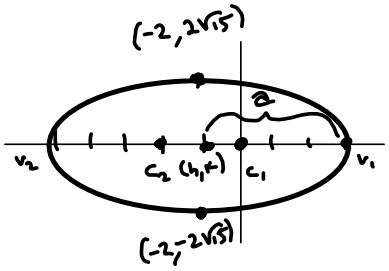
Student Question

§6.9 #20

Ellipse Vertices (6,0), (10, π)



$\frac{6-10}{2} = \frac{-4}{2} = -2$

$c = 2$, $a = 8$ $\Rightarrow e = \frac{c}{a} = \frac{2}{8} = \frac{1}{4} = e$

$$\frac{ep}{1+e\cos\theta} = \frac{\frac{1}{4}p}{1+\frac{1}{4}\cos\theta} = \frac{p}{4+\cos\theta}$$

& we know $r(0) = 6$:

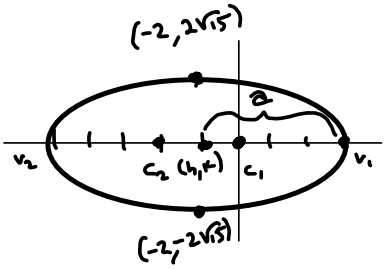
$$\frac{p}{4+\cos(0)} = \frac{p}{4+1} = \frac{p}{5} = 6 \Rightarrow p = 30$$

$$\therefore r = \frac{ep}{1+e\cos\theta} = \frac{(\frac{1}{4})(30)}{1+\frac{1}{4}\cos\theta} = \frac{30}{4+\cos\theta}$$

Rectangular Version

$a = 8,\ c = 2$

$b = \sqrt{a^2 - c^2} = \sqrt{8^2 - 4} = \sqrt{64-4} = \sqrt{60} = 2\sqrt{15}$

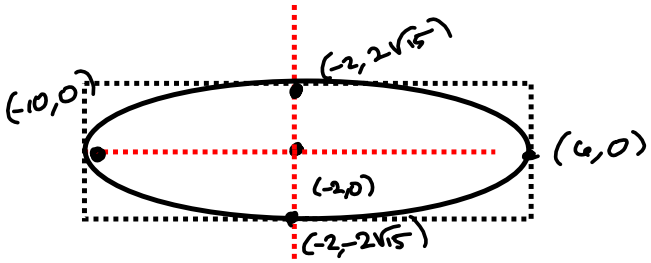


$(h,k) = (-2,0)$ in rectangular, so

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x+2)^2}{8^2} + \frac{y^2}{(2\sqrt{15})^2} = 1$$

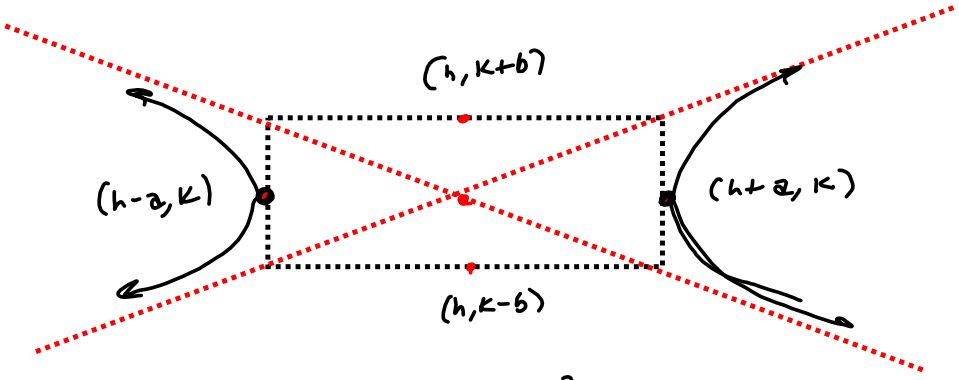
$$\boxed{\frac{(x+2)^2}{64} + \frac{y^2}{60} = 1}$$



