1. Arc Length and Area of Sector. Suppose we have a circle of radius $r = 10$.

   a. (5 pts) Find the arc length on the circle, that is intercepted by an angle of 1317^0. Round to 3 decimal places.

$$s = r\theta = 10(1317^0)\left(\frac{\pi}{180^0}\right) = \frac{439\pi}{6} \approx 229.8598625$$

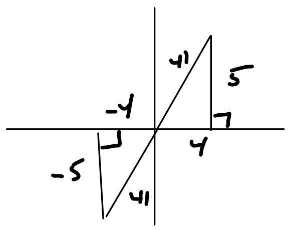
$$\approx \boxed{229.860}$$

b. (5 pts) Find the *exact* area of the sector that is intercepted (swept through) by an angle of $\theta = \dfrac{3\pi}{4}$

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}(10^2)\left(\frac{3\pi}{4}\right) = \boxed{\frac{75\pi}{2}} \approx 117.8097245$$

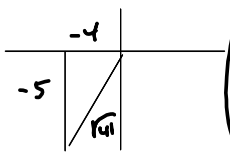
2. Answer the questions about the equation $\tan(\theta) = \dfrac{5}{4}$.

   a. (5 points) Sketch two triangles that satisfy $\tan(\theta) = \dfrac{5}{4}$.



$$\sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$$

b. (5 pts) Assume the terminal side of the angle θ lies in the 3rd quadrant. Find the other five trigonometric functions of θ.

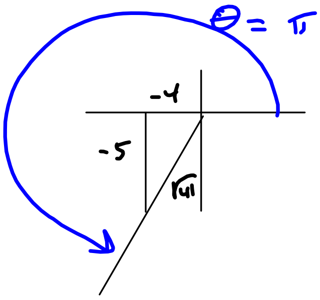


$\sin\theta = -\dfrac{5}{\sqrt{41}}$ OR $-\dfrac{5\sqrt{41}}{41}$ $\qquad \csc\theta = -\dfrac{\sqrt{41}}{5}$

$\cos\theta = -\dfrac{4}{\sqrt{41}}$ OR $\dfrac{-4\sqrt{41}}{41}$ $\qquad \sec\theta = -\dfrac{\sqrt{41}}{4}$

$\tan\theta = \dfrac{5}{4}$ $\qquad \cot\theta = \dfrac{4}{5}$

c. (5 pts) Again, assuming θ's terminal side lies in Q III, and $0 \le \theta < 2\pi$, find θ, in radians *and* degrees, rounded to 3 decimal places.



$\theta = \pi + \arctan\left(\frac{5}{4}\right) \approx 4.037648039$

$\approx \boxed{4.038 \approx \theta}$

$180^\circ +$

$(4.037648039)\left(\frac{180^\circ}{\pi}\right)$

$\approx 231.340917^\circ$

$\approx \boxed{231.341^\circ} \approx \theta$

$\arctan\left(\frac{5}{4}\right) \in Q\,I.$

θ has same ref. angle $\therefore$ Q III

d. (5 pts) Give *all* solutions to the equation $\tan(\theta) = \frac{5}{4}$, in degrees *and* radians, rounded to three (3) decimal places.

$$\theta = \left\{ \arctan\left(\tfrac{5}{4}\right) + n\pi \;\middle|\; n \in \mathbb{Z} \right\}$$

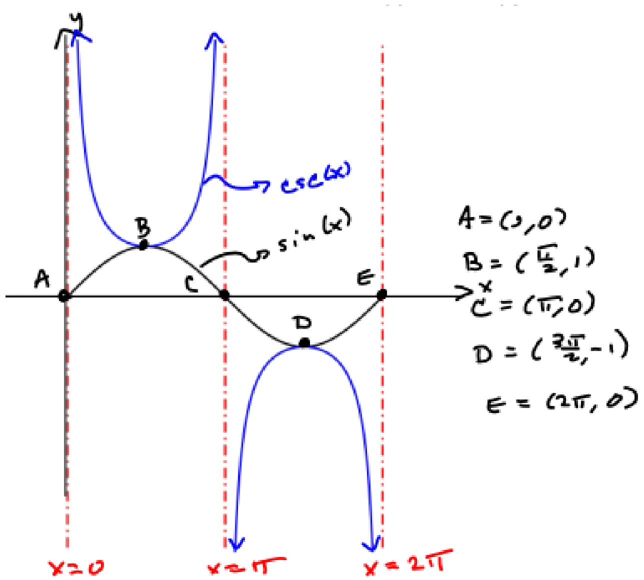
$$\approx \left\{ 0.8960553846 + n\pi \;\middle|\; n \in \mathbb{Z} \right\}$$

