I did some shuffling around of Chapter 4 Assignments. Needed to clean up the overlap between sections of Chapter 4. I think it's because I built the course off the 9th Edition and they changed just enough to call it a 10th Edition, accomplishing nothing but messing up existing homework assignments.

Sorry about that. They *said* they would update the assignments for the next edition, and for the most part, they did, but they really messed with 4.3 - 4.5. 9th edition didn't even have a 4.5. DeMoivre was covered in 4.4.

I re-scheduled 4.3 - 4.5, and pushed Test 4 back to Tuesday from the Sunday deadline.

There are questions in the 4.3 WebAssign that are from 4.4 in the 10th Edition. Sorry!

Click Here for Course Schedule for the Latest Revision.
(Link also pasted into chat.)

4.3 - Due Thursday - I did no distance or midpoint, yet.

4.4 - Due Sunday

4.5 - Due Monday

Test 4 - Due Tuesday.

Sorry about the 10th-edition mix-ups, but the pedagogy and sequencing are still the same, although distributed to different sections.

You have seen most of 4.4 material and heard mention of DeMoivre's Theorem. DeMoivre's Theorem is Section 4.5 stuff.

So, today, examples from 4.3, 4.4, and discussion of 4.5, with examples.

A - 79-91 (91,100) (79,90) $y = \frac{100-90}{91-79}(x-91)+100$

$= \frac{10}{12}(x-91)+100 = \boxed{\frac{5}{6}(x-91)+100}$ A

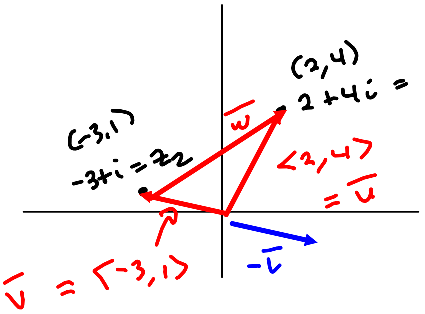
B - 65-78 (78,89) (65,80) $y = \frac{89-80}{78-65}(x-78)+89 = \boxed{\frac{9}{13}(x-78)+89}$ B

C - 45-64 (64,79) (45,70) $\frac{79-70}{64-45}(x-64)+79 = \boxed{\frac{9}{19}(x-64)+79}$ C

D - 30-41 (41,69) (30,60) $\frac{69-60}{41-30}(x-41)+69 = \boxed{\frac{9}{11}(x-41)+69}$ D

F - 0-29 (0,0) (29,59) $\boxed{\frac{59}{29}x}$ F

§4.3 Points in the plane
vectors
complex #s



$z_1 - z_2 = w$

$\bar{u} - \bar{v} = \bar{w}$

Distance between z_1 & z_2 is just what you'd hope & expect.

Real: $D = \sqrt{(2-(-3))^2 + (4-1)^2}$

$= \sqrt{5^2 + 3^2} = \sqrt{34}$

Complex: The modulus of the difference, i.e., $|z_1 - z_2|$

$= |2-(-3) + (4-1)i| = |5+3i|$

$\sqrt{5^2+3^2} = \sqrt{34}$ OR

$|w| = \sqrt{w\tilde{w}} = \sqrt{(5+3i)(5-3i)}$

$= \sqrt{5^2+3^2} = \sqrt{34}$

As a vector, it would be

$\|\bar{u} - \bar{v}\| = \|\langle 2,4\rangle - \langle -3,1\rangle\| = \|\bar{w}\|$

$= \|\langle 5, 3\rangle\|$

$= \sqrt{\bar{w}\cdot\bar{w}} = \sqrt{\langle 5,3\rangle \cdot \langle 5,3\rangle}$

