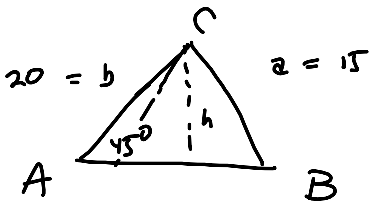
Suppose $A = 45°$, $b = 20$, $a = 15$ WP #2



ASS
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Need $a \geq h$

$\frac{h}{b} = \frac{h}{20} = \sin 45°$

$h = 20 \sin 45° = 20\left(\frac{\sqrt{2}}{2}\right) = 10\sqrt{2} \approx 14 < a$ ≈14.14 AT LEAST ONE SOL'N.

& $a = 15 < 20 = b$ → So 2 sol'ns

What would it take for only 1 solution?

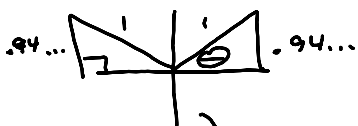
$a > b$ So $a = 21 \longrightarrow$ 1 sol'n.

Law of sines for this

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow \sin B = \frac{b \sin A}{a} = \frac{20 \sin(45°)}{15} = \frac{4\left(\frac{\sqrt{2}}{2}\right)}{3} = \frac{2\sqrt{2}}{3}$

$\approx$ 0.94280904158

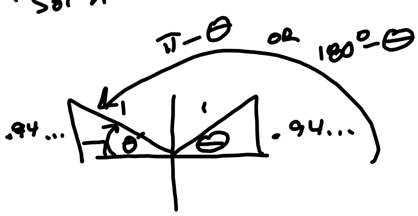


1st SOL'N: $\arcsin(\sin B) = \arcsin(.94280904158)$

$\approx$ 1.23095942 $\approx B$

$\approx$ 70.5287795179° $\approx B$ ACUTE

2nd Sol'n



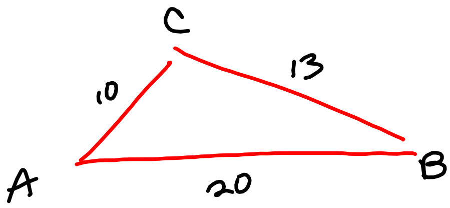
So, 180° − 70.5287795179°

$\approx$ 109.471220482 $\approx B$ OBTUSE

Angle $C = 180° - 45° - B$ for the 2 different B's

Law of Cosines for 3 sides and no angles given.

Solve



$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

$$b^2 = a^2 + c^2 - 2ac\cos\theta$$

$$-2ac\cos\theta = a^2 + c^2 - b^2$$

+

$$\cos\theta = \frac{a^2+c^2-b^2}{-2ac}$$

$$\theta = \cos^{-1}\left(\frac{a^2+c^2-b^2}{-2ac}\right)$$

$$= \cos^{-1}\left(\frac{13^2+20^2-10^2}{-2(13)(20)}\right)$$

$$\approx \boxed{25.58799012^\circ \approx B}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

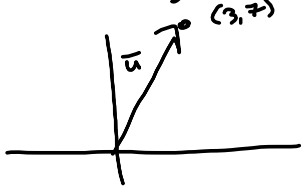
$$\sin A = \frac{a\sin B}{b} = \frac{13\sin(25.\ldots^\circ)}{10} \approx \boxed{34.15722248^\circ \approx A}$$

$$C = 180^\circ - B - A \quad \text{etc.}$$

(3) $\bar{u} = \langle 3,7 \rangle$, $\bar{v} = \langle -1,4 \rangle$

(a) $\|\bar{u}\| = \sqrt{\bar{u} \cdot \bar{u}} = \sqrt{3^2+7^2} = \sqrt{9+49} = \boxed{\sqrt{58} = \|\bar{u}\|}$ $2 \,\underline{)58}$ 29

(b) Direction angle for $\bar{u}$!



$\theta = \arctan\left(\frac{7}{3}\right)$

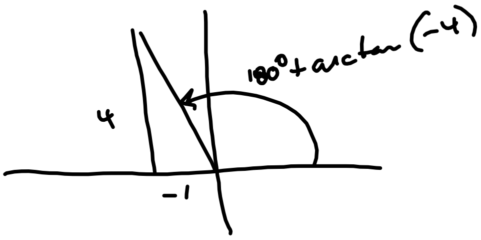
$\approx 66.80140949^\circ$

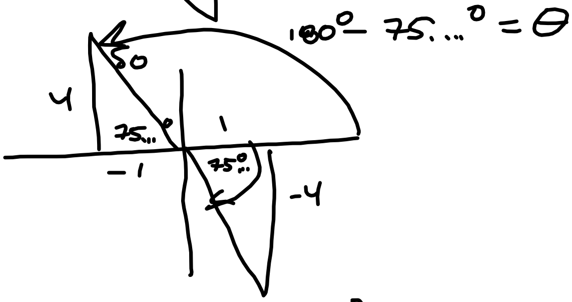
$\approx \boxed{66.801^\circ \approx \theta}$

What about direction angle for $\bar{v}$?

