10. 0/3 points LarTrig

Consider the following.

$x = 1 + \cos\theta$
$y = 1 + 7\sin\theta$

(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

(b) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

$$\frac{(x-1)^2}{1} + \frac{(y-1)^2}{49} = 1$$

Adjust the domain of the rectangular equation, if necessary.

- [−6, 8]
- (−8, 6)
- (0, 2)
- [0, 2]
- not necessary

?

$x = 1 + \cos\theta \implies x - 1 = \cos\theta$

$y = 1 + 7\sin\theta \qquad y - 1 = 7\sin\theta$

$\frac{y-1}{7} = \sin\theta$

$\sin^2\theta + \cos^2\theta = (x-1)^2 + \left(\frac{y-1}{7}\right)^2 = 1$

$\iff (x-1)^2 + \frac{(y-1)^2}{49}$

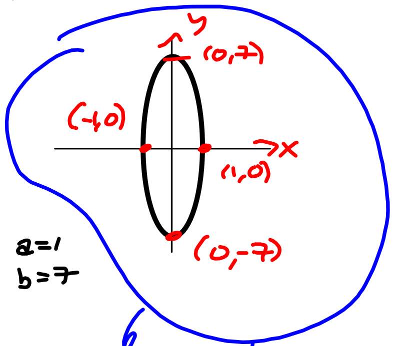
7^2

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$(h,k) = (h,k)$

$a = \longleftrightarrow$ from (h,k)

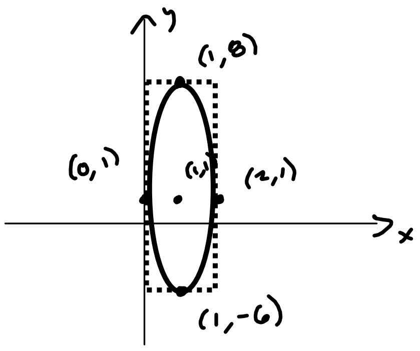
$b = \updownarrow$ " "



$a = 1$
$b = 7$

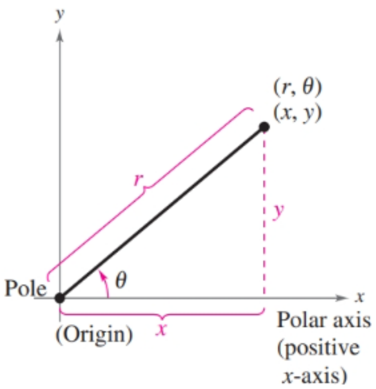
Right shape. wrong center!

$(h,k) = (1,1)$



Coordinate Conversion

The polar coordinates (r, θ) and the rectangular coordinates (x, y) are related as follows.



Polar-to-Rectangular

$x = r\cos\theta$

$y = r\sin\theta$

$\frac{x}{r} = \cos\theta \Rightarrow$

$\frac{y}{r} = \sin\theta \Rightarrow$

Rectangular-to-Polar

$\tan\theta = \frac{y}{x}$

$r^2 = x^2 + y^2$

$TAN^{-1}\left(\frac{y}{x}\right)$

$= \arctan\left(\frac{y}{x}\right)$ isn't necessarily in the right quadrant.

7. 0/7 points S6.7

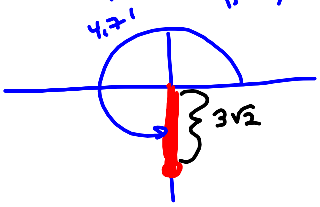
Plot the point given in polar coordinates.

$(3\sqrt{2}, 4.71) = (r, \theta)$

$r = 3\sqrt{2} \approx 4.2$ ish $(3(1.4)$ ish$)$

$4.71 \approx \frac{3\pi}{2}$ How?

$(4.71)\left(\frac{180^\circ}{\pi}\right) \approx 270^\circ$




```
4.71*180/π
    269.8631215
```

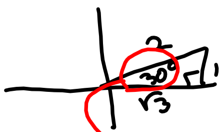
Tricky ones involving arctangent

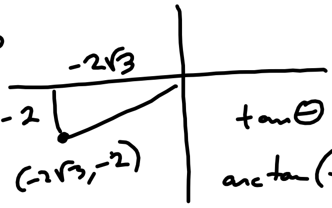
Convert $(-2\sqrt{3}, -2)$ to Polars

Calculator:

$\arctan\left(\frac{2}{2\sqrt{3}}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