$$\approx \boxed{\left\{ .896 + n\pi \;\middle|\; n \in \mathbb{Z} \right\}}$$

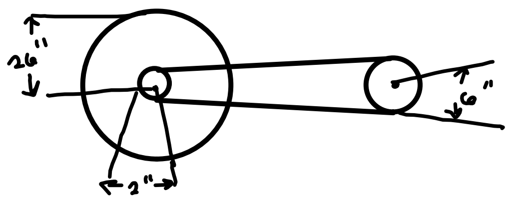
$$\approx \left\{ 51.34019173^\circ + 180^\circ n \;\middle|\; n \in \mathbb{Z} \right\}$$

$$\approx \boxed{\left\{ 51.340 + 180^\circ n \;\middle|\; n \in \mathbb{Z} \right\}}$$

3. (5 pts) Sketch one period of the graphs of $y = \sin(x)$ and $y = \csc(x)$ on the same set of coordinate axes.



4. The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 6 inches, 2 inches and 26 inches, respectively. A cyclist is pedaling at a rate of 1.5 revolutions per second.



a. (5 pts) Find the speed of the bicycle in feet per second.

$$\left(\frac{1.5 \text{ revs front}}{\text{sec}}\right)\left(\frac{6 \text{ revs rear}}{2 \text{ revs front}}\right)\left(\frac{2\pi}{1 \text{ rev}}\right)\left(26 \text{ inches}\right)\left(\frac{1 \text{ ft}}{12 \text{ inches}}\right)$$

$$= \left(\frac{3}{2}\right)(3)(2\pi)(26)\left(\frac{1}{12}\right)\frac{\text{ft}}{\text{sec}} = \frac{9(26\pi)}{12}\frac{\text{ft}}{\text{sec}} = \frac{3(13\pi)}{2}\frac{\text{ft}}{\text{sec}}$$

$$= \boxed{\frac{39\pi}{2}\ \text{ft/sec}} \approx 61.26105675\ \frac{\text{ft}}{\text{sec}} \approx 61.3\ \frac{\text{ft}}{\text{sec}}$$

b. (5 pts) Convert your answer, above, to miles per hour. Round final answers to 1 decimal plac

$$\left(\frac{39\pi}{2}\right)\left(\frac{\text{ft}}{\text{sec}}\right)\left(\frac{60\ \frac{\text{mi}}{\text{hr}}}{88\ \frac{\text{ft}}{\text{sec}}}\right) = \frac{585\pi}{44}\ \frac{\text{mi}}{\text{hr}} \approx 41.76890233\ \frac{\text{mi}}{\text{hr}}$$

$$\approx \boxed{41.8\ \frac{\text{mi}}{\text{hr}}}$$

5. (5 pts) Sketch the graph of $f(x)=3\sin\left(\frac{\pi}{6}x+18\pi\right)+6$.

$\frac{18\pi}{\frac{\pi}{6}} = 6(18)=108$

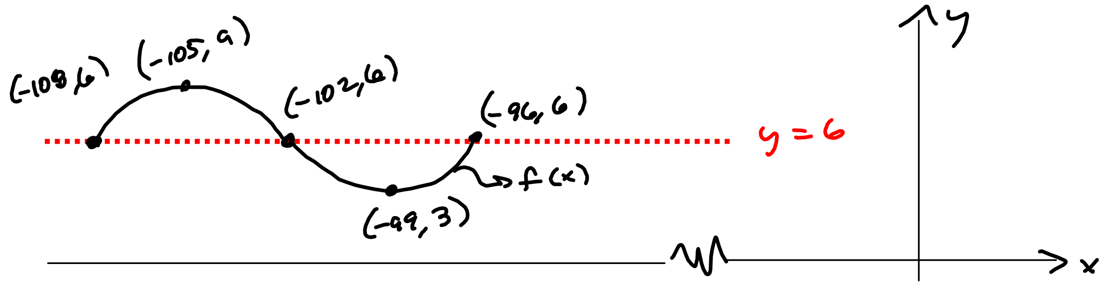
$=3\sin\left(\frac{\pi}{6}(x+108)\right)+6 \leftarrow y=6$ midline

Amp $=3$

$\frac{\pi}{6}x = 2\pi \Rightarrow$

$x=\frac{2\pi}{\frac{\pi}{6}}=(2\pi)\left(\frac{6}{\pi}\right)=\boxed{12=\text{period}}$

start @ $x=-108$



6. (5 pts) Write the cosine function that achieves its maximum height of $y = 11$ centimeters at time $t = 18$ seconds and its minimum height of $y = -4$ centimeters at $t = 30$ seconds.

