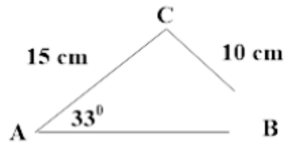
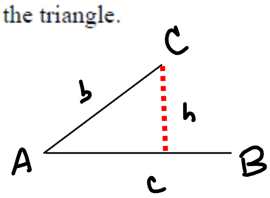
1. Ch 3 Suppose $A = 33^0$, $a = 10$ cm, and $b = 15$ cm. (See figure.)



a. (5 pts) Show that there are two solutions to this triangle, before solving the triangle.



$\frac{h}{15} = \sin\theta$

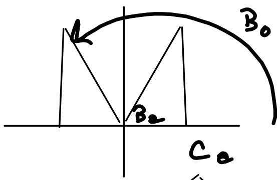
$h = 15\sin 33^0 \approx 8.169585525 < 10 = a < 15 = b$

$\Longrightarrow$ 2 solns.

b. (5 pts) Find both solutions. For one, B will be acute. For the other, B will be obtuse. Round final answers to 3 decimal places.

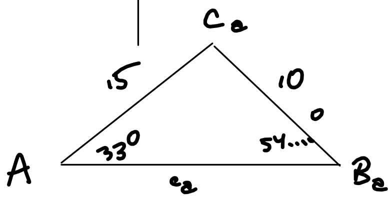
$\frac{\sin B}{b} = \frac{\sin A}{a} \Longrightarrow \sin B = \frac{b \sin A}{a} = \frac{15 \sin 33^0}{10} \approx 0.8169585525$

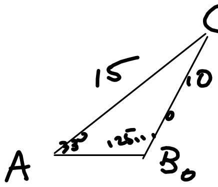
$\approx$



B_a = B acute" $\approx \arcsin(.8169585525) \approx \boxed{54.781^0 \approx B_a}$

"B_o = B obtuse" $= 180^0 - B_a \approx \boxed{125.219^0 \approx B_o}$





C_a, c_a corresponds to B_a

C_o, c_o corresponds to B_o solution.

$C_a = 180^0 - A - B_a = 180^0 - 33^0 - 54...^0$

$\approx 92.21851802^0 \approx \boxed{92.219^0 \approx C_a}$

$c_a \approx \sqrt{a^2 + b^2 - 2ab\cos(C_a)}$

$\approx \sqrt{10^2 + 15^2 - 2(10)(15)\cos(92...^0)}$

$\approx 18.34702240 \approx \boxed{18.347 \approx c_a}$

$C_o = 180^0 - A - B_o$

$= 180^0 - 33^0 - 125.211...^0$

$\approx \boxed{21.781^0 \approx C_a}$

$c_o = \sqrt{a^2 + b^2 - 2ab\cos(C_a)}$

$\approx \boxed{6.813 \approx c_o}$

$c_a = \frac{a \sin(C_a)}{\sin(A)} = \frac{10 \sin(92.2...^0)}{\sin(33^0)} \approx 18.34702240 \approx \boxed{18.347 \approx c_a}$

2. Ch 3 Suppose $a = 10$, $b = 15$, $c = 20$. Solve the triangle. Round final answers to 3 decimal places. See figure.

$a^2 = b^2 + c^2 - 2bc\cos A = c^2$ **The one thing I have memorized.**

$-2bc\cos A = a^2 - b^2 - c^2$

$\cos A = \dfrac{a^2 - b^2 - c^2}{-2bc} = \dfrac{b^2 + c^2 - a^2}{2bc}$ Use this.