$= \frac{\pi}{6}$



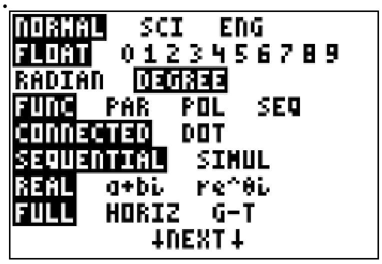


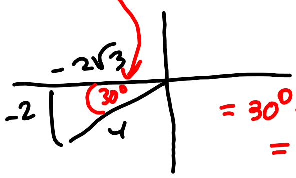
$\tan\theta = \frac{-2}{-2\sqrt{3}}$

$\arctan(\tan\theta) = \arctan\left(\frac{1}{\sqrt{3}}\right)$

$= 30^\circ$

I prefer to use degrees, because they're easier to see.





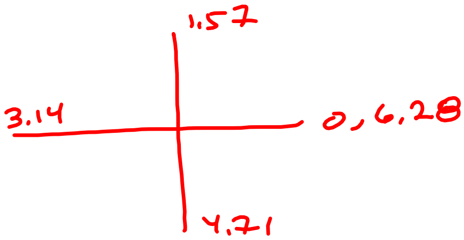
$= 30^\circ + 180^\circ = 210^\circ$

$= \frac{7\pi}{6}$

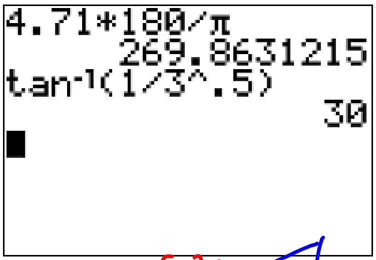
$\tan\theta = \frac{-2}{-2\sqrt{3}}$

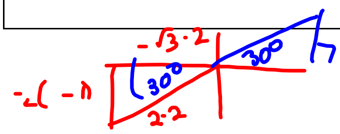
$\arctan(\tan\theta) = \arctan\left(\frac{1}{\sqrt{3}}\right)$

$= 30^\circ$



$(x,y) = (-2\sqrt{3}, -2)$



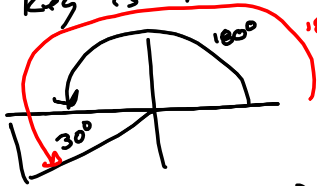


But we know we're in QIII

$180° < \theta < 270°$

Key is reference angle will be 30°

$180° + 30° = 210° = \boxed{\frac{7\pi}{6} = \theta}$



$r^2 = 2^2 + (2\sqrt{3})^2$

$= 4 + 4\cdot 3 = 4 + 12 = 16$

EXACT: $\left(4, \pi + \arctan\left(\frac{1}{\sqrt{3}}\right)\right)$ $\Rightarrow r = \pm\sqrt{16} = \pm 4.$

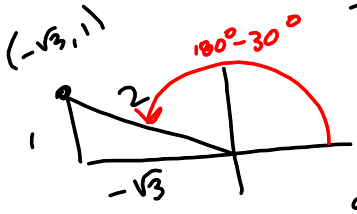
We take $r = 4$.

Also: $\frac{\pi}{6}$ & $r = -4$ would put you there.

Arctangent's in the wrong quadrant for points in QII

Recall

$R(\text{arctangent}) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = (-90°, 90°)$



$\arctan\left(-\frac{1}{\sqrt{3}}\right) = -30° \leadsto$ QII w/ 30°

reference is:

$(2, 150°) = (2, 180° - 30°) = \left(2, \pi - \frac{\pi}{6}\right) = \left(2, \frac{5\pi}{6}\right)$

$= \left(2, \pi + \arctan\left(-\frac{1}{\sqrt{3}}\right)\right)$

Don't be formulaic.

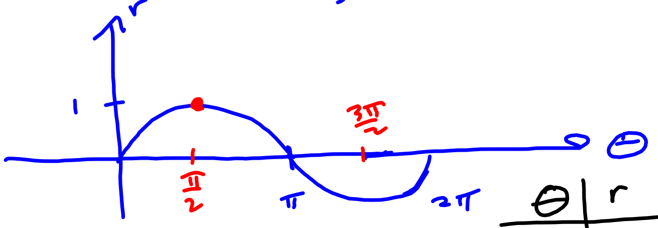
Section 6.8 - Graphs in Polar Coordinates.

