Written Final

Monday, May 6th: 7:30 am - 6 pm start times

Tuesday, May 7th 7:30 am - 4:30 pm start time

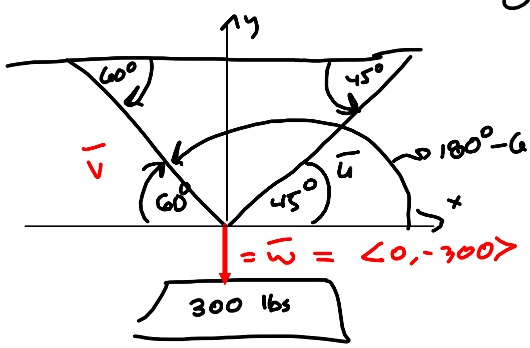
I asked for 3 hours. If I get that, then move the start times from 6 pm to 5 pm and the 4:30 pm to 3:30 pm.

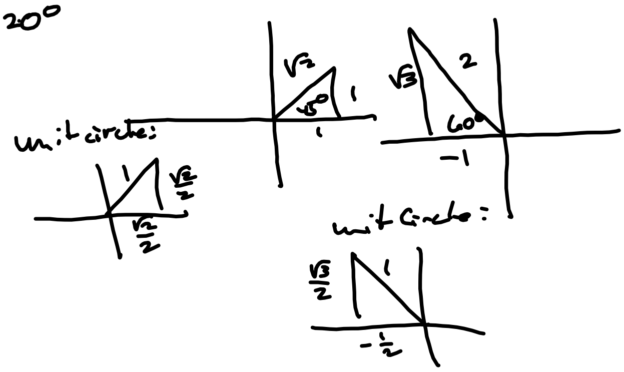
Horizon Hall, I think Ray's office is Rm 107.

Osteogenesis Imperfecta.

want $\bar{u} + \bar{v} = \langle 0, 300 \rangle$

$\bar{u} + \bar{v} = -\bar{w}$ to balance forces





$\|\bar{u}\| \langle \cos(45°), \sin(45°) \rangle + \|\bar{v}\| \langle \cos(120°), \sin(120°) \rangle$

want $\boxed{\begin{aligned} \|\bar{u}\| &= x \\ \|\bar{v}\| &= y \end{aligned}}$

$x\langle a,b\rangle + y\langle c,d\rangle = \langle 0,300\rangle \longrightarrow$

$\langle ax,bx\rangle + \langle cy,dy\rangle = \langle 0,300\rangle \Longrightarrow$

$\langle ax+cy, bx+dy\rangle = \langle 0,300\rangle \Longrightarrow$

$ax+cy=0 \Longrightarrow cy=-ax \Longrightarrow \boxed{y=\frac{-ax}{c}}$

$bx+dy=300$

$\cos 45^\circ = \frac{1}{\sqrt{2}} = a$

$\sin 45^\circ = \frac{1}{\sqrt{2}} = b$

$\cos 120^\circ = -\frac{1}{2} = c$

$\sin 120^\circ = \frac{\sqrt{3}}{2} = d$

$bx+d\left(\frac{-ax}{c}\right) = bx - \frac{adx}{c} = x\left(b-\frac{ad}{c}\right) = \left(\frac{bc-ad}{c}\right)x = 300$

Adjoavi Yankey!

$\Longrightarrow x = \frac{300c}{bc-ad} = \frac{300\cos(120^\circ)}{\sin(45^\circ)\cos(120^\circ) - \cos(45^\circ)(\sin(120^\circ))}$

$= \frac{300\left(-\frac{1}{2}\right)}{\frac{\sqrt{2}}{2}\cdot\left(-\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}}\cdot\frac{4}{4} = \frac{-600}{-\sqrt{2}-\sqrt{2}\sqrt{3}} = \frac{600}{\sqrt{2}(1+\sqrt{3})} = \boxed{\frac{600}{1+\sqrt{3}} = x}$