$b^2 + c^2 - a^2$

$= \dfrac{15^2 + 20^2 - 10^2}{2(15)(20)} = \dfrac{225 + 400 - 100}{600} = \dfrac{525}{600} \Rightarrow A = \arccos\left(\frac{525}{600}\right)$

$= \arccos\left(\frac{7}{8}\right) \approx 0.5053605103 \approx \boxed{A \approx .505}$

OR

OR $28.95502437^\circ \approx \boxed{28.955^\circ \approx A}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{10^2 + 15^2 - 20^2}{2(10)(15)} = \dfrac{100 + 225 - 400}{300} = \dfrac{-75}{300} = -\frac{1}{4}$

$\Rightarrow C = \arccos\left(-\frac{1}{4}\right) \approx 1.823476582 \approx \boxed{C \approx 1.823}$

OR

OR $104.4775122^\circ \approx \boxed{C \approx 104.478^\circ}$

$\Rightarrow B = 180^\circ - A - C \approx 46.56746347^\circ \approx \boxed{B \approx 46.567^\circ}$

OR $B = \pi - A - C \approx 0.812755562 \approx \boxed{B \approx 0.813}$

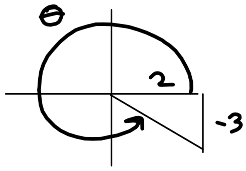
3. Ch 3 Let $\overline{u} = \langle 2, -3 \rangle$ and $\overline{v} = \langle -1, 5 \rangle$. Compute or sketch, as directed:

a. (5 pts) $\|\overline{u}\|$ (Use dot product).

$$\|\overline{u}\| = \sqrt{\overline{u} \cdot \overline{u}} = \sqrt{2^2+3^2} = \boxed{\sqrt{13} = \|\overline{u}\|}$$

b. (5 pts) The direction angle for $\overline{u}$. Use degrees. Round final answer to 3 decimal places.

$\arctan\left(-\frac{3}{2}\right) \approx -56.30993247^\circ$



$$\theta = 360^\circ - 56.3\ldots^\circ \approx \boxed{303.690^\circ \approx \theta}$$

c. (5 pts) The angle between $\overline{u}$ and $\overline{v}$. Use degrees. Round final answer to 3 decimal places.

Let θ be " " " " " ". Then

$$\cos\theta = \frac{\overline{u} \cdot \overline{v}}{\|\overline{u}\|\|\overline{v}\|} = \frac{\langle 2,-3 \rangle \cdot \langle -1,5 \rangle}{\sqrt{13}\sqrt{1^2+5^2}} = \frac{-2-15}{\sqrt{13}\sqrt{26}} = \frac{-17}{13\sqrt{2}}$$

$$\approx -0.9246780985 \Longrightarrow \theta \approx 157.6198649^\circ \approx \boxed{157.620^\circ \approx \theta}$$

d. (5 pts) The resultant of $\overline{u}$ and $\overline{v}$.

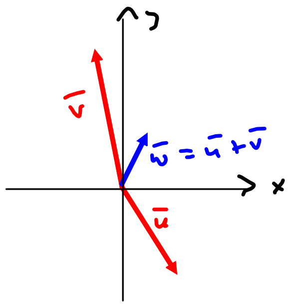
$$\overline{u} + \overline{v} = \langle 2,-3 \rangle + \langle -1,5 \rangle = \boxed{\langle 1,2 \rangle = \overline{w}}$$

e. (5 pts) $\operatorname{proj}_{\overline{v}}(\overline{u})$. $= \frac{\overline{u}\cdot\overline{v}}{\|\overline{v}\|^2}\overline{v} = \frac{-17}{\sqrt{26}^2}\langle -1,5\rangle =$

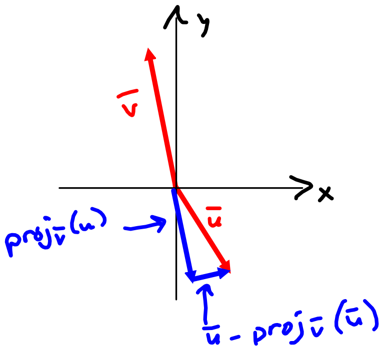
$= -\frac{17}{26}\langle -1,5\rangle = \langle \frac{17}{26}, \frac{-85}{26}\rangle = \operatorname{proj}_{\overline{v}}(\overline{u})$

f. (5 pts) Sketch the vectors $\overline{u}$, $\overline{v}$, and $\overline{u}+\overline{v}$ on the same set of coordinate axes.