I want you to READ the symmetry tests. We'll use them next time.

For now, I'm looking for the basic skill.

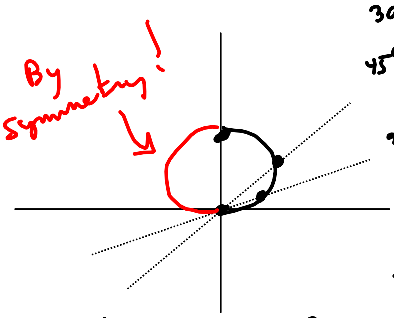
Graph $r = \sin\theta$

Sketch in Rectangular Form:



θ		r
30°	$\frac{\pi}{6}$	$\frac{1}{2}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
90°	$\frac{\pi}{2}$	1
	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$

By symmetry!



Turns out to be a circle.

$(-r,-\theta)$: $-r = \sin(-\theta)$
$r = \sin\theta$ yes

$(r, \pi-\theta)$ Not Needed.
Already have Positive Test.

$r = \sin(\pi - \theta)$
$= \sin(\pi)\cos(-\theta)$
$+ \sin(-\theta)\cos(\pi)$
$= 0\cdot\cos(-\theta) - (\sin\theta)(-1)$
$r = +\sin\theta$
SAME!

Symmetry thru the pole:

$r = \sin\theta$
$r = \sin(-\theta)$? $\Rightarrow r = -\sin(\theta)$ Not Same

Try $(r, \theta+\pi)$ $r = \sin(\theta+\pi) = \sin\theta\cos\pi + \sin\pi\cos\theta$
$= -\sin\theta$ Not Same.

No symmetry thru the pole.

Polar Axis Symmetry

$(r,\theta) \longmapsto (r, -\theta)$
OR
$(r,\theta) \longmapsto (-r, \pi-\theta)$

$r = \sin\theta$
$r = \sin(-\theta)$?
$r = -\sin\theta$ Not SAME

No polar-axis symmetry for $r = \sin\theta$

$-r = \sin(\pi - \theta)$
$= \sin\pi\cos(-\theta) + \sin(-\theta)\cos(\pi)$
$= 0 + -\sin\theta(-1)$
$\Rightarrow -r = \sin\theta$
Not Same!

Do you know what Spirograph is?

Symmetry thru the pole:

$r = \sin\theta$

$r = \sin(-\theta)$? $\Rightarrow r = -\sin(\theta)$ Not Same

Try $(r, \theta+\pi)$ $\quad r = \sin(\theta+\pi) = \sin\theta\cos\pi + \sin\pi\cos\theta$

$= -\sin\theta$ Not Same.

No symmetry thru the pole.

Polar Axis Symmetry

$(r,\theta) \longmapsto (r, -\theta)$

OR

$(r,\theta) \longmapsto (-r, \pi-\theta)$

$r = \sin\theta$

$r = \sin(-\theta)$?

$r = -\sin\theta$ Not SAME

No polar-axis symmetry for $r = \sin\theta$

$-r = \sin(\pi-\theta)$

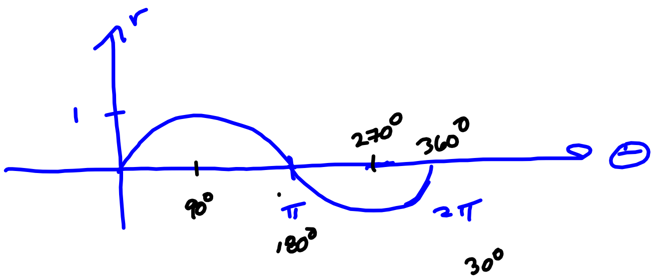
$= \sin\pi\cos(-\theta) + \sin(-\theta)\cos(\pi)$

$= 0 + -\sin\theta(-1)$

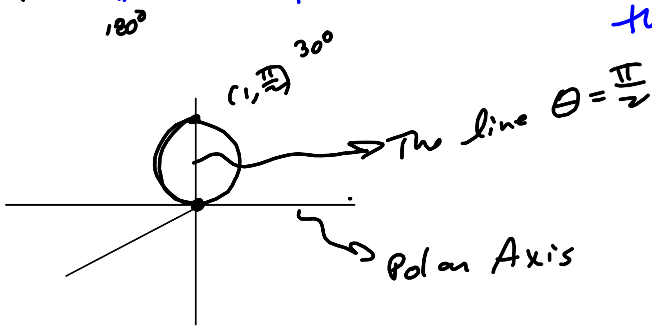
$\Rightarrow -r = \sin\theta$

Not Same!