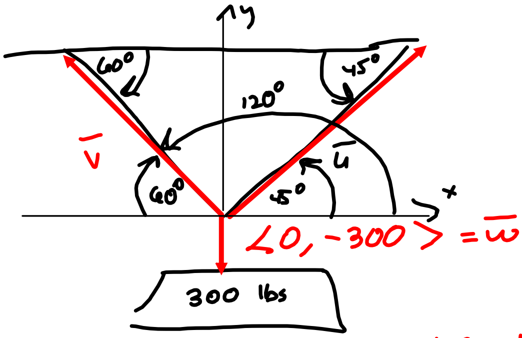
$y = \frac{-ax}{c} = \frac{\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{600}{1+\sqrt{3}}\right)}{\left(-\frac{1}{2}\right)}\cdot\frac{2}{2} = \boxed{\frac{-600\sqrt{2}}{1+\sqrt{3}}}$

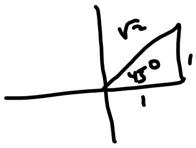
That's the idea

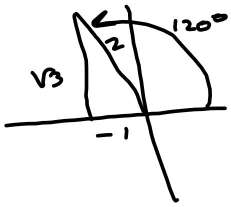
I made a mistake, somewhere, but this would be a 9 out of 10 points effort, because the undersanding is there.

$\|\bar{u}\| = \frac{-600}{1+\sqrt{3}}$?!

$\|\bar{v}\| = \frac{600}{1+\sqrt{3}}$ ACCORDING TO Computer Algebra System CAS.







$\bar{u} + \bar{v} = <0, 300>$ to perfectly balance/cancel $\bar{w} = <0,-300>$

$\|\bar{u}\| <\cos 45°, \sin 45°> + \|\bar{v}\| <\cos 120°, \sin 120°>$

Want $\|\bar{u}\|$ & $\|\bar{v}\|$

Let $x = \|\bar{u}\|$, $y = \|\bar{v}\|$,

$\cos 45° = \frac{1}{\sqrt{2}} = a$

$\sin 45° = \frac{1}{\sqrt{2}} = b$

$\cos 120° = -\frac{1}{2} = c$

$\sin 120° = \frac{\sqrt{3}}{2} = d$

$\|\bar{u}\| <\cos 45°, \sin 45°> + \|\bar{v}\| <\cos 120°, \sin 120°>$

$x <a,b> + y <c,d> = <0, 300>$

$<ax, bx> + <cy, dy> = <0, 300>$

$ax + cy = 0 \longrightarrow cy = -ax \Longrightarrow y = \frac{-ax}{c}$

$bx + dy = 300$

0

$$\Rightarrow bx + d\left(\frac{-ax}{c}\right) = 300$$

$$x\left(b - \frac{ad}{c}\right) = 300$$

$$x\left(\frac{bc - ad}{c}\right) = 300$$

$$x = \frac{300c}{bc - ad} = \frac{300\left(-\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)} = \frac{-150}{\frac{-1-\sqrt{3}}{2\sqrt{2}}} = \frac{150(2\sqrt{2})}{1+\sqrt{3}} = \frac{300\sqrt{2}}{1+\sqrt{3}}$$

$$\Rightarrow y = \frac{-ax}{c} = \frac{-a\left(\frac{300c}{bc-ad}\right)}{c} = \frac{-\frac{1}{\sqrt{2}}\left(\frac{300\left(-\frac{1}{2}\right)}{\frac{-1-\sqrt{3}}{2\sqrt{2}}}\right)}{-\frac{1}{2}} = \frac{-300(2)}{-1-\sqrt{3}} = \frac{600}{1+\sqrt{3}}$$

$$= \frac{300(2)}{3} = \boxed{200 = y = \|\vec{v}\|}$$

Check:

$$\langle ax, bx\rangle + \langle cy, dy\rangle$$

$$= \langle(\cos 45^\circ)(100\sqrt{2}), (\sin 45^\circ)(100\sqrt{2})\rangle + \langle(\cos 120^\circ)(200), (\sin 120^\circ)(200)\rangle$$

$$= \left\langle\left(\frac{1}{\sqrt{2}}\right)(100\sqrt{2}), \frac{1}{\sqrt{2}}(100\sqrt{2})\right\rangle + \left\langle \frac{1}{2}(200), \frac{\sqrt{3}}{2}(200)\right\rangle$$

Ellipses and Hyperbolas in Rectangular Coordinates.

