4.1

Perform the operation and write the result in standard form.

10. $\left(-2+\sqrt{-8}\right)+\left(7-\sqrt{-54}\right)$

$= -2+\sqrt{8}\,i + 7 - \sqrt{54}\,i$

$= -2+2\sqrt{2}\,i + 7 - 3\sqrt{6}\,i$

$5 + (2\sqrt{2}-3\sqrt{6})i$

$8 = 2 \cdot 2 \cdot 2 \Rightarrow 2\sqrt{2}$

$54 = 2 \cdot 3 \cdot 3 \cdot 3 \Rightarrow 3\sqrt{6}$

$\boxed{5+i\left(2\sqrt{2}-3\sqrt{6}\right)}$

4.3

Use a graphing utility to write the complex number in standard form. (Round all numerical values to four decimal places.)

13. $2\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right) = 2\cos\frac{\pi}{5} + 2i\sin\left(\frac{\pi}{5}\right)$

$\approx 1.618033989 + 1.175570505\,i \approx \boxed{1.6180 + 1.1756i}$

$\boxed{1.618 + 1.1756i}$

3.3 Applications

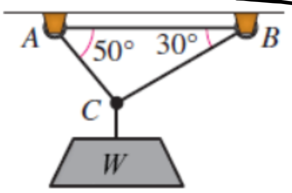
Use the figure to determine the tension in each cable supporting the load. (Round your answers to one decimal place.)

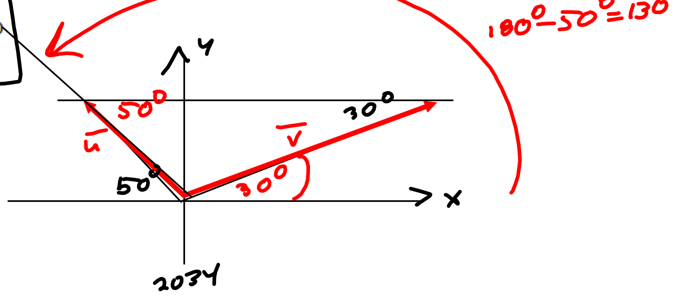
2. W = 2034 lb

tension in $\overline{AC}$ ✗ 1788.7 lb

tension in $\overline{BC}$ ✗ 1327.6 lb

Section 3.3 Applications deadline is not Wednesday, April 10th.





$\bar{u} + \bar{v} = \langle 0, 2034 \rangle$

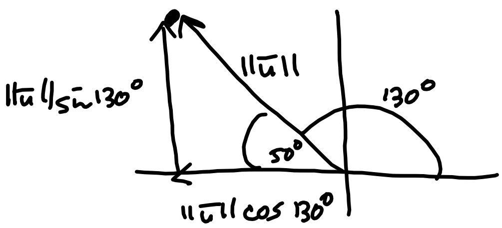
$\langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle 0, 2034 \rangle$

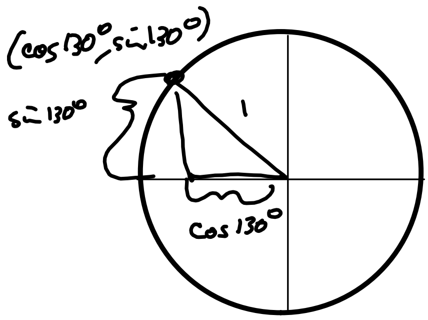
$u_1 + v_1 = 0$

$u_2 + v_2 = 2034$

$\bar{u} = (\|\bar{u}\|)$(unit vector in the direction of $\bar{u}$)

$= \|\bar{u}\| \langle \cos 130°, \sin 130° \rangle$





Want $\|\bar{u}\|$ & $\|\bar{v}\|$

$\bar{v} = \|\bar{v}\| \langle \cos 30°, \sin 30° \rangle$

Let $x = \|\bar{u}\|$, $y = \|\bar{v}\|$,

$\cos(130°) = a$, $\sin(130°) = b$

$\cos(30°) = c$, $\sin(30°) = d$

$\bar{u} + \bar{v} = \langle 0, 2034 \rangle$

$\langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle 0, 2034 \rangle$

$\left. \begin{array}{l} x \langle \cos 130°, \sin 130° \rangle = x \langle a, b \rangle = \bar{u} \\ y \langle \cos 30°, \sin 30° \rangle = y \langle c, d \rangle = \bar{v} \end{array} \right\} \Rightarrow$

$\bar{u} + \bar{v} = x \langle a, b \rangle + y \langle c, d \rangle = \langle 0, 2034 \rangle$

i.e. $\langle xa, xb \rangle + \langle yc, yd \rangle = \langle 0, 2034 \rangle$

$= \langle ax, bx \rangle + \langle cy, dy \rangle = \langle 0, 2034 \rangle$

$\Rightarrow \begin{array}{l} ax + cy = 0 \Rightarrow cy = -ax \Rightarrow \boxed{y = -\frac{ax}{c}} \\ bx + dy = 2034 \Rightarrow bx + d\left(-\frac{ax}{c}\right) = 2034 \end{array}$

$$\Rightarrow bx - \frac{ad}{c}x = 2034$$

$$\Rightarrow \left(b - \frac{ad}{c}\right)x = 2034$$

$$x = \frac{2034}{b - \frac{ad}{c}} = \frac{2034}{\sin(130^\circ) - \frac{\cos(130^\circ)\sin(30^\circ)}{\cos(30^\circ)}}$$

