1. Arc Length and Area of Sector. Suppose we have a circle of radius $r = 7$.

   a. (5 pts) Find the arc length on the circle, that is intercepted by an angle of 1220^0. Round to 3 decimal places.

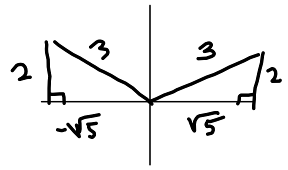
$$s = r\theta = 7(1220^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{427\pi}{9} \approx \boxed{149.051}$$

   b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = \frac{7\pi}{6}$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(7^2)\left(\frac{7\pi}{6}\right) = \boxed{\frac{343\pi}{12}} \approx 89.79719002$$

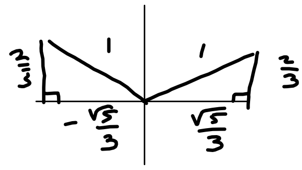
2. Answer the questions about the equation $\sin(\theta) = \frac{2}{3}$.

   a. (5 points) Sketch two triangles that satisfy $\sin(\theta) = \frac{2}{3}$.

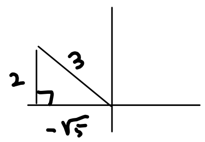


$$\sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$$

OR



   b. (5 pts) Assume the terminal side of the angle θ lies in the 2nd quadrant. Find the other five trigonometric functions of θ.



$\boxed{\sin\theta = \frac{2}{3}}$

$\boxed{\cos\theta = -\frac{\sqrt{5}}{3}}$

$\boxed{\tan\theta = -\frac{2}{\sqrt{5}} \text{ or } -\frac{2\sqrt{5}}{5}}$

$\boxed{\csc\theta = \frac{3}{2}}$

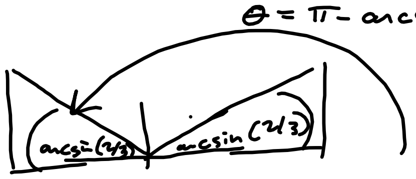
$\boxed{\sec\theta = -\frac{3}{\sqrt{5}} \text{ or } -\frac{3\sqrt{5}}{5}}$

$\boxed{\cot\theta = -\frac{\sqrt{5}}{2}}$

   c. (5 pts) Again, assuming θ's terminal side lies in Q II, and $0 \le \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.

$$\theta = \pi - \arcsin(2/3)$$

$$\theta = \pi - \arcsin(2/3) \approx 2.411864998 \approx \boxed{2.412 \approx \theta}$$



OR $\theta \approx 138.1896851^\circ \approx \boxed{138.190^\circ \approx \theta}$

   d. (5 pts) Give *all* solutions to the equation $\sin(\theta) = \frac{2}{3}$, in degrees *and* radians, rounded to three (3) decimal places.

$$\{\arcsin(2/3) + 2\pi n \mid n \in \mathbb{Z}\} \approx \{\overset{.730}{0.7297276563} + 2\pi n \mid n \in \mathbb{Z}\}$$

$$\approx \{.730 + 2\pi n \mid n \in \mathbb{Z}\} \cup \{2.412 + 2\pi n \mid n \in \mathbb{Z}\}$$