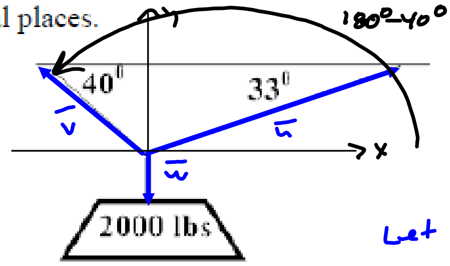
$\overline{u} = \langle 2,-3\rangle$, $\overline{v} = \langle -1,5\rangle$



g. (5 pts) Sketch the vectors $\overline{u}$, $\overline{v}$, $\operatorname{proj}_{\overline{v}}(\overline{u})$, and $\overline{u}-\operatorname{proj}_{\overline{v}}(\overline{u})$ on the same set of coordinate axes. Don't over-write your sketch from part f. Start a new sketch.



4. Ch 3 (15 pts) A weight of 2000 lbs is suspended from two hooks in the ceiling. Find the tension in each cable. (See figure.) Round final answers to 3 decimal places.



$\bar{u} + \bar{v} = -\bar{w} = \langle 0, 2000 \rangle$

$\bar{u} = \|\bar{u}\| \langle \cos(33^\circ), \sin(33^\circ) \rangle$

$\bar{v} = \|\bar{v}\| \langle \cos(140^\circ), \sin(140^\circ) \rangle$

Let $x = \|\bar{u}\|$, $a = \cos 33^\circ$, $b = \sin(33^\circ)$

$y = \|\bar{v}\|$, $c = \cos(140^\circ)$, $d = \sin(140^\circ)$

Then $x\langle a, b\rangle + y\langle c, d\rangle = \langle 0, 2000\rangle \longrightarrow$

$ax + cy = 0 \longrightarrow cy = -ax \longrightarrow y = -\frac{ax}{c}$

$bx + dy = 2000 \longrightarrow$

$bx + d\left(-\frac{ax}{c}\right) = \left(\frac{bc - ad}{c}\right)x = 2000$

$\longrightarrow x = \frac{2000c}{bc - ad} = \frac{2000(\cos(140^\circ))}{\sin(33^\circ)\cos(140^\circ) - \cos(33^\circ)(\sin(140^\circ))}$

≈ 1602.092719 lbs $\approx \|\bar{u}\| \approx \boxed{1602.093 \text{ lbs}}$

$y = \|\bar{v}\| = -\frac{ax}{c} \approx -\frac{\cos(33^\circ)(1602.092719)}{\cos(140^\circ)}$

$\approx 1753.981799 \approx \|\bar{v}\| \approx \boxed{1753.982 \text{ lbs}}$

Check:

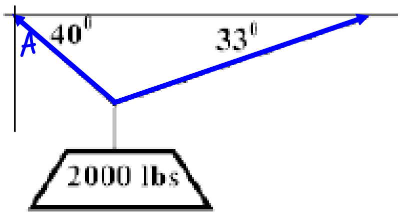
$1602.093\langle \cos(33^\circ), \sin(33^\circ)\rangle + 1753\langle \cos(140^\circ), \sin(140^\circ)\rangle$

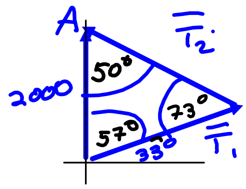
$\approx \langle 0, 2000 \rangle$ ✓

The more elegant way, using Law of Sines, courtesy of Sven Lunas

4. Ch 3 (15 pts) A weight of 2000 lbs is suspended from two hooks in the ceiling. Find the tension in each cable. (See figure.) Round final answers to 3 decimal places.