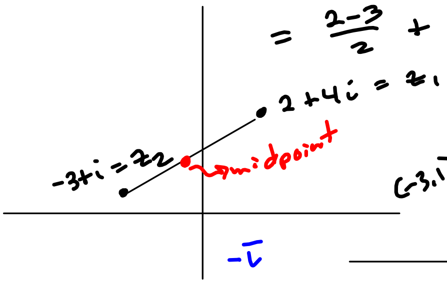
$= \sqrt{5^2+3^2} = \sqrt{34}$

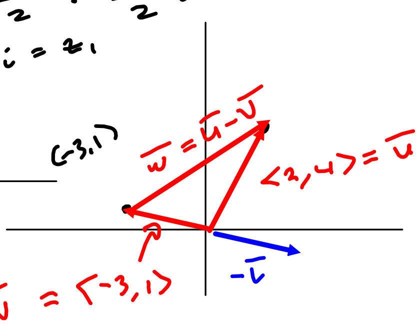
All the same.
Just different notation & ideas

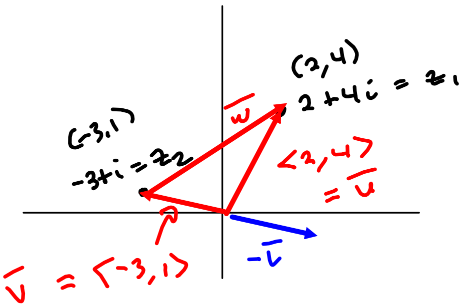
Midpoint: Same as in the plane:

$2+4i$ & $-3+i$ midpoint

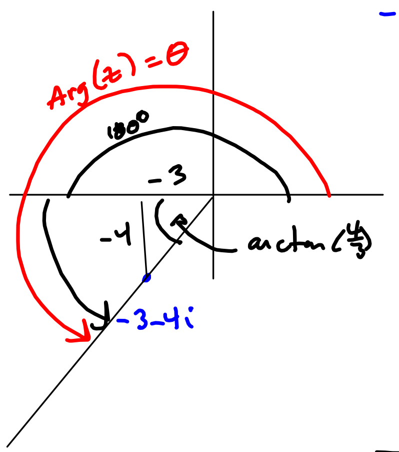
$= \frac{2-3}{2} + \frac{4+1}{2}i = -\frac{1}{2} + \frac{5}{2}i$







4.4 - Trig Form of Complex Numbers

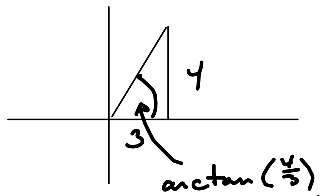


$-3-4i = z$

Write in trig form.

$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$

Now, $\arctan\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{4}{3}\right) \approx 53.13010235^\circ$



$\Longrightarrow \theta = 180^\circ + \arctan\left(\frac{4}{3}\right) \approx 180^\circ - 53.13010235^\circ$

$= \boxed{126.8698976^\circ \approx \theta}$

Now we need $|z|$:

$|z| = \sqrt{3^2+4^2} = \sqrt{25} = 5 = |z|$

$z = |z|\left(\cos(\operatorname{Arg}(z)) + i\sin(\operatorname{Arg}(z))\right)$

$= |z|(\cos\theta + i\sin\theta)$

$\approx 5\left(\cos(126.8699^\circ) + i\sin(126.8699^\circ)\right)$

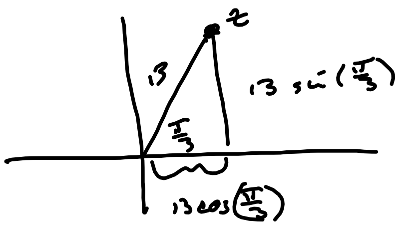
to $\underbrace{\text{4 digits to the right of decimal}}_{\text{4 decimal places.}}$

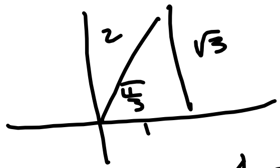
Go the other way:

$$z = 13\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 13\cos\frac{\pi}{3} + \left(13\sin\frac{\pi}{3}\right)i$$

$$= 13\cdot\frac{1}{2} + 13\cdot\frac{\sqrt{3}}{2}\,i$$

$$= \boxed{\frac{13}{2} + \frac{13\sqrt{3}}{2}\,i = z}$$





§4.4 Multiplying & Dividing in Trig Form

Multiply/Divide Moduli

&

Add/Subtract Arguments (angles), respectively

$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ $\Longrightarrow$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 z_2 = |z_1||z_2|\left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$

$= r_1 r_2 \left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$

r's are always positive until C 6, when things get weird.

