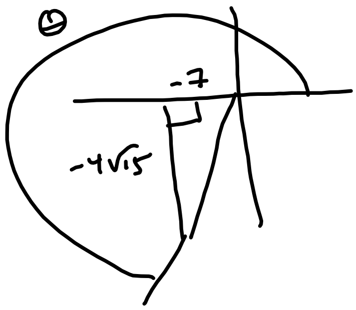
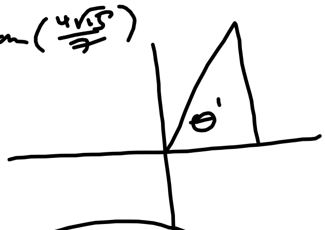
4.4 #14

$z = -7 - 4\sqrt{15}\, i$



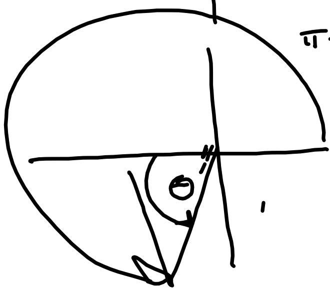
$\tan\theta = \frac{-4\sqrt{15}}{-7}$ & $\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{4\sqrt{15}}{7}\right)$
$= \theta'$

$\arctan\left(\frac{4\sqrt{15}}{7}\right)$



$\pi + \arctan\left(\frac{4\sqrt{15}}{7}\right)$

$\approx$ 4.287999272



$|z| = \sqrt{(4\sqrt{15})^2 + 7^2}$
$= \sqrt{16 \cdot 15 + 49} = \sqrt{289} = 17$

$\Longrightarrow z \approx 17(\cos(4.29) + i \sin(4.29))$

$z = r(\cos\theta + i\sin\theta) \longrightarrow$

$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

&

$z^{\frac{1}{n}} = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)\right)$, $k = 0, 1, 2, \ldots, n-1$

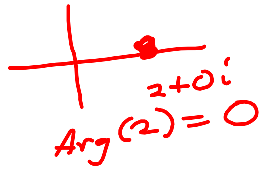
$k=0$ is the principal n^{th} root of all the rest are built off it, by adding the increment $= \frac{2\pi}{n}$ to the argument.

$k=0$: $\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n}\right) + i\sin\left(\frac{\theta}{n}\right)\right)$

E Find all solutions of $z^4 = 16$. (or $z^n - 16 = 0$)

real solution: $\sqrt[4]{z^4} = \sqrt[4]{16} = 2 = 2 + 0i = 2(\cos(0) + i\sin(0))$

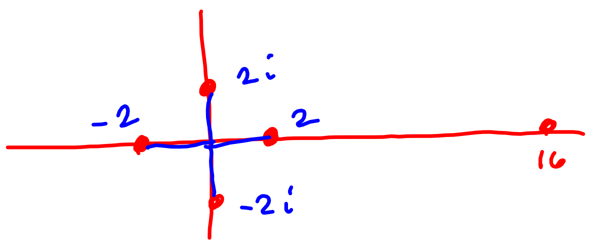
$k=0$ $= 2\left(\cos\left(\frac{0}{4}\right) + i\sin\left(\frac{0}{4}\right)\right) = \sqrt[4]{z}$.



$k=1$: $0 + \frac{2\pi}{4} = \frac{\pi}{2}$ $\quad 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$k=2$: $\frac{\pi}{2} + \frac{2\pi}{4} = \pi$ $\quad 2(\cos(\pi) + i\sin(\pi))$

$k=3$: $\pi + \frac{\pi}{2} = \frac{3\pi}{2}$ $\quad 2\left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right)$



Another way
$z^2 + 4 = z^2 - (2i)^2$

OLD-SCHOOL

$z^4 - 16 = (z^2)^2 - 4^2 = (z^2 - 4)(z^2 + 4)$

$= (z-2)(z+2)(z-2i)(z+2i)$

$\Rightarrow z = \pm 2, \pm 2i$

Another way: $z^2 + 4 = 0$

$z^2 = -4$

$z = \pm\sqrt{-4} = \pm 2i$

Another example:

Find all 4^{th} roots of $z = 32\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$

$$\sqrt[4]{z} = \sqrt[4]{32}\left(\cos\left(\frac{\frac{\pi}{3}}{4}\right) + i\sin\left(\frac{\frac{\pi}{3}}{4}\right)\right)$$