$A = 50^\circ$
(complement)





$$\frac{\|\vec{T}_1\|}{\sin(50^\circ)} = \frac{2000}{\sin(73^\circ)}$$

$$\|\vec{T}_1\| = \frac{2000\sin(50^\circ)}{\sin(73^\circ)} \approx 1602.092719$$

$$\|\vec{T}_2\| = \frac{2000\sin(57^\circ)}{\sin(73^\circ)} \approx 1753.981799$$

Ch 4 (10 pts) Use the fact that $z = 2 + 3i$ is a zero of $f(x) = x^4 - 9x^3 + 19x^2 - 9x - 182$ to obtain a linear factorization of $f(x)$, that is, split f into linear factors by finding all its zeros.

Book Way (Clever)

$$\boxed{(x-(a+bi))(x-(a-bi))}$$
$$= x^2 - x(a-bi) - x(a+bi) + (a^2+b^2)$$
$$= \boxed{x^2 - 2ax + a^2 + b^2 \text{ is HANDY}}$$

we divide by $(x-(2+3i))(x-(2-3i))$
$$= x^2 - 2(2)x + 2^2 + 3^2$$
$$= x^2 - 4x + 13$$

Keeps it real

$$\begin{array}{r|l} & x^2 - 5x - 14 + 0 \\ x^2-4x+13 & x^4 - 9x^3 + 19x^2 - 9x - 182 \\ & -(x^4 - 4x^3 + 13x^2) \\ & -5x^3 + 6x^2 - 9x - 182 \\ & -(-5x^3 + 20x^2 - 65x) \\ & -14x^2 - 56x - 182 \\ & -(-14x^2 - 56x - 182) \\ & 0 \end{array}$$

$x^2 - 5x - 14$
$= (x-7)(x+2)$, so

$$\boxed{f(x) = (x-(2+3i))(x-(2-3i))(x-7)(x+2)}$$

My way: (Brutish)

$$\begin{array}{r|rrrrr} 2+3i & 1 & -9 & 19 & -9 & -182 \\ & & 2+3i & -23-15i & 37-42i & 182 \\ \hline 2-3i & 1 & -7+3i & -4-15i & 28-42i & 0 \\ & & 2-3i & -10+15i & -28+42i & \\ \hline & 1 & -5 & -14 & 0 & \end{array}$$

Messes with imaginaries.

$x^2 - 5x - 14 = (x-7)(x+2) \longrightarrow$

$$\boxed{f(x) = (x-(2+3i))(x-(2-3i))(x-7)(x+2)}$$

scratch!
$(-7+3i)(2+3i) = -14 - 21i + 6i + (3i)^2 = -14 - 15i - 9 = -23 - 15i$
$(-4-15i)(2+3i) = -8 - 12i - 30i + 45 = 37 - 42i$
$14(2-3i)(2+3i) = 14(2^2+3^2) = 14(13) = 196 - 14 = 182$

6. Ch 4 Let $z = 3\left(\cos\left(33^0\right) + i\sin\left(33^0\right)\right)$ and $w = 7\left(\cos\left(11^0\right) + i\sin\left(11^0\right)\right)$. Compute the following.

a. (5 pts) $\boxed{zw = 21\left(\cos(44^0) + i\sin(44^0)\right)}$

b. (5 ps) $\boxed{\frac{z}{w} = \frac{3}{7}\left(\cos(22^0) + i\sin(22^0)\right)}$

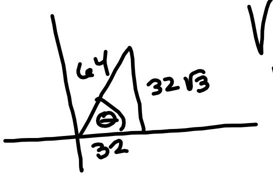
7. Ch 4 Let $z = 32 + 32i\sqrt{3}$.

a. (10 pts) Find all 5th roots of z. Leave your answers in trigonometric form. Use exact radians (i.e., π - radians).

