Assume $t > 0$

6.6 Parametric Equations.

gravity $y = -t^2 + 1$

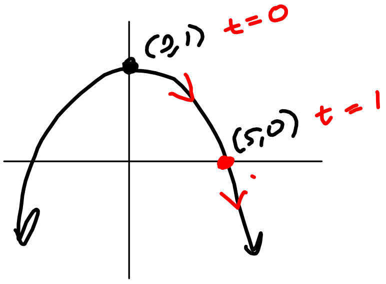
wind $x = 5t$

Falling ball

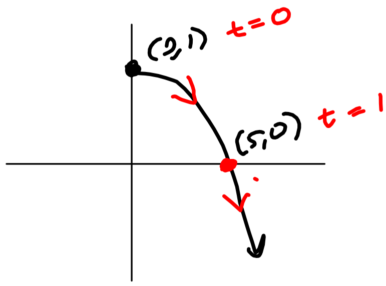
$\downarrow$ $\rightarrow$ Falling body : $h = -\frac{1}{2}gt^2 + v_0 t + h_0$

$\Rightarrow \frac{1}{5}x = t$

$\Rightarrow y = -\left(\frac{1}{5}x\right)^2 + 1 = -\frac{1}{25}x^2 + 1$



But $t < 0$ is silly, so



Rocket ship in $\mathbb{R}^3$

$\langle x, y, z \rangle = \langle 3t+1, 5t^2 - 7, 8t \rangle$ Vector-valued function parametrized by t = time.

$$\sin^2(t)+\cos^2(t)=1$$

$x=1+2\cos(t)$
$y=1+5\sin(t)$ describes an ellipse!

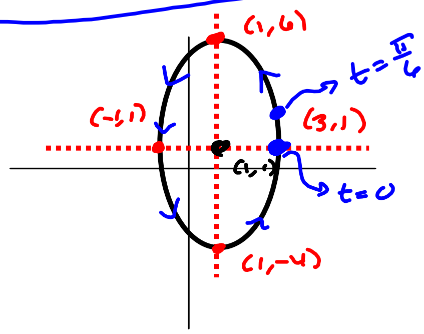
$x-1=2\cos(t)$ $\qquad$ $y-1=5\sin(t)$

$\frac{x-1}{2}=\cos(t)$ $\qquad$ $\frac{y-1}{5}=\sin(t)$ $\longrightarrow$ Eliminating the parameter.

$$\left(\frac{x-1}{2}\right)^2+\left(\frac{y-1}{5}\right)^2=\sin^2(t)+\cos^2(t)=1$$

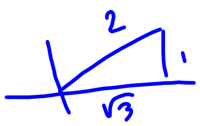
$$\Rightarrow \frac{(x-1)^2}{2^2}+\frac{(y-1)^2}{5^2}=1$$

$(h,k)=(1,1)$



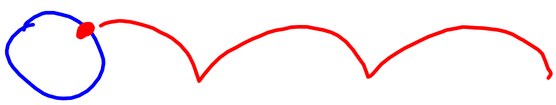
Rectangular equation has no orientation. For that, we need t.

t	x	y
0	3	1
$\frac{\pi}{6}$		



$x\left(\frac{\pi}{6}\right)=1+2\cos\frac{\pi}{6}=1+2\cdot\frac{\sqrt{3}}{2}=1+\sqrt{3}$

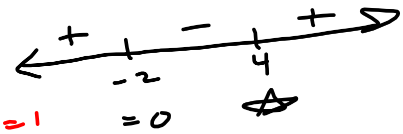
$y\left(\frac{\pi}{6}\right)=1+5\sin\frac{\pi}{6}=1+5\left(\frac{1}{2}\right)=\frac{7}{2}$

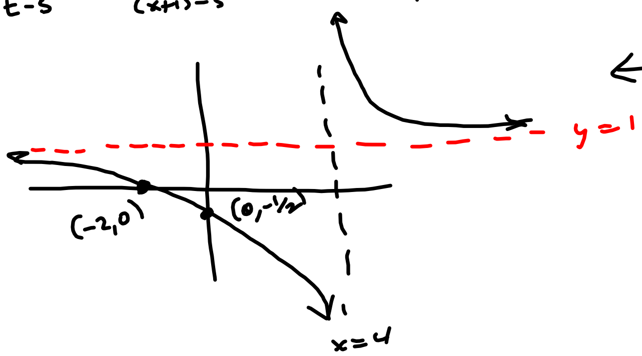


cycloid?

$x = t - 1 \longrightarrow x + 1 = t$

$$y = \frac{t+1}{t-5} = \frac{(x+1)+1}{(x+1)-5} = \frac{x+2}{x-4} = \frac{x\left(1+\frac{2}{x}\right)}{x\left(1-\frac{4}{x}\right)} = \frac{1+\underbrace{\frac{2}{x}}_{\to 0}}{1-\underbrace{\frac{4}{x}}_{\to 0}} \xrightarrow{x \to \pm\infty} 1 = y$$



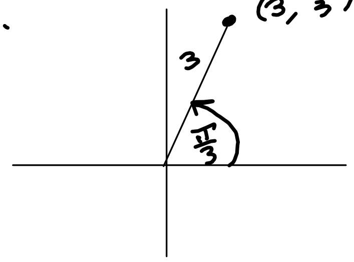


Section 6.7 - Polar coordinates

(x,y) – Rectangular

(r,θ) – Polar coordinates!

