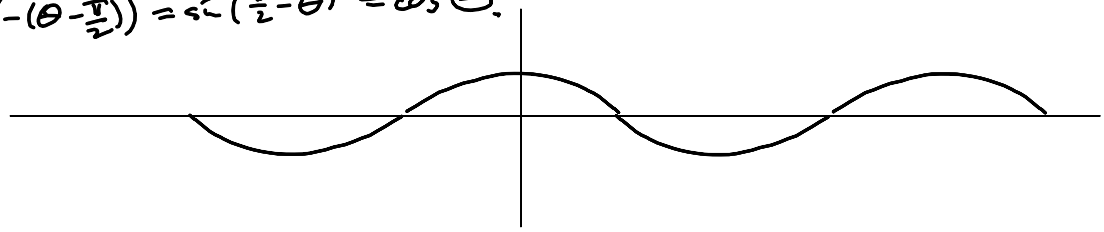
$\sin\theta$



$\sin(-\theta)$



$\sin\left(-\left(\theta-\frac{\pi}{2}\right)\right) = \sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$!



$\sin\left(\frac{\pi}{2}-\theta\right) = \sin\left(-\theta+\frac{\pi}{2}\right) = \sin\left(-\left(\theta-\frac{\pi}{2}\right)\right)$

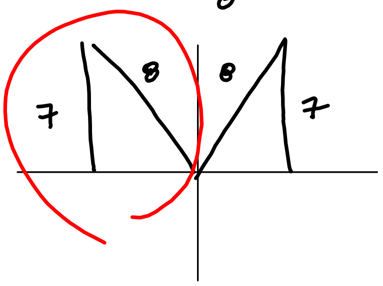
$\sin\theta \longmapsto \sin(-\theta) \longmapsto \sin\left(-\left(\theta-\frac{\pi}{2}\right)\right)$

$\theta \longmapsto -\theta \qquad \theta \longmapsto \theta+\frac{\pi}{2}$

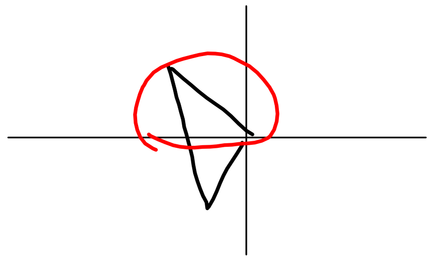
Reciprocal Identities

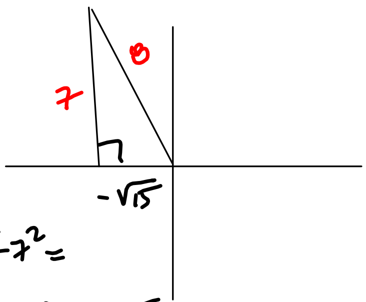
$$\sin\theta = \frac{1}{\csc\theta}$$

$\sin\theta = \frac{7}{8}$, $\cos\theta < 0 \longrightarrow$



$\cos\theta < 0$





$8^2 - 7^2 =$

$64 - 49 = 15$

$\leadsto \sqrt{15}$

$\sin\theta = \frac{7}{8}$

$\cos\theta = -\frac{\sqrt{15}}{8}$

$\tan\theta = -\frac{7}{\sqrt{15}}$

$\csc\theta = \frac{8}{7}$

$\sec\theta = -\frac{8}{\sqrt{15}}$

$\cot\theta = -\frac{\sqrt{15}}{7}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\cot^2\theta + 1 = \csc^2\theta$

$$\cot^2\theta + 1 = \frac{\cos^2\theta}{\sin^2\theta} + \frac{1}{1}\cdot\frac{\sin^2\theta}{\sin^2\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}$$

$$= \frac{1^2}{\sin^2\theta} = \left(\frac{1}{\sin\theta}\right)^2 = (\csc\theta)^2 = \csc^2\theta$$