$z = 32(1 + \sqrt{3}i)$

$|z| = 64$

$\theta = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3} = 60^0$



$\sqrt{(32\sqrt{3})^2 + (32)^2}$

$\sqrt{32^2\sqrt{3}^2 + 32^2}$

$= 32\sqrt{3+1}$

$= 32\sqrt{4} = 32 \cdot 2 = 64$

$\Rightarrow z = 64\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

$k=0$: $\sqrt[5]{z} = \sqrt[5]{64}\left(\cos\left(\frac{\pi}{15}\right) + i\sin\left(\frac{\pi}{15}\right)\right)$

$= \boxed{2\sqrt[5]{2}\left(\cos\left(\frac{\pi}{15}\right) + i\sin\left(\frac{\pi}{15}\right)\right)}$ via $\frac{\frac{\pi}{3}}{5} = \frac{\pi}{15}$

$k=1$: $\text{Inc} = \frac{2\pi}{5} = \frac{6\pi}{15} \rightarrow \frac{\pi + 6\pi}{15} = \frac{7\pi}{15} \Rightarrow \boxed{2\sqrt[5]{2}\left(\cos\left(\frac{7\pi}{15}\right) + i\sin\left(\frac{7\pi}{15}\right)\right)}$

$k=2$: $7+6=13 \rightarrow \boxed{2\sqrt[5]{2}\left(\cos\left(\frac{13\pi}{15}\right) + i\sin\left(\frac{13\pi}{15}\right)\right)}$

$k=3$: $13+6=19 \Rightarrow \boxed{2\sqrt[5]{2}\left(\cos\left(\frac{19\pi}{15}\right) + i\sin\left(\frac{19\pi}{15}\right)\right)}$

$k=4$: $19+6=25 \rightarrow \frac{25}{15} = \frac{5}{3} \rightarrow \boxed{2\sqrt[5]{2}\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)}$

Check $k=5$. $25+6=31$, $\frac{31\pi}{15} = \frac{30\pi}{15} + \frac{\pi}{15} = 2\pi + \frac{\pi}{15}$ is coterminal with $k=0$.

b. (5 pts) Convert your answers in part a to standard form. Round results to 3 decimal places.

$-1.537258453 - 1.707298477\,I,\ 1.148698355 - 1.989603914\,I$

$k=0$: $2.247 + 0.478i$

$k=1$: $0.246 + 2.285i$

$k=2$: $-2.099 + .934i$

$k=3$: $-1.537 - 1.407i$

$k=4$: $1.149 - 1.990i$

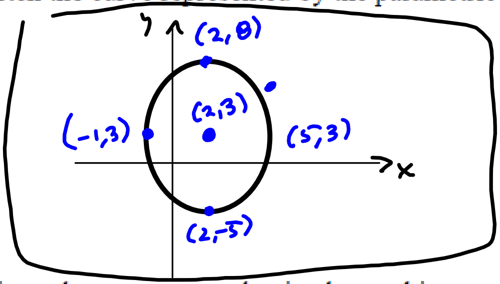
8. Ch 4 Consider the following:

$x = 2 + 3\cos(\theta)$

$y = 3 - 5\sin(\theta)$

Easier to do part b, first!

a. (5 pts) Sketch the curve represented by the parametric equations. Indicate the orientation of the curve.



b. (5 pts) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

$x = 2+3\cos\theta \rightarrow x-2 = 3\cos\theta \rightarrow \frac{x-2}{3} = \cos\theta$

$y = 3-5\sin\theta \rightarrow y-3 = -5\sin\theta \rightarrow \frac{y-3}{-5} = \sin\theta$

$\rightarrow \boxed{\frac{(x-2)^2}{3^2} + \frac{(y-3)^2}{5^2} = 1}$

$\rightarrow a = 3,\ b = 5,$ ellipse.