$(3, \frac{\pi}{3}) \longleftrightarrow (\frac{3}{2}, \frac{3\sqrt{3}}{2})$ in rect.



convert to rectangular

$\tan(\frac{\pi}{3}) = \frac{y}{x}$

$r = 3 = \sqrt{x^2+y^2}$

$\sin(\frac{\pi}{3}) = \frac{y}{r} \longrightarrow y = r\sin(\frac{\pi}{3}) = \frac{3\sqrt{3}}{2}$

$= r\sin\theta$

$\cos(\frac{\pi}{3}) = \frac{x}{r} \longrightarrow x = r\cos(\frac{\pi}{3})$

$x = r\cos\theta = \frac{3}{2}$

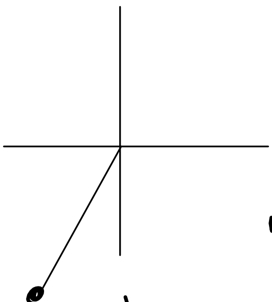
$\boxed{(x,y) = (\frac{3}{2}, \frac{3\sqrt{3}}{2})}$

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

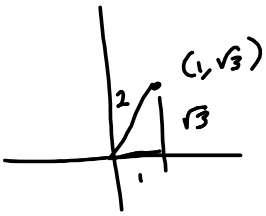
But $(-3, \frac{\pi}{3})$ is *also* a legit "point."



r can be negative?!

$(-3, \frac{\pi}{3})$!

Convert $(1, \sqrt{3})$ to polar coords



$$\theta = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

$$r = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$$

$$\Longrightarrow (r, \theta) = \left(2, \frac{\pi}{3}\right)$$

$$= \left(-2, \frac{4\pi}{3}\right)$$

$$= \left(2, \frac{7\pi}{3}\right)$$

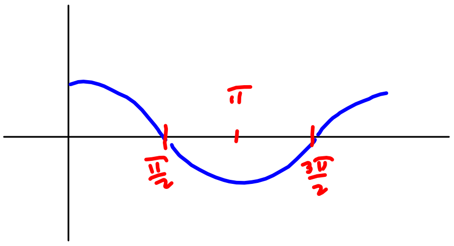
arctangent is used for converting from rect. to polar. So is

$$x^2 + y^2 = r^2$$

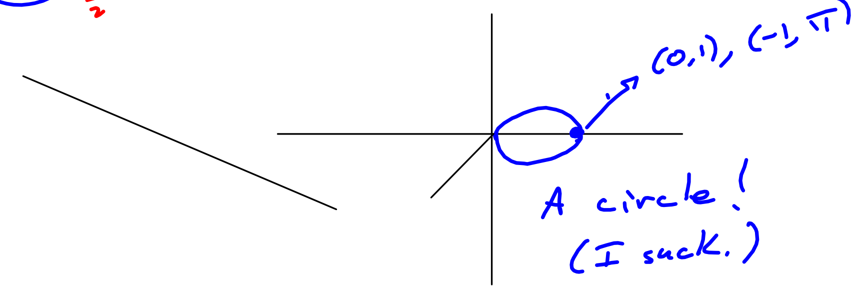
Infinitely many ways to represent in polar coordinates.

Rectangular coords

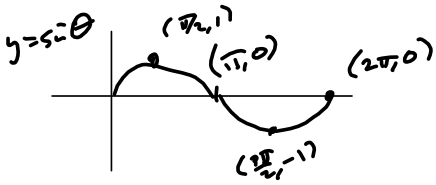
$r = \cos\theta$

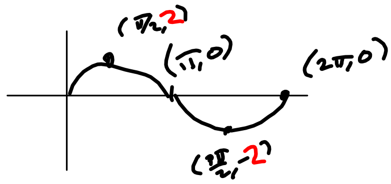


Polar

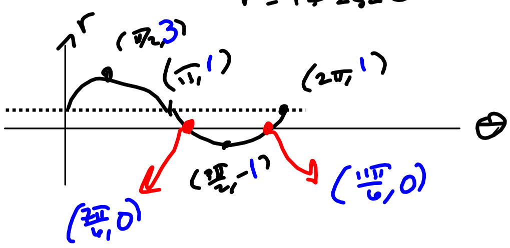


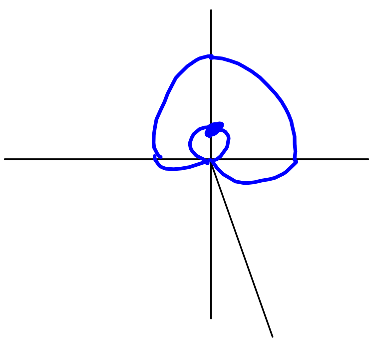
$1 + 2\sin\theta$





$r = 1 + 2\sin\theta$

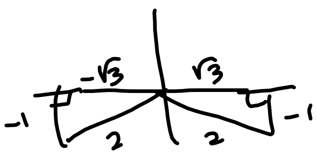




$1 + 2\sin\theta = 0 \Rightarrow$

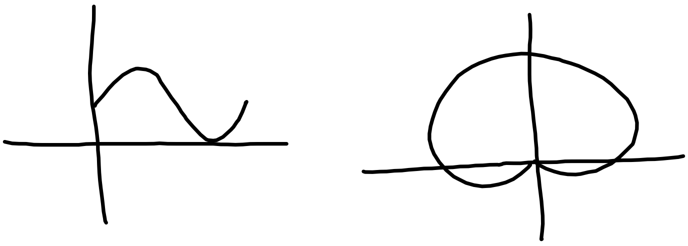
$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$



$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ $(210°, 330°)$

$r = 1 + 1\sin\theta$



Don't be shy about graphing these on a graphing utility, but you don't want to crutch it.