(18,11)

$30-18=12=\frac{1}{2}$ period.

$\boxed{c=-18 \text{ max}}$

$\frac{11-(-4)}{2}=\boxed{\frac{15}{2}=a}$

midline: $\frac{11+(-4)}{2}=\boxed{\frac{7}{2}=d}$

$bx=2\pi$ when $x=24$

$24b=2\pi \Rightarrow b=\frac{2\pi}{24}=\boxed{\frac{\pi}{12}=b}$

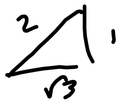
(30,−4)

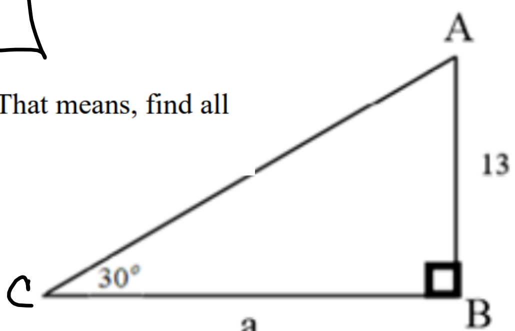
$f(x)=a\cos(b(x-c))+d$

$=\boxed{\frac{15}{2}\cos\left(\frac{\pi}{12}(x-18)\right)+\frac{7}{2}}$

7. (5 pts) Solve the triangle in the figure on the right. That means, find all lengths and angles. Exact answers required.

$\frac{13}{a}=\tan 30^\circ=\frac{1}{\sqrt{3}} \Rightarrow$





$\boxed{13\sqrt{3}=a}$

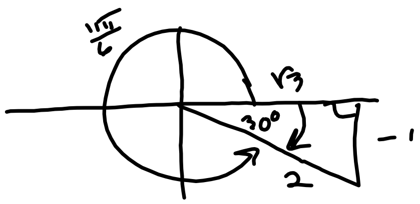
$b=\sqrt{a^2+c^2}=\sqrt{13^2\cdot 3+13^2}=13\sqrt{3+1}=13\cdot 2=26$

$\boxed{b=26}$

$\boxed{C=60^\circ \quad B=90^\circ}$

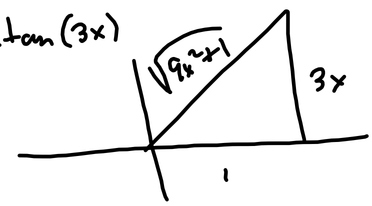
8. (5 pts) Find the exact value of $\arccos\left(\sin\left(\frac{11\pi}{6}\right)\right)$ $= \theta$

$= \arccos\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3} \text{ or } 120^{\circ}}$



9. (5 pts) Draw the sketch and use it to find an algebraic expression that is equivalent to $\sin(\arctan(3x))$.

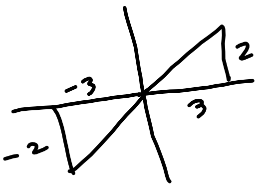
$\theta = \arctan(3x)$ $\qquad = \sin\theta$

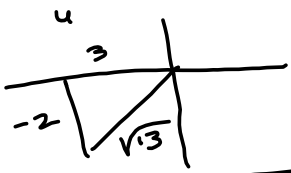


$\Rightarrow \sin\theta = \boxed{\frac{3x}{\sqrt{9x^2+1}}}$

10. (5 pts) Find the values of all six trigonometric functions, given $\tan(u) = \frac{2}{3}$ and $\sin(u) < 0$.