$\sin^{-1}(\sim) = \arcsin(\sim)$

$\cancel{(x+1)(2x-5) = 2x^2-3x-5}$

FACTOR

$2\cos^2\theta - 3\cos\theta - 5$

$2u^2 - 3u - 5$ $\qquad (2)(-5) = -10$

$2 - 5 = -3$

$2u^2 + 2u - 5u - 5$

$= 2u(u+1) - 5(u+1)$

$= (u+1)(2u-5)$

$= (\cos\theta + 1)(2\cos\theta - 5)$

$60x^2 - 157x - 112$

$2u^2 - 3u - 5$

$a = 2, b = -3, c = -5$

$b^2 - 4ac = 3^2 - 4(2)(-5)$

$= 9 + 40$

$= 49 \rightsquigarrow \sqrt{49} = 7$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{3 \pm 7}{2(2)} = \dfrac{3 \pm 7}{4}$ $\qquad \dfrac{10}{4} = \dfrac{5}{2}$ $\qquad \dfrac{-4}{4} = -1$

FACTOR THEOREM $\quad x = c$ is a zero $\Rightarrow$ $x - c$ is a factor.

$f(x) = 2\left(x-\frac{5}{2}\right)(x-(-1))$

$= (2x-5)(x+1)$

$60x^2 - 157x - 112 = 0 \Rightarrow x = \frac{16}{5}, -\frac{7}{12}$

By Quadratic Formula

$60\left(x-\frac{16}{5}\right)\left(x+\frac{7}{12}\right)$

$5 \cdot 12\left(x-\frac{16}{5}\right)\left(x+\frac{7}{12}\right)$

$= (5x-16)(12x+7)$

$b^2 - 4ac$

$= 157^2 - 4(60)(-112)$

$= 51529 \rightarrow$

$\sqrt{51529} = 227$

$x = \frac{157 \pm 227}{2(60)}$, etc.

Sledgehammer Factoring

Factor $x^2 + 16$

$x^2 = -16$

$x = \pm\sqrt{-16} = \pm 4i$

$x^2 + 16 = (x-4i)(x+4i)$

$= x^2 - (-16)$

$= x^2 - (4i)^2$

$= (x-4i)(x+4i)$

$x^2 + 17 = x^2 - (-\sqrt{17}i)^2$

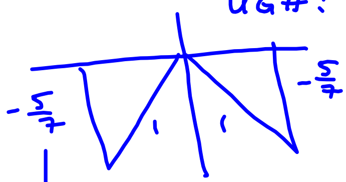
$= (x - \sqrt{17}i)(x + \sqrt{17}i)$

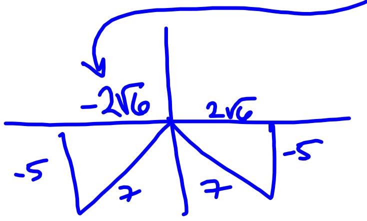
$\csc(x) = -\frac{7}{5}$ $\tan(x) > 0$

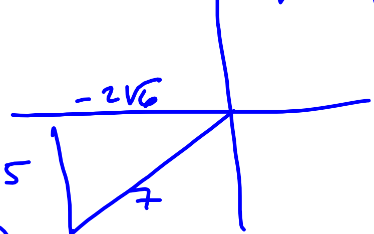
$\sin(x) = -\frac{5}{7}$

$\sin\theta = -\frac{5}{7}$

UGH!







$7^2 - (-5)^2$

$= 7^2 - 5^2 = 49 - 25 = 24$

$\sqrt{24} =$

$2 \mid 24$
$2 \mid 12$
$2 \mid 6$
3

$2\sqrt{2 \cdot 3} = 2\sqrt{6}$

$\sin\theta = -\frac{5}{7}$ $\csc\theta = -\frac{7}{5}$

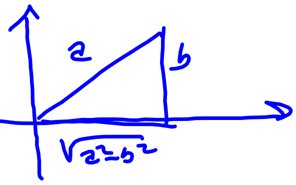
$\cos\theta = \frac{-2\sqrt{6}}{7}$ $\sec\theta = -\frac{7}{2\sqrt{6}}$

$\tan\theta = \frac{-5}{-2\sqrt{6}} = \frac{5}{2\sqrt{6}}$ $\cot\theta = \frac{2\sqrt{6}}{5}$

Trigonometric Substitution.