BONUS MATERIAL

Ellipse :

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

center is (h,k)

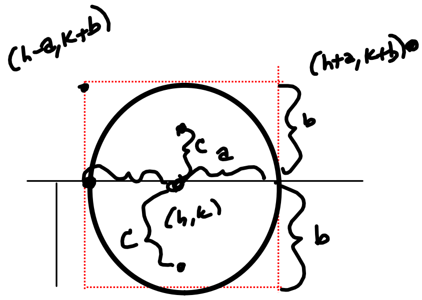
$c = \sqrt{a^2 - b^2}$ = focal length

= distance from focus to center

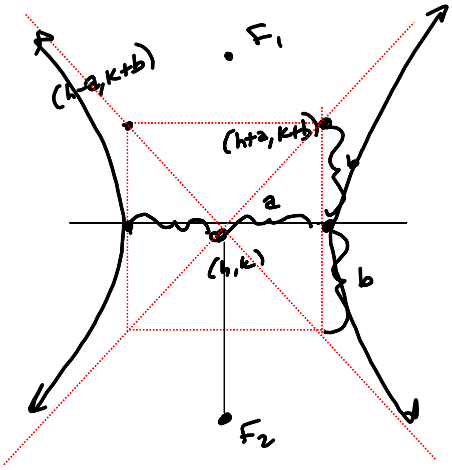
* $a > b$ for this one

If $b > a$, then $c = \sqrt{b^2 - a^2}$

RECALL $e = \frac{c}{a} < 1$



Directrix



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

center = (h,k)

$c = \sqrt{a^2 + b^2}$

Write the Polar equation for

$$\frac{(x+5)^2}{3^2} - \frac{y^2}{4^2} = 1$$

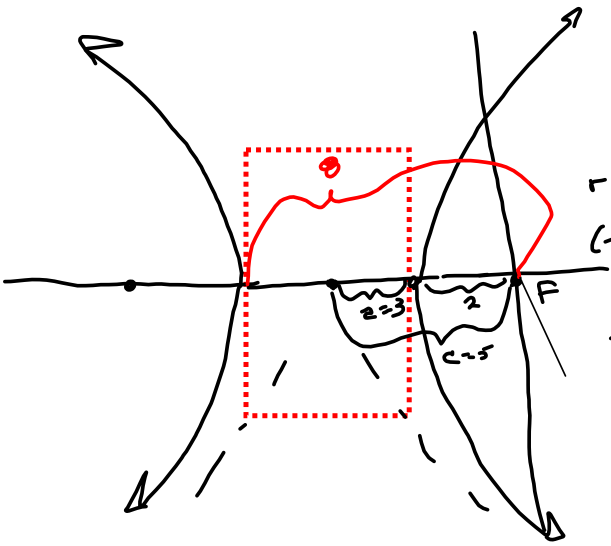
$(h,k) = (-3,0)$

) (

$a=3,\ b=4$

$c = \sqrt{a^2+b^2} = \sqrt{3^2+4^2} = \sqrt{25} = \boxed{5 = c}$

Now, if we can establish that the (a) focus is @ the pole, Then $r = \frac{ep}{1}$. It is !!



Directrix to left of pole

$$r = \frac{ep}{1 - e\cos\theta}$$

$r(0) = -2$? $\Rightarrow r(\pi) = 8$

$(-5, 0)^R$

$r(0) = -8 \Rightarrow r(\pi) = +2$!

Try $r(0) = -2 \Rightarrow$

$r = \frac{ep}{1 - e\cos\theta} = -2$ & we know e.

$e = \frac{c}{a} = \frac{5}{3}$

-2

$$= \frac{\frac{5}{3}p}{1-\frac{5}{3}\cos(0)} = \frac{5p}{3-5(1)} = \frac{5p}{-2} =$$

$\Rightarrow \boxed{p = \frac{4}{5}}$

Check: $r(\pi) = 8$?

$$\frac{\frac{5}{3}p}{1-\frac{5}{3}\cos\pi} = 8$$

$$\Rightarrow \frac{5p}{3-5(-1)} = \frac{5p}{8} = 8$$

$\Rightarrow \boxed{p = \frac{64}{5}}$

$\Rightarrow$ Nope!