$\arctan\left(\frac{4}{-1}\right) \approx -75.96375653$

But tangent will spit out





So $\theta = 180^\circ - 75.\ldots^\circ$

≈ 104.0362435

$\approx 104.036^\circ$

(c) Angle between $\bar{u}$ & $\bar{v}$:

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \frac{\bar{u}\cdot\bar{v}}{\sqrt{\bar{u}\cdot\bar{u}}\sqrt{\bar{v}\cdot\bar{v}}}$$

$\bar{u} = \langle 3,7\rangle$, $\bar{v} = \langle -1,4\rangle$

$$\frac{\bar{u}\cdot\bar{v}}{\sqrt{(\bar{u}\cdot\bar{u})}\sqrt{\bar{v}\cdot\bar{v}}} = \frac{(3)(-1)+(7)(4)}{\sqrt{3^2+7^2}\sqrt{1^2+4^2}} = \frac{-3+28}{\sqrt{9+49}\sqrt{17}} = \frac{25}{\sqrt{58}\sqrt{17}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{25}{\sqrt{58}\sqrt{17}}\right)$$

$\approx$ 64.69948197^0 $\approx \boxed{64.699^0 \approx \theta}$

(d) The resultant of $\bar{u}$ & $\bar{v}$ is $\bar{u}+\bar{v} = \langle 3-1, 7+4\rangle$
$= \boxed{\langle 2,11\rangle = \bar{u}+\bar{v}}$

(e) $$\text{proj}_{\bar{v}}\bar{u} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{v} = \frac{\bar{u}\cdot\bar{v}}{\bar{v}\cdot\bar{v}}\bar{v}$$

$\bar{u} = \langle 3,7\rangle$, $\bar{v} = \langle -1,4\rangle$

$$= \frac{-3+28}{\sqrt{1^2+4^2}^2}\langle -1,4\rangle$$

$$= \boxed{\frac{25}{\sqrt{17}}\langle -1,4\rangle}$$ or $\left\langle -\frac{25}{\sqrt{17}}, \frac{100}{\sqrt{17}}\right\rangle$

Perfect for written work

$$= \left\langle -\frac{25\sqrt{17}}{17}, \frac{100\sqrt{17}}{17}\right\rangle = \text{proj}_{\bar{v}}\bar{u}$$

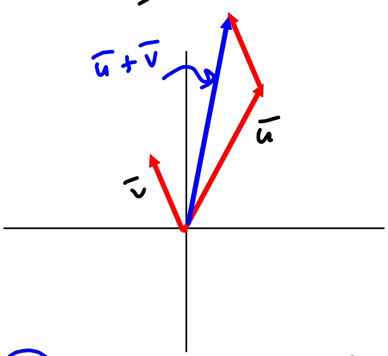
BONUS write $\bar{u}$ as the sum of 2 orthogonal vectors, one of which being parallel to $\bar{v}$:

$$\bar{u} = (\bar{u} - \text{proj}_{\bar{v}}\bar{u}) + \text{proj}_{\bar{v}}(u)$$

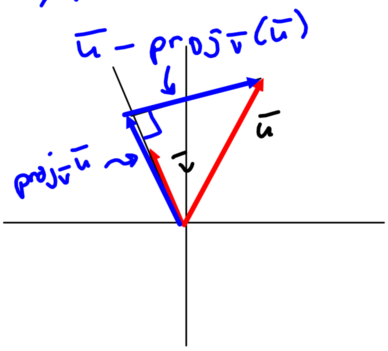
$\bar{u}_1 \quad + \quad \bar{u}_2$ & $\bar{u}$ is $\perp$ to

$\perp$ to $\bar{v}$ + $\parallel$ to $\bar{v}$

$\bar{u} = \langle 3, 7 \rangle$, $\bar{v} = \langle -1, 4 \rangle$



(9) $\bar{u}$, $\bar{v}$, $\text{proj}_{\bar{v}}(\bar{u})$, $\bar{u} - \text{proj}_{\bar{v}}(\bar{u})$



