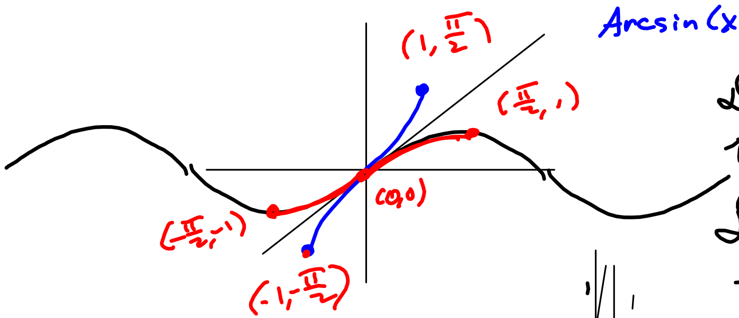
Inverse Functions in Book & Notes on harryzaims.com

Restricted sine function

Arcsin(x) = $\sin^{-1}(x)$

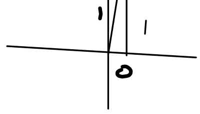


$\mathcal{D}(\text{sine}^*) = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\mathcal{R}(\text{sine}^*) = [-1, 1]$

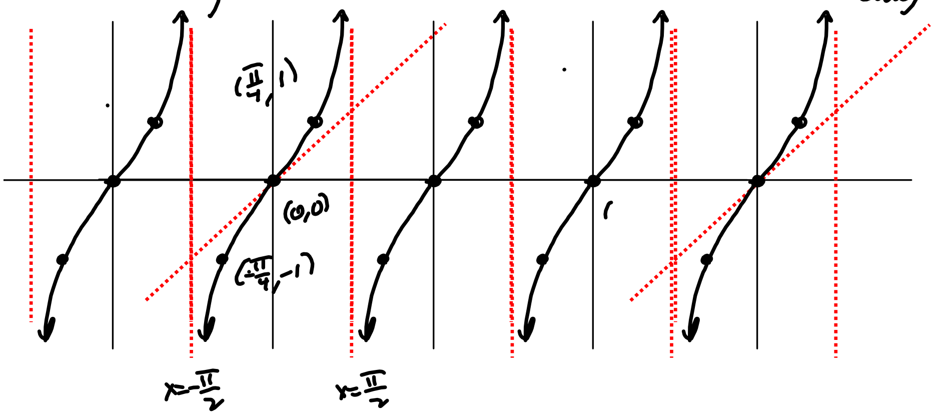
$\mathcal{D}(\text{arcsine}) = [-1, 1]$

$\mathcal{R}(\text{arcsine}) = [-\frac{\pi}{2}, \frac{\pi}{2}]$



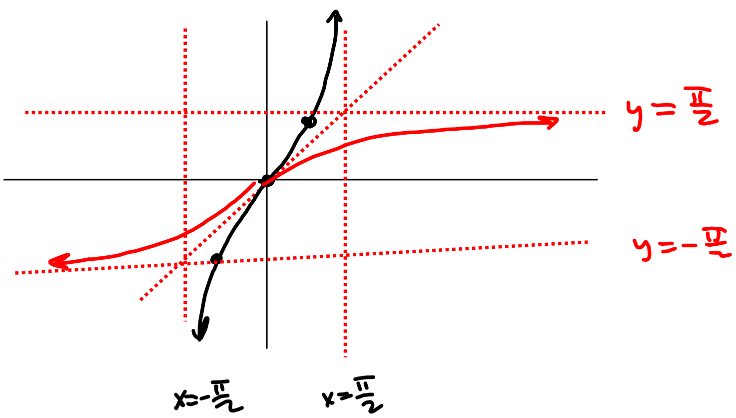
arcsine never "sees" anything beyond $-\frac{\pi}{2}$ or $\frac{\pi}{2}$

Tangent :



Restricted Tangent and its Inverse

arctan(x) = $\tan^{-1}(x)$



Restricted tangent

$\mathcal{D} = (-\frac{\pi}{2}, \frac{\pi}{2})$

$\mathcal{R} = (-\infty, \infty)$

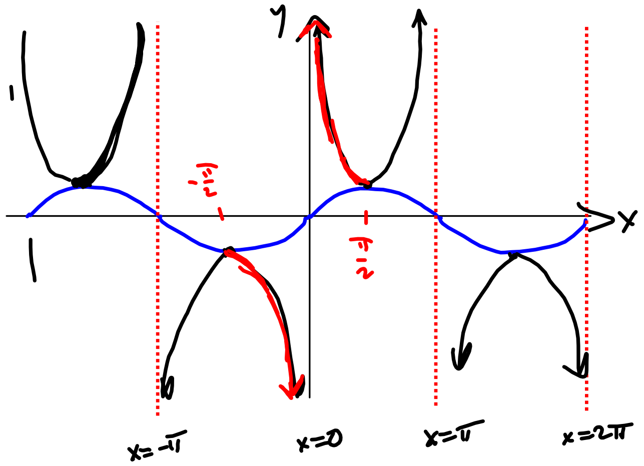
Arctangent

$\mathcal{D} = (-\infty, \infty)$

$\mathcal{R} = (-\frac{\pi}{2}, \frac{\pi}{2})$

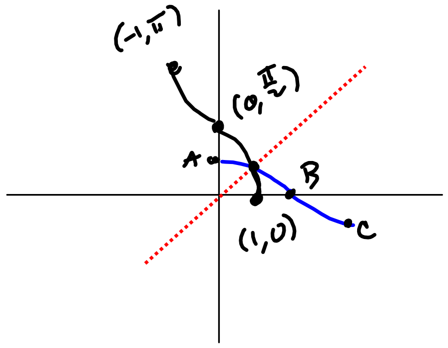
Cosecant = $\frac{1}{\text{sine}}$

cosecant, using sine as a guide.