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try $r(0) = -8$ & $r(\pi) = 2$

$r(0) = -8$

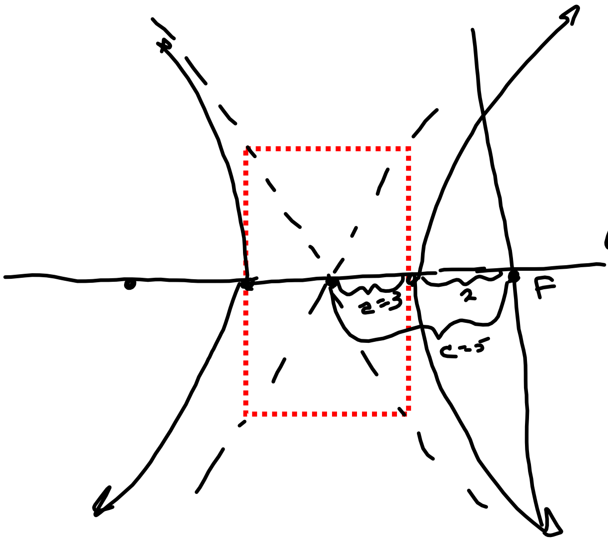
$$\frac{\frac{5}{3}p}{1-\frac{5}{3}\cos(0)} = \frac{5p}{3-5} = \frac{5p}{-2} = -8 \Rightarrow \boxed{p = \frac{16}{5}}$$

Check:

$r(\pi) = 2$

$$\frac{5p}{3-5\cos\pi} = \frac{5p}{8} = 2 \Rightarrow \boxed{p = \frac{16}{5}}$$

Yes!



$e=5$
$a=5$
$b=4$

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

vert $c=5$
$a=3$
$b=4$

$$r=\frac{ep}{1-e\cos\theta}=\frac{\frac{5}{3}p}{1-\frac{5}{3}\cos\theta}=\frac{5p}{3-5\cos\theta}$$

$$\frac{(x+5)^2}{3^2}-\frac{y^2}{4^2}=1$$

2 guesses
1 works.

$\boxed{r(0)=-8 \qquad r(\pi)=2}$

$\frac{5p}{-2}=-8$ $\qquad \frac{5p}{3+5}=2$

$p=\frac{16}{5}$ $\qquad 5p=\frac{2(8)}{5}=\frac{16}{5}$

$\boxed{r(0)=-5 \qquad r(\pi)=8}$

$\frac{5p}{3-5}=-5$ $\qquad \frac{5p}{3+5}=8$

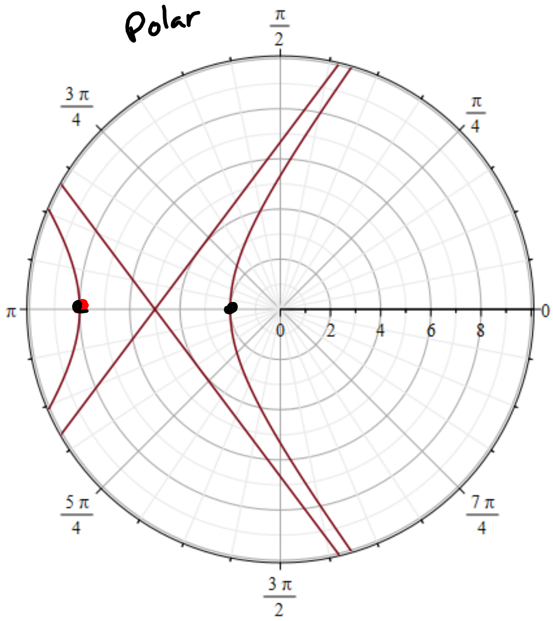
$\frac{5p}{-2}=-5$ $\qquad \frac{5p}{8}=8$

$p=\frac{-2(-5)}{5}=2$ $\qquad p=\frac{64}{5}$?!

$$\frac{ep}{1-e\cos\theta}=\frac{\frac{5}{3}(16/5)}{1-\frac{5}{3}\cos\theta}=\frac{16}{3-5\cos\theta}$$

$r = \frac{16}{3 - 5\cos\theta}$

Polar



$\frac{x^2}{9} - \frac{y^2}{16} = 1$

Rectangular