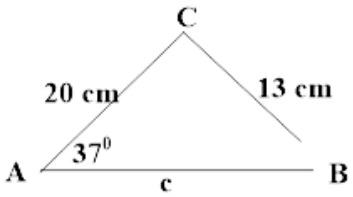
MAT 1420-R11 Written Final, Spring, 2024 Name_____
200 Points Emphasizes Chapters 3, 4, and 6.

Show All Work. Circle Final Answers.

1. (10 pts) Find the value of the 6 trigonometric functions, given that $\sin(u) = -\frac{3}{4}$ and $\frac{3\pi}{2} \le u < 2\pi$.

2. (10 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\sin(u) = -\frac{3}{4}$ and $\frac{3\pi}{2} \le u < 2\pi$.

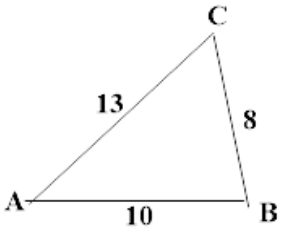
3. Suppose $A = 37^0$, $a = 13$ cm, and b = 20 cm. (See figure.)



   a. (5 pts) Show that there are two solutions to this triangle, before solving the triangle.

   b. (5 pts) Find both solutions. For one, *B* will be acute. For the other, *B* will be obtuse. Round final answers to 3 decimal places.

4. (10 pts) Suppose $a = 8$, $b = 13$, $c = 10$. Solve the triangle. Round final answers to 3 decimal places. See figure.



5. Let $\overline{u} = \langle 7,1 \rangle$ and $\overline{v} = \langle 2,6 \rangle$. Compute or sketch, as directed:

   a. (10 pts) $\|\overline{u}\|$ (Use dot product).

   b. (5 pts) The direction angle for $\overline{u}$. Use degrees. Round final answer to 3 decimal places.

   c. (5 pts) The angle between $\overline{u}$ and $\overline{v}$. Use degrees. Round final answer to 3 decimal places.

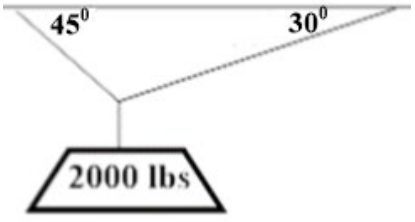
   d. (5 pts) The resultant of $\overline{u}$ and $\overline{v}$.

   e. (5 pts) $\text{proj}_{\overline{v}}(\overline{u})$.

   f. (5 pts) Sketch the vectors $\overline{u}$, $\overline{v}$, and $\overline{u} + \overline{v}$ on the same set of coordinate axes.

   g. (5 pts) Sketch the vectors $\overline{u}$, $\overline{v}$, $\text{proj}_{\overline{v}}(\overline{u})$, and $\overline{u} - \text{proj}_{\overline{v}}(\overline{u})$ on the same set of coordinate axes. Don't over-write your sketch from part f. Start a new sketch,

6. (10 pts) A weight of 2000 lbs is suspended from two hooks in the ceiling. Find the tension in each cable. (See figure.) Round final answers to 3 decimal places.



7. (10 pts) Use the fact that $z = 1 + 2i$ is a zero of

$$f(x) = x^4 - x^3 - 3x^2 + 17x - 30$$

to obtain a linear factorization of $f(x)$, that is, split f into linear factors by finding all its zeros.

8. Let $z = 3\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$ and $w = 7\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$. Compute the following.

   a. (10 pts) zw

   b. (10 pts) $\frac{z}{w}$

9. Let $z = 81i$.

   a. (10 pts) Convert z to trigonometric form.

   b. (10 pts) Find all 5th roots of z. Leave your answers in trigonometric form. Use exact radians (i.e., π-radians).

   c. (10 pts) Convert $z = 3\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$ to standard (rectangular) form.

10. Consider the following:

$$x = 2 + 3\cos(\theta)$$
$$y = 3 - 5\sin(\theta)$$

   a. (10 pts) Sketch the curve represented by the parametric equations. Indicate the orientation of the curve.

   b. (10 pts) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

11. (10 pts) Convert the equation $2x - 3y - 15 = 0$ to polar form. Solve your equation for r so that it is of the form $r = f(\theta)$.

12. (10 pts) Sketch the graph of the polar equation $r = \sin(3\theta)$

13. (10 pts) Find the polar equation of the ellipse with vertices $\left(2, \frac{\pi}{2}\right), \left(4, \frac{3\pi}{2}\right)$.

14. (Bonus 10 pts) Convert the equation $\frac{(x-4)^2}{25} + \frac{y^2}{9} = 1$ to polar form of a conic $r = \frac{ep}{1 - e\cos(\theta)}$.