OR

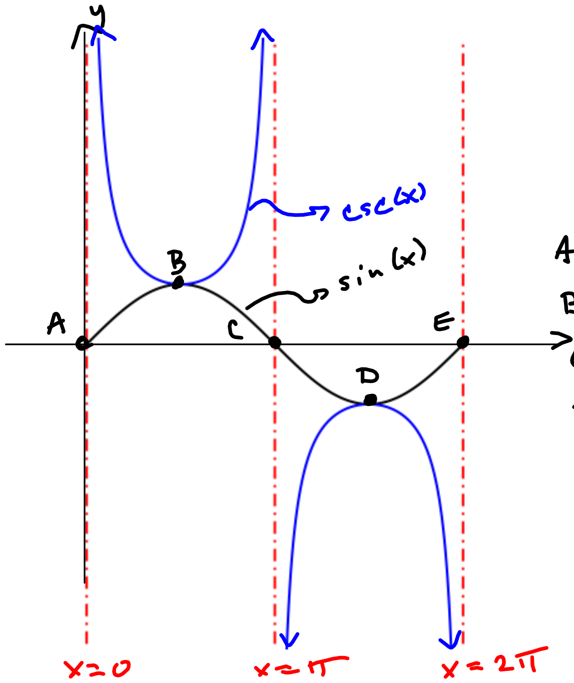
$$\{41.810^\circ + 360^\circ n \mid n \in \mathbb{Z}\} \cup \{138.190^\circ + 2\pi n \mid n \in \mathbb{Z}\}$$

Other ways of writing it:

$$\{x + 2\pi n \mid x = .730 \text{ or } 2.412,\ n \in \mathbb{Z}\}$$

$$\{x + 360^\circ n \mid x = 41.810^\circ \text{ or } 138.190^\circ,\ n \in \mathbb{Z}\}$$

3. (5 pts) Sketch one period of the graphs of $y = \sin(x)$ and $y = \csc(x)$ on the same set of coordinate axes.



$A = (0, 0)$

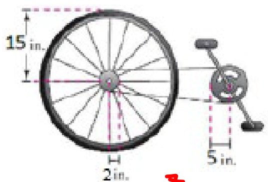
$B = (\frac{\pi}{2}, 1)$

$C = (\pi, 0)$

$D = (\frac{3\pi}{2}, -1)$

$E = (2\pi, 0)$

4. The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 5 inches, 2 inches and 15 inches, respectively. A cyclist is pedaling at a rate of 1.4 revolutions per second.



a. (5 pts) Find the speed of the bicycle in feet per second.

$$\left(\frac{1.4 \text{ rev pedal}}{\text{sec}}\right)\left(\frac{5 \text{ rev wheel}}{2 \text{ rev pedal}}\right)\left(\frac{2\pi}{1 \text{ rev}}\right)(15 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = \left(\frac{\cancel{14}^{\,7}}{\cancel{10}}\right)\left(\frac{\cancel{10}\pi}{\cancel{2}}\right)\left(\frac{\cancel{15}^{\,5}}{\cancel{12}_{\,4}}\right)$$

$$= \frac{35\pi}{4} \frac{\text{ft}}{\text{s}} \approx 27.48893572 \frac{\text{ft}}{\text{s}} \approx \boxed{27.5 \frac{\text{ft}}{\text{s}}}$$

b. (5 pts) Convert your answer, above, to miles per hour. Round final answers to 1 decimal place.

$$\left(\frac{35\pi}{4} \frac{\text{ft}}{\text{s}}\right)\left(\frac{60 \frac{\text{mi}}{\text{hr}}}{88 \frac{\text{ft}}{\text{s}}}\right) = \frac{525\pi}{88} \frac{\text{mi}}{\text{hr}} \approx 18.74245617 \approx \boxed{18.7 \frac{\text{mi}}{\text{hr}}}$$

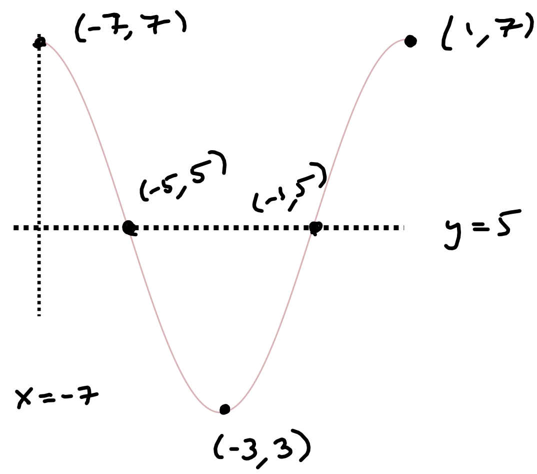
5. (5 pts) Sketch the graph of $f(x) = 2\cos\left(\frac{\pi}{4}x + \frac{7\pi}{4}\right) + 5$. $= 2\cos\left(\frac{\pi}{4}(x+7)\right) + 5$