Restricted Cosine & its INVERSE

arccos(x) = $\cos^{-1}(x)$



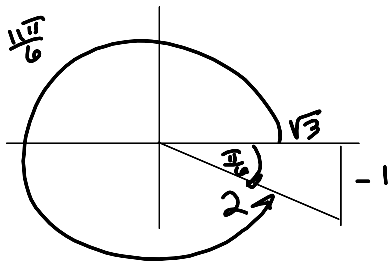
D(cosine*) = [0, π]

R(cosine) = [-1, 1]

D(arccosine) = [-1, -1]

R(arccosine) = [0, π]

$\arcsin\left(\sin\left(\frac{11\pi}{6}\right)\right) = \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6} = -30^\circ$



Reference angle $\frac{\pi}{6}$

$\arcsin\left(\frac{1}{2}\right) = -\frac{\pi}{6}$

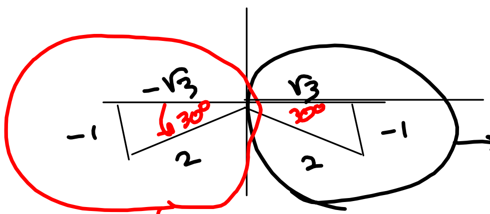
Your calculator sees $-\frac{\pi}{6}$ & your job is to interpret that.

$\sin\theta = -\frac{1}{2}$ assume $0 < \theta < 2\pi$

$\arcsin\left(-\frac{1}{2}\right) = -30^\circ \notin (0, 2\pi)$

$\rightsquigarrow 360^\circ - 30^\circ = \boxed{330^\circ}$

Where's the other one.



$\to \frac{11\pi}{6} = 330^\circ$ from -30° on calculator

This other one is $180^\circ + 30^\circ = 210^\circ$

$\theta = 330^\circ, 210^\circ$ OR

$= \frac{11\pi}{6}, \frac{7\pi}{6}$

Find ALL Sol'ns to $\sin\Theta = -\frac{1}{2}$ in radians

$x = \frac{11\pi}{6} + 2n\pi$ or $x = \frac{7\pi}{6} + 2n\pi \ \forall n \in \mathbb{Z}$

$= 330^\circ + 360^\circ n$ or $x = 210^\circ + 360^\circ n \ \forall n \in \mathbb{Z}$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\}$

$\forall$ = For all, for each, for every

$\exists$ = There is or there exists

$\Longrightarrow$ = Implies

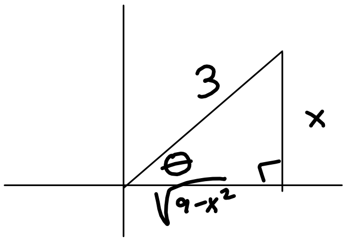
$\Longleftrightarrow$ = "if and only if" = iff

$\ni$ = such that, so that

I $\heartsuit$ u $\forall t \in \mathbb{R}$

Come up with algebraic expression

$\tan(\arcsin(\frac{x}{3})) = \tan\theta = \frac{x}{\sqrt{9-x^2}}$

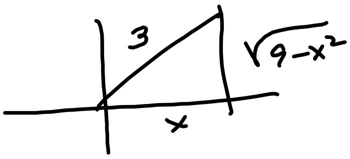


Assume $\theta \in (0, \frac{\pi}{2})$

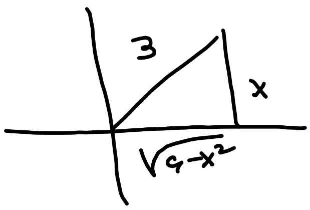
$\sqrt{3^2 - x^2}$

Calc II

$\int \frac{x\,dx}{\sqrt{9-x^2}}$



OR



Let $\frac{x}{3} = \sin\theta$

$x = 3\sin\theta$

$\sqrt{9-x^2}$

$= \sqrt{9 - 3^2\sin^2\theta}$

$= \sqrt{9(1-\sin^2\theta)}$

$= 3\sqrt{1-\sin^2\theta}$

$= 3\sqrt{\cos^2\theta}$

$= 3|\cos\theta|$

$\int \frac{3\sin\theta\,dx}{3|\cos\theta|}$

This is called Trig Substitution

$x = 3\sin\theta \Longrightarrow$

$dx = 3\cos\theta\,d\theta$

$\int \frac{3\sin\theta \cdot 3\cos\theta\,d\theta}{3|\cos\theta|}$

on $[0, \frac{\pi}{2}]$ $\cos\theta \geq 0$, so

$\int \frac{3\sin\theta\,\cancel{3}\,\cancel{\cos}\theta\,d\theta}{\cancel{3}\,\cancel{\cos}\theta}$