Do you know what Spirograph is?

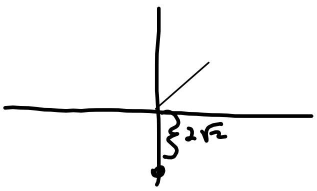


0 to 2π traces out the circle twice!



6.7 #7

$(2\sqrt{2}, 4.71)$



$4.71 \approx \frac{3\pi}{2} = 270^\circ$

$\boxed{(-2\sqrt{2}, \frac{\pi}{2})} \approx \boxed{(-2\sqrt{2}, 1.57) \ \#2}$

Add/subtract 2π

$\frac{\pi}{2} + 2\pi > 2\pi$ ✗

$\frac{\pi}{2} - 2\pi = \frac{\pi}{2} - \frac{4\pi}{2} = -\frac{3\pi}{2}$

$\boxed{(-2\sqrt{2}, -\frac{3\pi}{2})} \approx \boxed{(-2\sqrt{2}, -4.71) \ \#1}$

$(2\sqrt{2}, \frac{3\pi}{2})$

$\frac{3\pi}{2} - 2\pi = \frac{3\pi - 4\pi}{2} = -\frac{\pi}{2}$

$(2\sqrt{2}, -\frac{\pi}{2}) \approx \boxed{(2\sqrt{2}, -1.57) \ \#3}$

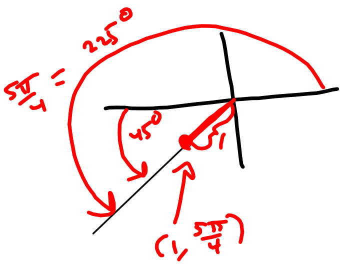
S6.7

5. 0/7 points

Plot the point given in polar coordinates.

$\left(1, \frac{5\pi}{4}\right) = (r, \theta) = (1, 225^\circ) \in Q\,III$

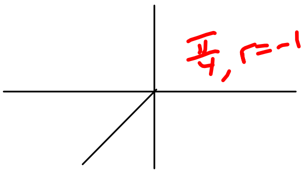
45° reference angle

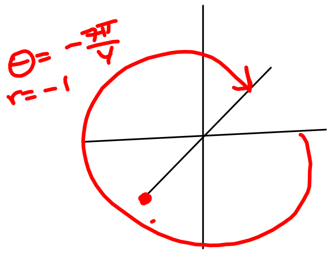


For more representations

$r = 1,\ \theta = \frac{5\pi}{4}$

$r = -1,\ \theta = \frac{\pi}{4}$





Find three additional polar representations of the point, using $-2\pi < \theta < 2\pi$. (Enter your answers in order from smallest to largest first by r-value, then by θ-value.)

$(r, \theta) = \left(\boxed{} \ \boxed{-1}, \boxed{} \ \boxed{\frac{-7\pi}{4}} \right)$

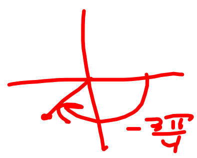
$(r, \theta) = \left(\boxed{} \ \boxed{-1}, \boxed{} \ \boxed{\frac{\pi}{4}} \right)$

$(r, \theta) = \left(\boxed{} \ \boxed{1}, \boxed{} \ \boxed{\frac{-3\pi}{4}} \right)$

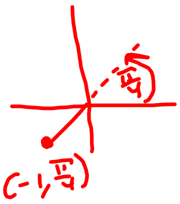
$\boxed{-\frac{3\pi}{4} = \theta \quad r = 1}$

NOTE $\frac{5\pi}{4} + 2\pi > 2\pi$, so we have to be clever.

$\frac{5\pi}{4} - 2\pi = \frac{5\pi - 8\pi}{4} = -\frac{3\pi}{4} \rightsquigarrow \left(1, -\frac{3\pi}{4}\right)$



Now, use $r = -1$ (same length)



$\left(-1, \frac{\pi}{4}\right)$!

See how $-r$ works?

Now, subtract 2π to get the 3rd one:

$\frac{\pi}{4} - 2\pi = \frac{\pi - 8\pi}{4} = -\frac{7\pi}{4} \rightsquigarrow \left(-1, -\frac{7\pi}{4}\right)$