$z_1 = 3\left(\cos\frac{\pi}{6} + i\sin\left(\frac{\pi}{6}\right)\right) = 3\left(\cos(30^\circ) + i\sin(30^\circ)\right)$

$z_2 = 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right) = 4\left(\cos(120^\circ) + i\sin(120^\circ)\right)$

$\Longrightarrow z_1 z_2 = 12\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$

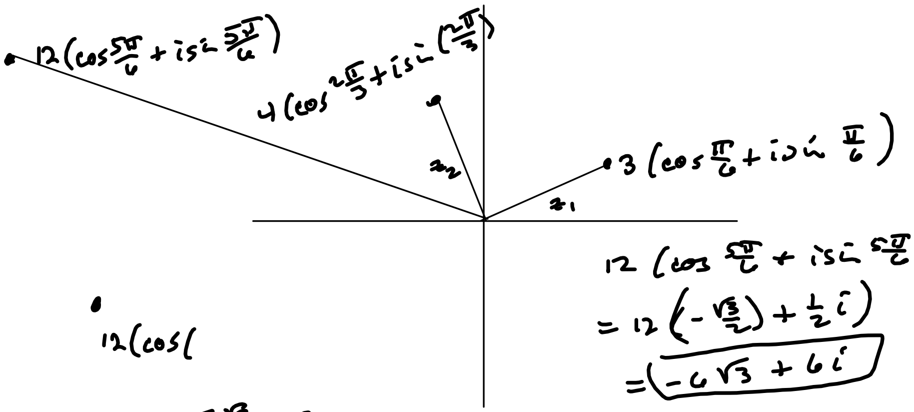
$\frac{2\pi}{3}\cdot\frac{2}{2} + \frac{\pi}{6} = \frac{5\pi}{6}$

$\frac{z_1}{z_2} = \frac{3}{4}\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right) = \frac{3}{4}(0 - i) = -\frac{3}{4}i$

$\frac{\pi}{6} - \frac{4\pi}{6} = -\frac{3\pi}{6} = -\frac{\pi}{2}$

$$z_1 = 3\left(\cos\frac{\pi}{6} + i\sin\left(\frac{\pi}{6}\right)\right) = 3\left(\cos(30^\circ) + i\sin(30^\circ)\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

$$z_2 = 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right) = 4\left(\cos(120^\circ) + i\sin(120^\circ)\right) = -2 + 2\sqrt{3}i$$



$$12\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$
$$= 12\left(\left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{2}i\right)$$
$$= \boxed{-6\sqrt{3} + 6i}$$

$$z_1 = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$$
$$z_2 = -2 + 2\sqrt{3}i$$

$$\Rightarrow z_1 z_2 = \left(\frac{3\sqrt{3}}{2} + \frac{3}{2}i\right)\left(-2 + 2\sqrt{3}i\right)$$
$$= (3\sqrt{3} + 3i)(-1 + \sqrt{3}i)$$
$$= -3\sqrt{3} + 3\cdot 3i - 3i + 3\sqrt{3}i^2$$
$$= -3\sqrt{3} + 6i - 3\sqrt{3}$$
$$= \boxed{-6\sqrt{3} + 6i}$$ See? It works!

If you're given complex numbers in trigonometric form, then multiplication and division are easy.

Section 4.5 - DeMoivre's Theorem.

$z = r(\cos\theta + i\sin\theta) \Rightarrow$

$z^2 = r(\cos\theta + i\sin\theta)(r(\cos\theta + i\sin\theta))$

$= (r)(r)(\cos(\theta+\theta) + i\sin(\theta+\theta))$

$= r^2(\cos(2\theta) + i\sin(2\theta))$

Suppose $z^K = r^K(\cos(K\theta) + i\sin(K\theta))$

Then $z^{K+1} = z^K \cdot z = r^K(\cos(K\theta) + i\sin(K\theta))(r(\cos\theta + i\sin\theta))$

$= r^{K+1}(\cos(K\theta+\theta) + i\sin(K\theta+\theta))$

$= r^{K+1}(\cos((K+1)\theta) + i\sin((K+1)\theta))$

$\Rightarrow$ $\boxed{z^n = r^n(\cos(n\theta) + i\sin(n\theta)) \ \forall\, n \in \mathbb{N} = \{1, 2, 3, \dots\}}$

ROOTS!