HYPERBOLA:

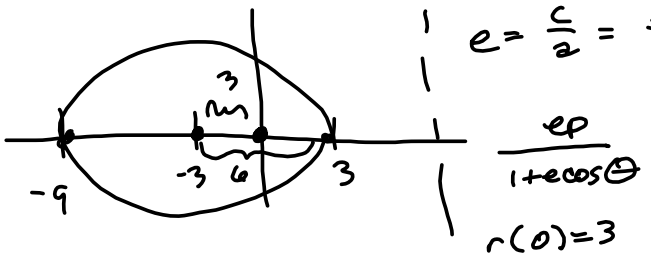
$b^2 = c^2 - a^2$

$b = \sqrt{c^2 - a^2}$



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$(3,0)$, $(9,\pi)$



$e = \frac{c}{a} = \frac{3}{6} = \frac{1}{2} = e$

$\frac{ep}{1+e\cos\theta}$

$r(0) = 3$

$$\frac{\frac{1}{2}p}{1+\frac{1}{2}\cos(0)} = 3$$

$$\frac{p}{2+\cos(0)} = \frac{p}{2+1} = \frac{p}{3} = 3 \Rightarrow \boxed{p = 9}$$

$$r = \frac{\frac{1}{2}(9)}{1+\frac{1}{2}\cos\theta} = \frac{9}{2+\cos\theta}$$

Alternate: Vertices $(6,0), (10,\pi)$

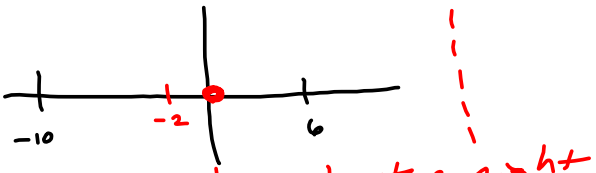
a = Dist from center to vertex = 8

c = " " " " focus = 2

$e = \frac{c}{a} = \frac{2}{8} = \frac{1}{4} = e$ same e.

$r = \frac{ep}{1 \pm \cos\theta}$

$r(0) = 6$



Directrix to the right

$(6 < 10)$

$r(0) = 6$

$\frac{ep}{1+e\cos(0)} = \frac{ep}{1+e} = 6 \Rightarrow$

$p = 6\left(\frac{1+e}{e}\right)$

$r(\pi) = 10 \Rightarrow$

$\frac{ep}{1+e\cos(\pi)} = \frac{ep}{1-e} = 10 \Rightarrow$

$p = 10\left(\frac{1-e}{e}\right)$

$p = p$: $\frac{6+6e}{e} = \frac{10-10e}{e}$

$\Rightarrow 6+6e = 10-10e$

$-10-6e = -10-6e$

$-4 = -16e$

$\frac{4}{16} = \frac{1}{4} = e$

$\Rightarrow p = 6\left(\frac{1+e}{e}\right) = 6\left(\frac{1+\frac{1}{4}}{\frac{1}{4}}\right)$

$= 6\left(\frac{4+1}{1}\right) = 6(5) = 30 = p$

Write eq'n, etc.

$r(0)=6$ & $r(\pi)=10 \Longrightarrow$ solve for e if you want

$\frac{ep}{1+e} = 6$ $\qquad$ $\frac{ep}{1-e} = 10 \longrightarrow$

$ep = 6+6e$ $\qquad$ $ep = 10-10e$

$ep-6e = 6$ $\qquad$ $ep+10e = 10$

$e(p-6) = 6$ $\qquad$ $e(p+10) = 10$

$e = \frac{6}{p-6}$ $\qquad$ $e = \frac{10}{p+10}$ SET $\frac{6}{p-6}$

$\Longrightarrow 10p-60 = 6p+60$

$4p = 120$

$\boxed{p=30}$

$e = \frac{6}{p-6} = \frac{6}{30-6} = \frac{6}{24} = \boxed{\frac{1}{4} = e}$