$\tan(u) = \frac{2}{3}$ $\qquad \tan(u) = \frac{2}{3}$ and $\sin(u) < 0$





$\sin(u) = -\frac{2}{\sqrt{13}}$ or $\frac{-2\sqrt{13}}{13}$ $\qquad \csc(u) = -\frac{\sqrt{13}}{2}$

$\cos(u) = -\frac{3}{\sqrt{13}}$ or $-\frac{3\sqrt{13}}{3}$ $\qquad \sec(u) = -\frac{\sqrt{13}}{3}$

$\tan(u) = \frac{2}{3}$ $\qquad \cot(u) = \frac{3}{2}$

11. (5 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u)=\frac{3}{4}$ and $\frac{3\pi}{2} \le u < 2\pi$.

$$\sin\left(\frac{u}{2}\right)=\sqrt{\frac{1-\cos(u)}{2}}=\sqrt{\frac{1-\frac{3}{4}}{2}}=\sqrt{\frac{4-3}{8}}=\frac{1}{\sqrt{8}}$$

$$=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{2\cdot 2}=\boxed{\frac{\sqrt{2}}{4}=\sin\left(\frac{u}{2}\right)}$$

$\Longrightarrow \frac{3\pi}{4} \le \frac{u}{2} < \pi$

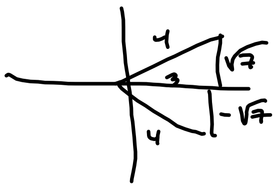
$\Longrightarrow$ $\boxed{\begin{array}{l} \frac{u}{2} \in Q\,\text{II} \\ \sin\left(\frac{u}{2}\right) > 0 \quad + \\ \cos\left(\frac{u}{2}\right) < 0 \quad - \end{array}}$

$$\cos\left(\frac{u}{2}\right)=-\sqrt{\frac{1+\cos(u)}{2}}=-\sqrt{\frac{1+\frac{3}{4}}{2}}=-\sqrt{\frac{4+3}{8}}=-\sqrt{\frac{7}{8}}=-\frac{\sqrt{7}}{2\sqrt{2}}=-\frac{\sqrt{14}}{4}$$

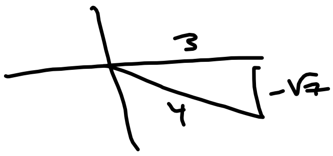
$$\boxed{\cos\left(\frac{u}{2}\right)=-\frac{\sqrt{14}}{4}}$$

$$\tan\left(\frac{u}{2}\right)=\frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)}=\frac{\sqrt{2}}{4}\cdot\frac{-4}{\sqrt{14}}=-\sqrt{\frac{2}{14}}=-\frac{\sqrt{1}}{\sqrt{7}}=\boxed{-\frac{\sqrt{7}}{7}=\tan\left(\frac{u}{2}\right)}$$

$\cos(u)=\frac{3}{4}$



$\cos(u)=\frac{3}{4}$ & $\frac{3\pi}{2} < u < 2\pi$



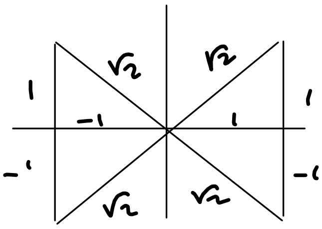
Alternate: $\tan\left(\frac{u}{2}\right)=\frac{1-\cos(u)}{\sin(u)}=\frac{1-\frac{3}{4}}{-\frac{\sqrt{7}}{4}}$

$$=\frac{4-3}{-\sqrt{7}}=-\frac{1}{\sqrt{7}}=-\frac{\sqrt{7}}{7}$$

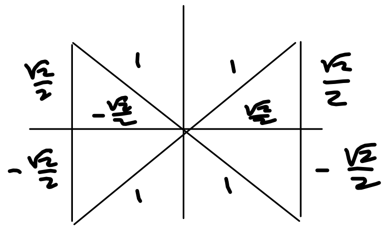
12. Consider the equation $2\sin^2(x)-1=0$.

a. (5 pts) Find all solutions x, in radians *and* degrees, to the equation in the interval $[0,2\pi)$.

$$2\sin^2(x)=1$$
$$\sin^2(x)=\frac{1}{2}$$
$$\sin(x)=\pm\sqrt{\frac{1}{2}}=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2}$$



OR



$$\arcsin\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4} \text{ OR } 45^\circ$$

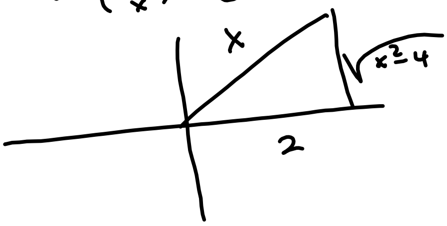
$$\boxed{x\in\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\} \text{ OR } \{45^\circ,135^\circ,225^\circ,315^\circ\}}$$
or $\left\{\frac{2n-1}{4}\pi \mid n=1,2,3,4\right\}$ or $\{(2n-1)45^\circ \mid n=1,2,3,4\}$

b. (5 pts) Find all real solutions x, in radians *and* degrees.