$2 \,|\, 32$, $2 \,|\, 16$, $2 \,|\, 8$, $2 \,|\, 4$, 2 → $2\sqrt[4]{2}$

$k=0$ $\quad = \boxed{2\sqrt[4]{2}\left(\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right)}$

Increment $= \frac{2\pi}{n} = \frac{2\pi}{4} = \frac{\pi}{2}$ or $\frac{6\pi}{12}$

$k=1$ $\quad \frac{\pi}{12} + \frac{\pi}{2} = \frac{\pi}{12} + \frac{6\pi}{12} = \frac{7\pi}{12}$

$k=2$ $\quad \frac{7\pi}{12} + \frac{6\pi}{12} = \frac{13\pi}{12}$

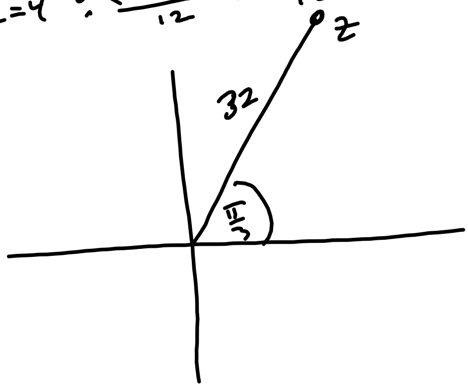
$k=3$ $\quad \frac{13\pi}{12} + \frac{6\pi}{12} = \frac{19\pi}{12}$

$2\sqrt[4]{2}\left(\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right)$

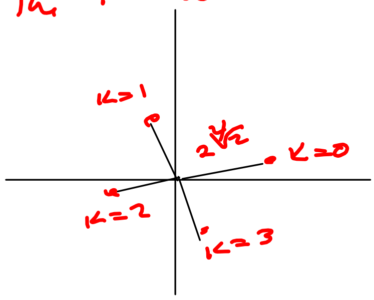
$2\sqrt[4]{2}\left(\cos\left(\frac{13\pi}{12}\right) + i\sin\left(\frac{13\pi}{12}\right)\right)$

$2\sqrt[4]{2}\left(\cos\left(\frac{19\pi}{12}\right) + i\sin\left(\frac{19\pi}{12}\right)\right)$

Check $k=4$: $\frac{(19+6)\pi}{12} = \frac{25\pi}{12} = \frac{24\pi}{12} + \frac{1\pi}{12} = 2\pi + \frac{\pi}{12}$ is coterminal with the where we started!



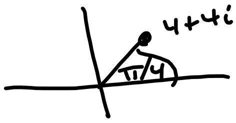
The 4^{th} Roots.



$z = 4+4i = 4\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$

Find z^6

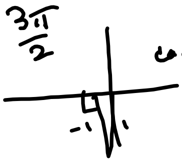
$\sqrt{4^2+4^2} = \sqrt{32} = 4\sqrt{2}$



$\arctan\left(\frac{4}{4}\right) = \arctan(1) = \frac{\pi}{4}$

& $\frac{\pi}{4} \in QI$

& $z \in QI$



$\cos\left(\frac{3\pi}{2}\right)=0$

$\sin\left(\frac{3\pi}{2}\right)=-1$

Scratch: $\left(\frac{\pi}{4}\right)(6) = \frac{6\pi}{4} = \frac{3\pi}{2}$

$\Rightarrow z^6 = 4^6\sqrt{2}^6\left(\cos\left(\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{2}\right)\right)$

$= 4^6 \cdot 2^3 (\ldots)$

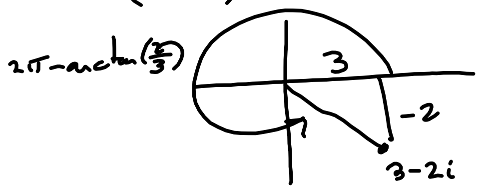
$= 2^{12} \cdot 2^3 (\ldots)$

$= 2^{15}(0 - i)$

$= -2^{15} i = -32768 i$

32768

$(3-2i)^6 = z^6$



$\sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13}$

$\sqrt{13}^6 = 13^3 = 13(169) =$ 2197

$\text{Arg}(z) = \arctan(-\frac{2}{3})$

If you want to keep
$\text{Arg}(z) \in [0, 2\pi)$
then use $2\pi - \arctan(\frac{2}{3})$
$= 2\pi + \arctan(-2/3)$