$\frac{\pi}{4}x = 2\pi \Rightarrow$

$x = 8$ = Period

Amp: = 2

start : $x = -7$



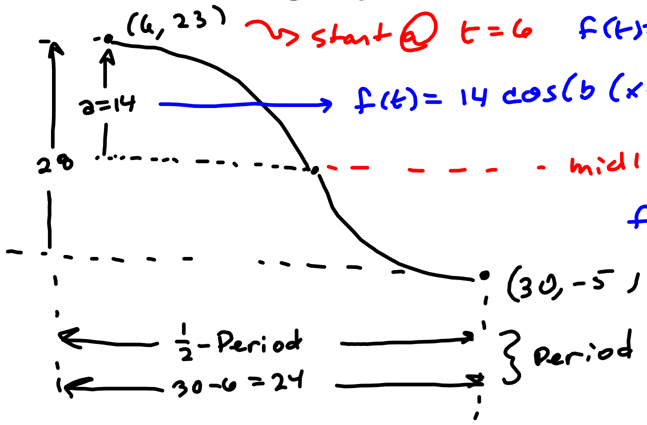
6. (5 pts) Write the cosine function that achieves its maximum height of $y = 23$ centimeters at time $t = 6$ seconds and its minimum height of $y = -5$ centimeters at $t = 30$ seconds.

start @ $t = 6$ $f(t) = a\sin(b(x-6)) + d$

$f(t) = 14\cos(b(x-6)) + d$

midline : $y = \frac{23 + -5}{2} = \frac{18}{2} = 9$

$f(t) = 14\cos(b(x-6)) + 9$



Period = (2)(24) = 48 seconds

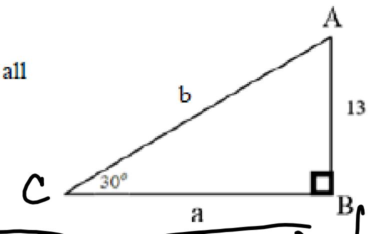
$bx = 2\pi$ when $t = 48$

$48b = 2\pi \Rightarrow$

$b = \frac{2\pi}{48} = \frac{\pi}{24}$

$$\boxed{f(t) = 14\cos\left(\frac{\pi}{24}(x-6)\right) + 9}$$

7. (5 pts) Solve the triangle in the figure on the right. That means, find all lengths and angles. Exact answers required.

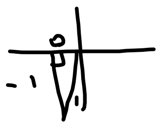


$\frac{13}{b} = \sin(30^\circ) = \frac{1}{2} \Rightarrow$

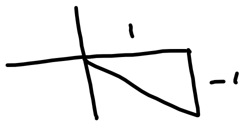
$\boxed{26 = b} \Rightarrow a = \sqrt{26^2 - 13^2} = \sqrt{2^2 \cdot 13^2 - 13^2} = \sqrt{13^2(2^2-1)} = \boxed{13\sqrt{3} = a}$

$\boxed{B = 90^\circ}$ $\boxed{A = 60^\circ}$

8. (5 pts) Find the exact value of $\arctan\left(\sin\left(\frac{3\pi}{2}\right)\right)$ $= \arctan\theta$



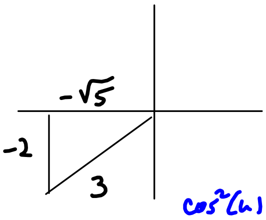
$= \arctan(-1) = \boxed{-\frac{\pi}{4}} = \boxed{-45^\circ}$



9. (5 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u) = -\frac{2}{3}$ and $\pi \le u < \frac{3\pi}{2}$. $\longrightarrow$