$\approx$ 1788.669581 lbs $\approx \|\vec{u}\| \approx$ 1788.7 lbs

$y = -\frac{a}{c}x \approx$ 1327.599213 lbs $\approx \|\vec{v}\| \approx$ 1327.6 lbs

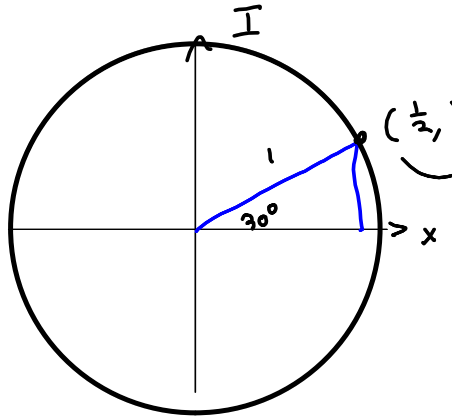
$$\frac{2034}{\sin(130) - \frac{\cos(130)\sin(30)}{\cos(30)}} = 1788.669581$$

$$-\frac{\cos(130)}{\cos(30)} \cdot \boxed{1788.669581} = 1327.599213$$

ans

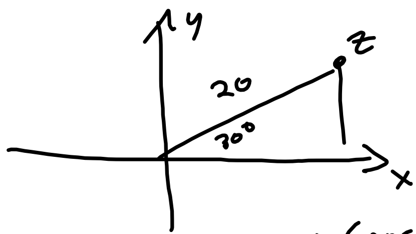
$1788.7\cos(130^\circ) \approx$

$1788.7\sin(30^\circ)$



$= \langle \cos 30°, \sin 30° \rangle$

$\underbrace{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}i\right) = (\cos 30°, i \sin 30°)}_{\text{Hybrid notation (Bridge from } \mathbb{R}^2 \text{ vectors)}} = \underbrace{\cos(30°) + i \sin(30°)}_{\text{Standard Notation}}$



$z = 20(\cos 30° + i \sin 30°)$

Length/Magnitude/Modulus of $z = a + bi = \sqrt{a^2 + b^2}$

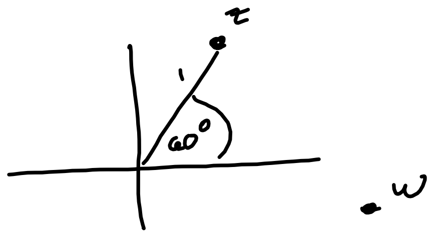
$\tilde{z} = a - bi$ = conjugate

The analogue of using dot product to find magnitude of a vector is

$|z| = \sqrt{z\tilde{z}} = \sqrt{(a+bi)(a-bi)} = \sqrt{a^2 + b^2}$

Arg(z) = angle its "vector" representation makes with the positive x-axis.

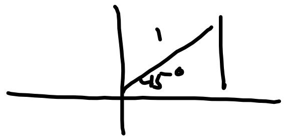
$z = |z|(\cos\theta + i \sin\theta)$
$w = |w|(\cos\phi + i \sin\phi)$

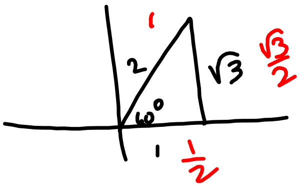


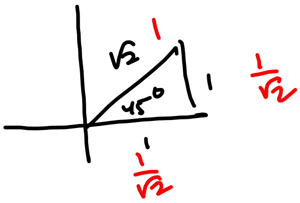
$$z = 1(\cos 60^\circ + i\sin 60^\circ) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

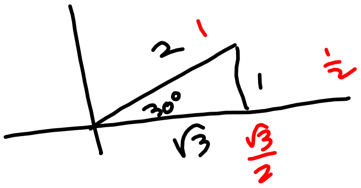
$$w = 1(\cos 45^\circ + i\sin(45^\circ)) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$









$$zw = ((\cos 60^\circ + i\sin 60^\circ)(\cos(45^\circ) + i\sin(45^\circ)))$$

$$= \cos(105^\circ) + i\sin(105^\circ)$$

I added the arg's.

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i + \frac{\sqrt{6}}{4}i + \frac{\sqrt{6}}{4}i^2$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} + \left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)i$$

$$\approx \boxed{-0.2588190451 + 0.9659258263\,i \approx zw}$$

$$(\cos(60^\circ) + i\sin(60^\circ))(\cos(45^\circ) + i\sin(45^\circ)) = \cos(105^\circ) + i\sin(105^\circ)$$

$$-0.2588190451 + 0.9659258263\,i$$

DeMoivre

If $z = |z|(\cos\theta + i\sin\theta)$

& $w = |w|(\cos\phi + i\sin\phi)$

$\Rightarrow zw = |z||w|(\cos(\theta+\phi) + i\sin(\theta+\phi))$

Thus, multiplication in the complex plane involves ROTATION. Multiplying complex numbers of magnitude 1 results in a pure rotation about the unit circle.

Roots of Unity $z = |z|(\cos\theta + i\sin\theta)$

$z^n = |z|^n(\cos(n\theta) + i\sin(n\theta))$

Powers!

Next Time Roots.

Get midterms and grade report back tonight/tomorrow morning.