Consider $z^3 - 8 = 0 \Rightarrow$

$z^3 = 8 \Rightarrow$

$z = \sqrt[3]{z^3} = \sqrt[3]{8} = \boxed{2 = z}$ That's not all!

Old way of finding All the roots/solutions:

Divide $z^3 - 8$ by $z - 2$

$$\begin{array}{r} z^2 + 2z + 4 \quad r\ 0 \\ z-2 \,\overline{)\, z^3 + 0z^2 + 0z - 8} \\ -(z^3 - 2z^2) \\ \hline 2z^2 + 0z - 8 \\ -(2z^2 - 4z) \\ \hline 4z - 8 \\ -(4z - 8) \\ \hline 0 \end{array}$$

$\frac{z^3}{z} = z^2$

$\frac{2z^2}{z} = 2z$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \\ & z^2 & z^1 & c & r \end{array}$$

$z^2 + 2z + 4 = 0$

$\Rightarrow z^2 + 2z + 1^2 - 1 + 4$

$= (z+1)^2 + 3 = 0 \Rightarrow$

$(z+1)^2 = -3 \Rightarrow$

$z + 1 = \pm\sqrt{-3} = \pm\sqrt{3}\, i \Rightarrow$

$\boxed{z = -1 \pm \sqrt{3}\, i}$ are the other 2 3rd roots of 8.

3rd roots of 8:

Solutions of $z^3 - 8 = 0$

$2, -1 - \sqrt{3}\, i, -1 + \sqrt{3}\, i$

$8 = 8(\cos(0) + i\sin(0))$

$2 = \sqrt[3]{8}\left(\cos\left(\frac{0}{3}\right) + i\sin\left(\frac{0}{3}\right)\right)$ principal 3rd root

to get the rest, keep adding $\frac{2\pi}{3}$

$0 + \frac{2\pi}{3} = \frac{2\pi}{3}$

$\frac{2\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3}$

$z^2+2z+4=0$

$\Rightarrow z^2+2z+1^2-1+4$

$=(z+1)^2+3=0 \Rightarrow$

$(z+1)^2=-3 \Rightarrow$

$z+1=\pm\sqrt{-3}=\pm\sqrt{3}\,i \Rightarrow$

$\boxed{z=-1\pm\sqrt{3}\,i}$ are the other 2 3rd roots of 8.

3rd roots of 8:

Solutions of $z^3-8=0$

$2, -1-\sqrt{3}\,i, -1+\sqrt{3}\,i$

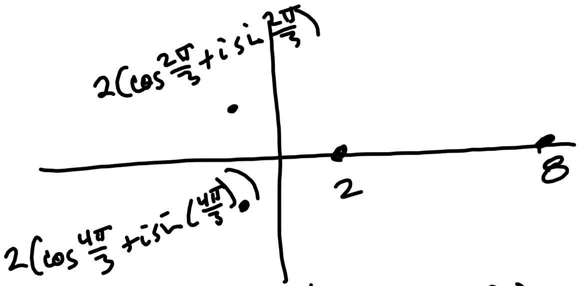
$8=8(\cos(0)+i\sin(0))$

$2=\sqrt[3]{8}\left(\cos\left(\frac{0}{3}\right)+i\sin\left(\frac{0}{3}\right)\right)$ principal 3rd root

to get the rest, keep adding $\frac{2\pi}{3}$

$0+\frac{2\pi}{3}=\frac{2\pi}{3}$

$\frac{2\pi}{3}+\frac{2\pi}{3}=\frac{4\pi}{3}$



$$z_0 = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n}\right) + i\sin\left(\frac{\theta}{n}\right)\right)$$

$$z_1 = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n} + \frac{2\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi}{n}\right)\right)$$

$$z_2 = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n} + 2\cdot\frac{2\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + 2\cdot\frac{2\pi}{n}\right)\right)$$

$$\vdots$$

$$z_k = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n} + \frac{2\pi}{n}k\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi}{n}k\right)\right)$$

$$k = 0, 1, \ldots, n-1$$