$\sin(u) = -\frac{2}{3}$, $u \in Q\,III$

$\frac{\pi}{2} < \frac{u}{2} < \frac{3\pi}{4} \Rightarrow \frac{u}{2} \in Q\,II$



$\sqrt{9-4} = \sqrt{5}$ $\longrightarrow \boxed{\cos(u) = -\frac{\sqrt{5}}{3}} \longrightarrow$ $\longrightarrow (-,+)$

Alternate:

$\cos^2(u) = 1 - \sin^2(u) = 1 - \left(-\frac{2}{3}\right)^2 = \frac{9-4}{9} = \frac{5}{9} \Rightarrow \cos(u) = -\frac{\sqrt{5}}{3}$ $(Q\,III)$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1+\frac{\sqrt{5}}{3}}{2}} = \sqrt{\frac{3+\sqrt{5}}{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \boxed{\frac{\sqrt{18+6\sqrt{5}}}{6} = \sin\left(\frac{u}{2}\right)}$$

$$= \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}.$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1-\frac{\sqrt{5}}{3}}{2}} = \boxed{-\frac{\sqrt{18-6\sqrt{5}}}{6} = \cos\left(\frac{u}{2}\right)}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u} = \frac{-\frac{2}{3}}{1-\frac{\sqrt{5}}{3}} = \frac{-2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = -\frac{6-2\sqrt{5}}{4} = \frac{-3-\sqrt{5}}{2}$$

OR

$$\frac{1-\cos(u)}{\sin(u)} = \frac{1+\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = \frac{3+\sqrt{5}}{-2} = \frac{-3-\sqrt{5}}{2}$$

OR

$$\tan\left(\frac{u}{2}\right) = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)} = \frac{\frac{\sqrt{18+6\sqrt{5}}}{6}}{-\frac{\sqrt{18-6\sqrt{5}}}{6}} = -\sqrt{\frac{18+6\sqrt{5}}{18-6\sqrt{5}}}$$

$$= -\sqrt{\frac{(18+6\sqrt{5})^2}{(18-6\sqrt{5})(18+6\sqrt{5})}} = -\frac{18+6\sqrt{5}}{\sqrt{18^2-36\cdot 5}} = -\frac{18+6\sqrt{5}}{\sqrt{6^2 3^2 - 6^2 5}}$$

$$= -\frac{18+6\sqrt{5}}{6\sqrt{9-5}} = -\frac{3+\sqrt{5}}{\sqrt{4}} = -\frac{3+\sqrt{5}}{2}$$

$$= \boxed{\frac{-3-\sqrt{5}}{2} = \tan\left(\frac{u}{2}\right)}$$

10. Consider the equation $4\sin^2(x)-3=0$.

a. (5 pts) Find all solutions x, in radians *and* degrees, to the equation in the interval $[0,2\pi)$.

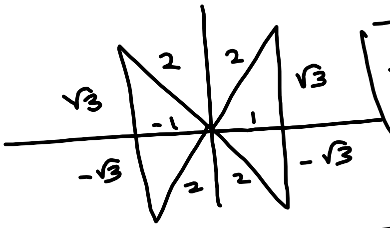
b. (5 pts) Find all real solutions x, in radians *and* degrees.

a

$\Rightarrow 4\sin^2(x)=3$

$\Rightarrow \sin^2(x)=\frac{3}{4}$

$\Rightarrow \sin(x)=\pm\frac{\sqrt{3}}{2}$



$$\boxed{x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}}$$

OR $x\in\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}\right\}$

$$\boxed{x\in\{60^\circ,120^\circ,240^\circ,300^\circ\}}$$

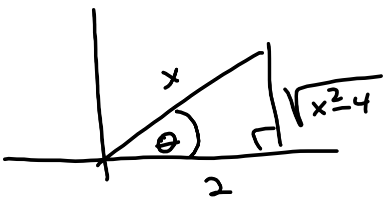
b $x\in\left\{x+n\pi \;\middle|\; x\in\left\{\frac{\pi}{3},\frac{2\pi}{3}\right\},\ n\in\mathbb{Z}\right\}$

OR

$x\in\left\{x+360^\circ n \;\middle|\; x\in\{60^\circ,120^\circ\},\ n\in\mathbb{Z}\right\}$

11. (5 pts) Draw the picture and use it to re-write $\sin\left(\cos^{-1}\left(\frac{2}{x}\right)\right)$ as an algebraic expression.