$x^4 - 4x^3 - x^2 + 10x - 150$

Given $x = 1+3i$ is a zero,

split $f(x)$ into linear factors

Divide by $x-(1+3i)$

$$
\begin{array}{r|rrrrr}
1+3i & 1 & -4 & -1 & 10 & -150 \\
 & & 1+3i & -12-6i & 5+33i & \\
\hline
 & 1 & -3+3i & -13-6i & 15+33i &
\end{array}
$$

Then divide by $x-(1-3i)$

$(-3+3i)(1+3i) = -3 - 9i + 3i - 9 = -12 - 6i$

$(1+3i)(-13-6i) = -13 - 6i - 39i + 18 = 5 - 45i$

$(1+3i)(5-45i) = 15 + 33i + 45i - 99$ NOPE

$(1+3i)(-13-6i) = -13 - 6i - 39i + 18$

$= 5 - 45i$

$(1+3i)(5-45i) = 5 - 45i + 15i + 135$

$x^4 - 4x^3 - x^2 + 10x - 150$

$x = 1+3i$ is a zero. Split $f(x)$ into linear factors

Divide by $x - (1+3i)$:

$$\begin{array}{c|ccccc} 1+3i & 1 & -4 & -1 & 10 & -150 \\ & & 1+3i & -12-6i & 5-45i & 150 \\ \hline & 1 & -3+3i & -13-6i & 15-45i & 0 \end{array}$$

$(1+3i)(-3+3i) = -3(1+3i)(1-i) = -3(1-i+3i-3i^2 = (4+2i)(-3)$
$= -12-6i$

$(1+3i)(-13-6i) = -13-6i-39i+18$
$= 5-45i$

$(1+3i)(15-45i) = 15-45i+45i+135$
$= 150$

GOT IT!

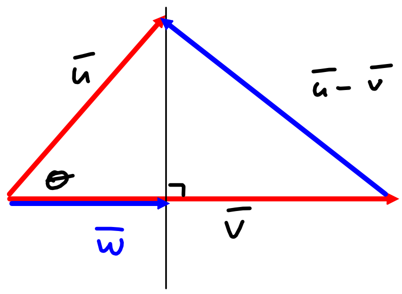
Next: Divide by
$x-(1-3i)$.

$$\begin{array}{c|cccc} 1-3i & 1 & -3+3i & -13-6i & 15-45i \\ \hline & & & & \end{array}$$

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$

$$\bar{w} = \text{proj}_{\bar{v}}\,\bar{u} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\,\bar{v}$$

$$= \frac{\bar{u}\cdot\bar{v}}{\bar{v}\cdot\bar{v}}\,\bar{v}$$



$$\|\bar{u}-\bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

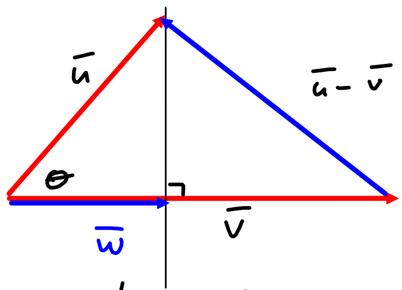
$$(\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v}) = \bar{u}\cdot\bar{u} + \bar{v}\cdot\bar{v} - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$\cancel{\bar{u}\cdot\bar{u}} - \bar{u}\cdot\bar{v} - \bar{u}\cdot\bar{v} + \cancel{\bar{v}\cdot\bar{v}} = \cancel{\bar{u}\cdot\bar{u}} + \cancel{\bar{v}\cdot\bar{v}} - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$-2(\bar{u}\cdot\bar{v}) = -2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$\bar{u}\cdot\bar{v} = \|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$\frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \cos\theta$$



Now, $\bar{w} = (\|u\|\cos\theta)$(unit vector in the direction of $\bar{v}$)

$$= (\cancel{\|u\|})\left(\frac{\bar{u}\cdot\bar{v}}{\cancel{\|\bar{u}\|}\|\bar{v}\|}\right)\left(\frac{1}{\|\bar{v}\|}\right)\bar{v}$$

$$= \boxed{\bar{w} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\,\bar{v} = \text{proj}_{\bar{v}}\,\bar{u}}$$

$$= \frac{\bar{u}\cdot\bar{v}}{\bar{v}\cdot\bar{v}}\,\bar{v}$$

If the angle between the vectors is greater than 90 degrees, then cosine will be negative, and the projection vector will be parallel to vbar, but in the opposite direction.