$(h,k) = (2,3)$

9. Ch 6 Convert the equations to polar form:

a. (5 pts) $x - 3y + 5 = 0$

$r\cos\theta - 3r\sin\theta + 5 = 0$

$r(\cos\theta - 3\sin\theta) = -5$

$\boxed{r = \dfrac{-5}{\cos\theta - 3\sin\theta}}$ or $\dfrac{5}{3\sin\theta - \cos\theta}$

b. (5 pts) $x^2 + y^2 = 25$

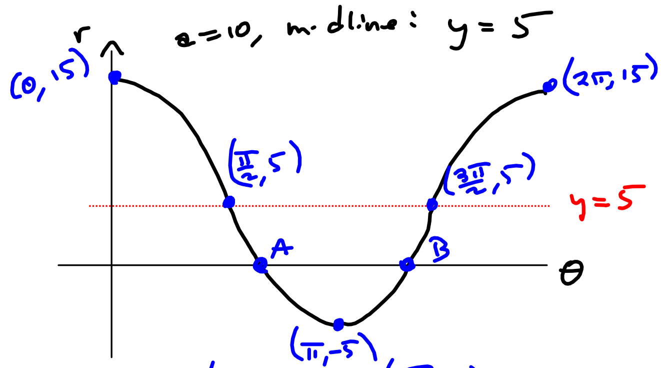
$r^2 = 25$

$\boxed{r = 5}$ $r = -5$ is also good.

10. Ch 6 (5 pts) Sketch the graph of the polar equation $r = 5(1 + 2\cos(\theta))$

$r = 5 + 10\cos\theta$ It's $10\cos\theta$ moved up 5 in rectangular

$a = 10$, m. dline: $y = 5$



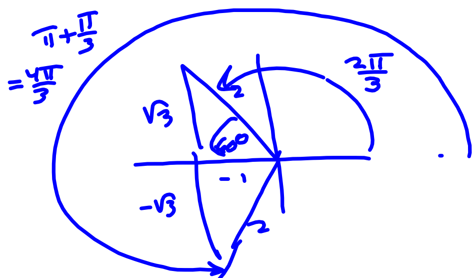
$A = \left(\frac{2\pi}{3}, 0\right)$, $B = \left(\frac{4\pi}{3}, 0\right)$

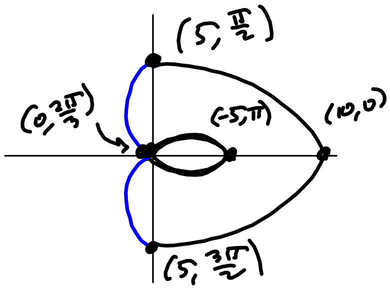
A & B: $5(1 + 2\cos\theta) = 0$

$1 + 2\cos\theta = 0$

$2\cos\theta = -1$

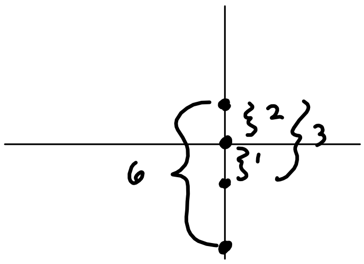
$\cos\theta = -\frac{1}{2}$





Not very good, but this is the idea.

11. Ch 6 (10 pts) Find the polar equation of the ellipse with vertices $\left(2, \frac{\pi}{2}\right), \left(4, \frac{3\pi}{2}\right)$.



$r = \frac{ep}{1 + e\sin\theta}$

$a = 3, \ c = 1$

$\Rightarrow e = \frac{c}{a} = \frac{1}{3}$

$r\left(\frac{\pi}{2}\right) = 2$

$\frac{\frac{1}{3}p}{1 + \frac{1}{3}\sin\frac{\pi}{2}} = \frac{p}{3+1} = \frac{p}{4} = 2 \Rightarrow p = 8$

$\Rightarrow r = \frac{\frac{1}{3}(8)}{1 + \frac{1}{3}\sin\theta} = \boxed{\frac{8}{3 + \sin\theta} = r}$

Check! $r\left(\frac{3\pi}{2}\right) = \frac{8}{3-1} = \frac{8}{2} = 4$ ✓