$z^6 = 2197\left(\cos\left(6\left(2\pi - \arctan\left(\frac{2}{3}\right)\right)\right) + i\sin\left(6\left(2\pi - \arctan\left(\frac{2}{3}\right)\right)\right)\right)$

$= 2197\left(\cos\left(12\pi - 6\arctan(2/3)\right) + i\sin\left(12\pi - 6\arctan(2/3)\right)\right)$

$= 2197\left(\cos\left(-6\arctan(2/3)\right) + i\sin\left(-\arctan(2/3)\right)\right)$

$= 2197\left(\cos\left(6\arctan(2/3)\right) - i\sin\left(6\left(\arctan(2/3)\right)\right)\right)$

$= 2197\left(-0.926263086 - i\left(-0.3768775603\right)\right)$

$-2035 + 828\,i$

Quadratic Formula

$2x^2 - 3x + 3 = 0 \longrightarrow$

$b^2 - 4ac = (-3)^2 - 4(2)(3) = 9 - 24 = -15 \longrightarrow$ 2 complex nonreal zeros.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-3) \pm \sqrt{-15}}{2(2)} = \boxed{\frac{3 \pm i\sqrt{15}}{4} = z}$

$= \frac{3}{4} \pm \frac{\sqrt{15}}{4} i$

"imaginary"

$z = 3i$ is pure imaginary
$z = 2+3i$ is complex, nonreal
(WebAssign says "imaginary")

$x^4 - 2x^2 - 3 = u^2 - 2u - 3 = 0 \Rightarrow u = x^2$

Quadratic in form

$b^2 - 4ac = (-2)^2 - 4(1)(-3) = 4 + 12 = 16$

$u = \frac{2 \pm \sqrt{16}}{2(1)} = \frac{2 \pm 4}{2} = \frac{1 \pm 2}{1} = 3$ OR -1

$u = x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

$u = x^2 = -1 \Rightarrow x = \pm\sqrt{-1} = \pm i$

$\pm\sqrt{3}, \pm i$

Write the polynomial as the product of linear factors.

$g(x) = x^3 + 3x^2 - 3x - 9$

$= x^2(x+3) - 3(x+3)$

$= (x+3)(x^2-3) = 0 \Rightarrow$

$\Rightarrow (x+3)(x-\sqrt{3})(x+\sqrt{3})$

$x^2 - 3 = x^2 - (\sqrt{3})^2 = a^2 - b^2$

$= (a-b)(a+b) = (x-\sqrt{3})(x+\sqrt{3})$

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the giver zero in your answer.)

Function	Zero
$g(x) = x^3 - 10x^2 + 36x - 40$	$4 + 2i$

BOOK WAY

Book way: $x = 4+2i$ is a zero $\Rightarrow$
$x = 4-2i$ " " ", also, by conjugate pairs theorem

$\Rightarrow (x-(4+2i))$ & $(x-(4-2i))$ are factors $\Rightarrow$

$(x-4-2i)(x-4+2i)$ is a factor

$x^2 - 4x + 2ix - 4x + 16 - 8i - 2ix + 8i - 4i^2$

$= x^2 - 8x + 20$

$$\begin{array}{r} x - 2 \\ x^2-8x+20 \overline{\smash{)}\, x^3 - 10x^2 + 36x - 40} \\ -(x^3 - 8x^2 + 20x) \\ \hline -2x^2 + 16x - 40 \\ -(-2x^2 + 16x - 40) \\ \hline 0 \end{array}$$

$\frac{x^3}{x^2} = x$

$-\frac{2x^2}{x^2} = -2$

zeros: $x = 4 \pm 2i, 2$

& $P(x) = (x-(4+2i))(x-(4-2i))(x-2)$

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the given zero in your answer.)

Function	Zero
$g(x) = x^3 - 10x^2 + 36x - 40$	$4 + 2i$

MY WAY

$4+2i$ is zero $\Rightarrow$

$x-(4+2i)$ is factor. Divide.

```
4+2i | 1     -10      36       -40
     |       4+2i   -28-4i     40
     ----------------------------
4-2i | 1    -6+2i   8-4i       0
     |       4-2i   -8+4i
     ----------------------------
       1     -2      0
```

$x-2$

$(x-2)(x-(4+2i))(x-(4-2i))$

$(4+2i)(-6+2i)$
$= -24+8i-12i+4i^2$
$= -24-4i-4$
$= -28-4i$

$(4+2i)(8-4i)$
$= 2\cdot 4(2+i)(2-i)$
$= 8(2^2+1^2)$
$= 40$