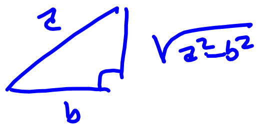
$\sqrt{81-x^2}$

$\sqrt{a^2-b^2}$



usually pick this.

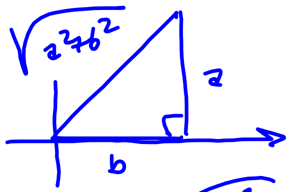
$\sin\theta = \frac{a}{b} \longrightarrow a = b\sin\theta$



$\frac{b}{a} = \cos\theta$

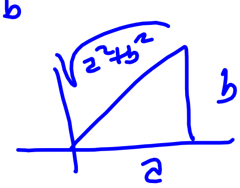
$b = a\cos\theta$

$\sqrt{a^2+b^2} \Rightarrow$

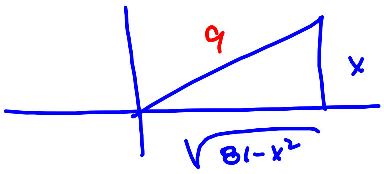


$\tan\theta = \frac{a}{b} \longrightarrow$

$b\tan\theta = a$

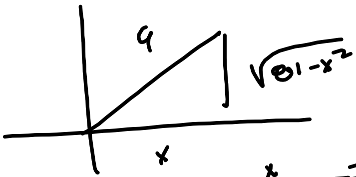


$\sqrt{81-x^2}$



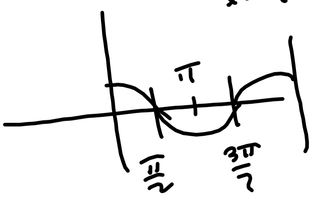
$\sin\theta = \frac{x}{9} \Rightarrow$

$9\sin\theta = x$



$\cos\theta = \frac{x}{9} \Rightarrow$

$x = 9\cos\theta$



$$\Rightarrow \sqrt{81-x^2}$$
$$= \sqrt{81-81\sin^2 x}$$
$$= \sqrt{81(1-\sin^2\theta)}$$
$$= \sqrt{81}\sqrt{1-\sin^2\theta}$$
$$= 9\sqrt{\cos^2\theta}$$
$$\boxed{= 9|\cos\theta|}$$

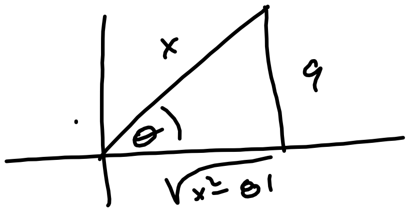
On $[0, 2\pi]$, this gives

$$9|\cos\theta| = \begin{cases} 9\cos\theta, & 0 \le \theta \le \frac{\pi}{2},\ \frac{3\pi}{2} \le \theta \le 2\pi \\ -9\cos\theta, & \frac{\pi}{2} < \theta < \frac{3\pi}{2} \end{cases}$$

Since the problem SAYS $0 < \theta < \frac{\pi}{2}$, we have

$9|\cos\theta| = 9\cos\theta$

$\sqrt{x^2-81}$

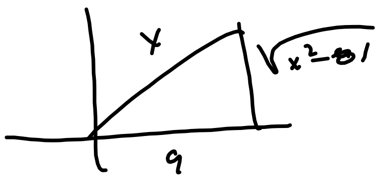


$\frac{9}{x} = \sin\theta$

$9 = x\sin\theta$

$x = \frac{9}{\sin\theta} = 9\csc\theta$

WebAssign Did this one



$\frac{9}{x} = \cos\theta \Rightarrow$

$\frac{9}{\cos\theta} = 9\sec\theta = x$