$=\sin\theta$



$$\Rightarrow \boxed{\sin\theta=\frac{\sqrt{x^2-4}}{x}}$$

12. Find the *exact* value of $\cos\left(\frac{5\pi}{12}\right)$ in two ways: (Hint: If degrees are easier for you, *use degrees*.)

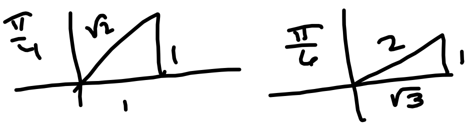
a. (5 pts) Use a Sum identity.

a $5=4+1=3+2$ ✓

$\cos\left(\frac{5\pi}{12}\right)=\cos\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)=\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$=\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)-\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\cdot\frac{1}{2}=\boxed{\frac{\sqrt{3}-1}{2\sqrt{2}} \text{ OR } \frac{\sqrt{6}-\sqrt{2}}{4}}$

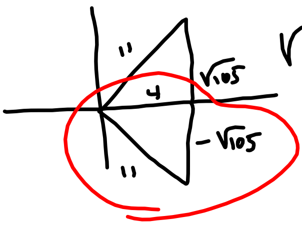


b. (5 pts) Use a Half-Angle identity

$\frac{5\pi}{12}=\frac{1}{2}\cdot\frac{5\pi}{6}=\frac{1}{2}u$, where $u=\frac{5\pi}{6}$

$\cos\left(\frac{u}{2}\right)=\sqrt{\frac{1+\cos(u)}{2}}=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}=\sqrt{\frac{2-\sqrt{3}}{4}}=\boxed{\frac{\sqrt{2-\sqrt{3}}}{2}=\cos\left(\frac{5\pi}{12}\right)}$

13. (5 pts) Find $\sin(2u)$, $\cos(2u)$ and $\tan(2u)$, given that $\cos(u) = \frac{4}{11}$ and $\sin(u) < 0$.



$\sqrt{11^2 - 4^2} = \sqrt{121 - 16} = \sqrt{105}$

$3 \,\underline{|\, 105}$
$5 \,\underline{|\, 35}$
7

-0.6774843484

$\sin(2u) = 2\sin u \cos u = 2\left(\frac{-\sqrt{105}}{11}\right)\left(\frac{4}{11}\right) = \boxed{\frac{-8\sqrt{105}}{121} = \sin(2u)}$

$\cos(2u) = 2\cos^2(u) - 1 = 2\left(\frac{4}{11}\right)^2 - 1 = \frac{32}{121} - \frac{121}{121} = \boxed{-\frac{89}{121} = \cos(2u)}$

-0.7355371903

$\tan(2u) = \dfrac{\frac{-8\sqrt{105}}{121}}{\frac{-89}{121}} = \boxed{\frac{8\sqrt{105}}{89} = \tan(2u)}$

0.9210742255

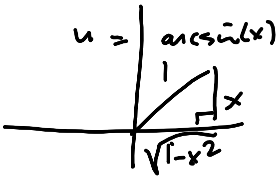
14. BONUS (5 pts) Re-write $\cos(\arcsin(x) + \arccos(x))$ as an algebraic expression. (Hint: Use Sum identity.)

$= \cos(u + v) = \cos u \cos v - \sin u \sin v$

$= \left(\sqrt{1-x^2}\right)(x) - x\sqrt{1-x^2}$

$= \boxed{0}!$

$u = \arcsin(x)$



$v = \arccos(x)$