$$\boxed{\left\{\frac{\pi}{4}+\frac{n\pi}{2} \mid n\in\mathbb{Z}\right\} \text{ or } \{45^\circ+90^\circ n \mid n\in\mathbb{Z}\}}$$

13. (5 pts) Re-write $\sin\left(\cos^{-1}\left(\frac{2}{x}\right)\right)$ as an algebraic expression.
$=\sin\theta$

$$\cos^{-1}\left(\frac{2}{x}\right)=\theta$$

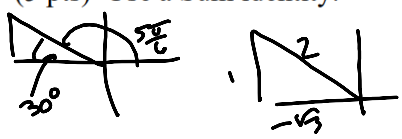


$$\longrightarrow \boxed{\sin\theta=\frac{\sqrt{x^2-4}}{x}}$$

14. Find the *exact* value of $\sin\left(\frac{5\pi}{6}\right)$ in two ways: (Hint: If degrees are easier for you, *use degrees*.)

a. (5 pts) Use a Sum identity.

Poorly Posed. Don't need sum identity or $\frac{1}{2}$-angle.



$\boxed{\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2} \text{ from basics}}$

b. (5 pts) Use a Half-Angle identity

15. (5 pts) Re-write $\cos(\arcsin(x) + \arccos(x))$ as an algebraic expression. (Hint: Use Sum identity.)

$= \cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$

$= \cos(\arcsin(x))\cos(\arccos(x)) - \sin(\arcsin(x))\sin(\arccos(x))$

$= \left(\sqrt{1-x^2}\right)(x) - x\sqrt{1-x^2} \; \boxed{= 0}$

$\arcsin(x) = u$

$\arccos(x) = v$

16. (5 pts) Find $\sin(2u), \cos(2u)$ and $\tan(2u)$, given that $\underbrace{\sin(u) = \frac{4}{7} \text{ and } \cos(u) < 0}$.

$\sin(2u) = 2\sin(u)\cos(u) = 2\left(\frac{4}{7}\right)\left(-\frac{\sqrt{33}}{7}\right)$

$\boxed{\sin(2u) = -\frac{8\sqrt{33}}{49}}$

$\cos(2u) = 1 - 2\sin^2(u) = 1 - 2\left(\frac{4}{7}\right)^2$

$\cos(2u) = \frac{49-32}{49} = \boxed{\frac{17}{49} = \cos(2u)}$

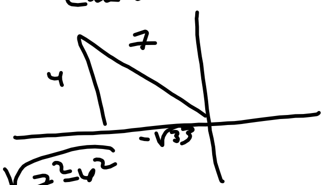
$\Rightarrow \tan(2u) = \left(\frac{-8\sqrt{33}}{49}\right)\left(\frac{49}{17}\right) = \boxed{-\frac{8\sqrt{33}}{17} = \tan(2u)}$

$u \in Q\,II$

$\frac{\pi}{2} < u < \pi \Rightarrow$

$\pi < 2u < 2\pi$

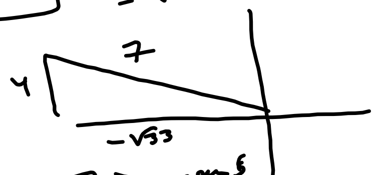
Can we estimate better?



$\sqrt{7^2 - 4^2}$

$= \sqrt{49-16}$

$= \sqrt{33} > 4$



This means

$\frac{3\pi}{4} < u < \pi \Rightarrow$

$\frac{3\pi}{2} < 2u < 2\pi \Rightarrow$

$2u \in Q\,IV$

(to check)

17. (5 pts) Find the arc length on a circle of radius $r = 8$ that is intercepted by an angle of 1356^0.

$s = r\theta = \overset{2}{\cancel{8}}(1356^0)\left(\frac{\pi}{\underset{45}{\cancel{180}}{}^0}\right) = \frac{904\pi}{15} \approx 189.3333173$

$= 2(\cancel{1356})\left(\frac{\pi}{\cancel{45}}\right) = 2(452)\left(\frac{\pi}{15}\right) = \frac{904\pi}{15}$