§4.1

$i = \sqrt{-1}$ = imaginary unit.

complex #s $\{a+bi \mid a,b \in \mathbb{R}\} = \mathbb{C} \supseteq \mathbb{R}$

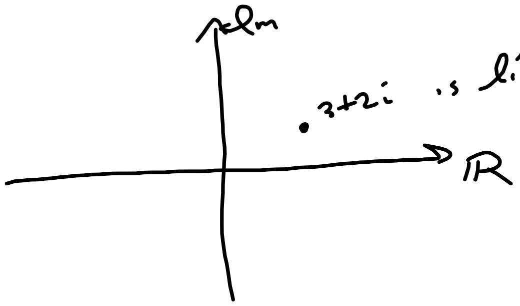
$\mathbb{R} = (-\infty, \infty)$ = Real numbers

$z = 3+2i$

Real part $= \mathrm{Re}(z) = 3$

Imaginary part $= \mathrm{Im}(z) = 2$

Complex Plane



$3+2i$ is like $(3,2)$

$\sqrt{-25} = \sqrt{(-1)(25)} = \sqrt{-1}\sqrt{25} = i\sqrt{25} = i(5) = 5i$

$\sqrt{-1348} = \sqrt{1348}\, i = 2\sqrt{337}\, i = \boxed{2i\sqrt{337}}$

$2 \,|\, 1348$
$2 \,|\, 674$
337

$2\sqrt{337}$

Primes 2, 3, 5, 7, 11, 13, 17, 19, 23,
29, 31, 37, 41,

Write $\frac{1}{2+i}$ in standard form. $(a+bi)$

$$\left(\frac{1}{2+i}\right)\left(\frac{2-i}{2-i}\right) = \frac{2-i}{5} = \frac{2}{5} - \frac{i}{5} = \boxed{\frac{2}{5} - \frac{1}{5}i}$$

$(a+b)(a-b) = a^2 - b^2$

$(2+i)(2-i) = 4 - i^2 = 4 - (-1) = 5$

Add/subtract:

$(3+2i) + (7-5i) = (3+7) + (2-5)i = 10 - 3i$

Like vectors:

$\langle 3,2\rangle + \langle 7,-5\rangle = \langle 10,-3\rangle$

multiplication:

$(3+2i)(5-7i) = 15 - 21i + 10i - \underline{14i^2} = 15 + 14 - 11i$

$= \boxed{29 - 11i}$

Complex Conjugate

$z = a+bi \Longrightarrow \bar{z} = a-bi = \tilde{z}$

Used to cancel out imaginary part, like I did with

$z = \frac{1}{2+i} = \left(\frac{1}{2+i}\right)\left(\frac{2-i}{2-i}\right) = \frac{2-i}{5} = \frac{2}{5} - \frac{1}{5}i$

$w = 2+i \Longrightarrow \tilde{w} = 2-i$

$w\tilde{w} = (2+i)(2-i) = 4 - 2i + 2i - i^2 = 4 - i^2 = 4+1 = 5$

(Conjugate Pairs Theorem soon!)

Quadratic Equations: $ax^2+bx+c=0 \Longrightarrow$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

b^2-4ac is the DISCRIMINANT

$3x^2+5x+20 = 0 \Longrightarrow a=3, b=5, c=20$

$b^2-4ac = 5^2-4(3)(20) = 25-240 = -215$

$5 \overline{)215}$ 43

43 is prime

$x = \frac{-5 \pm \sqrt{-215}}{2(3)} = \boxed{\frac{-5 \pm i\sqrt{215}}{6}} = -\frac{5}{6} \pm \frac{\sqrt{215}}{6}i$

Notice $x = -\frac{5}{6} + \frac{\sqrt{215}}{6}i$ & $-\frac{5}{6} - \frac{\sqrt{215}}{6}i$ are conjugates.

FACTOR THEOREM If $P(x)$ is a polynomial, and $x=c$ is a zero, then $x-c$ is a factor.

Fundamental Theorem of Algebra: A polynomial of degree n has n zeros

$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_2x^2 + a_1x + a_0$

$P(x) = 3x^2+5x-7$, e.g.

Then $P(x) = a_n(x-c_1)(x-c_2)\cdots(x-c_{n-1})(x-c_n)$

It's possible that one or more of the c's is repeated. We say it is a zero of muliplicity m > 1.

More generally:

$P(x) = a_n(x-c_1)^{m_1}(x-c_2)^{m_2}\cdots(x-c_k)^{m_k}$,

where $m_1+m_2+\dots+m_k = n$

Write a polynomial of minimal degree, with the following zeros with respective multiplicities:

$c_1 = 2, m_1 = 3; \quad c_2 = 5, m_2 = 1; \quad c_3 = -7, m_3 = 4$

$(x-2)^3(x-5)(x+7)^4 = P(x)$.

=

Fundamental Theorem of Algebra: A polynomial of degree n has n zeros

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$(x-2)^3 (x-5)(x+7)^4 = P(x)$.

$$x^8 + 17x^7 + 28x^6 - 754x^5 - 2207x^4 + 12341x^3 + 19306x^2 - 108388x + 96040$$

Wolfram Alpha can Really Help! The *WebAssign* always wants you to expand these, because it's mean and cruel!

Conjugate Pairs Theorem: If the polynomial has real coefficients, then any nonreal complex zeros occur in conjugate pairs.

Finding a Polynomial Function with Given Zeros In Exercises 59–64, find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

59. 1, $5i$

60. 4, $-3i$

61. 2, 2, $1 + i$

59 $(x-1)(x-5i)\overset{(x+5i)}{(x-(-5i))} = P(x)$!

$= (x-1)(x^2+5ix-5ix-5i(5i))$

$= (x-1)(x^2-25i^2) = (x-1)(x^2+25)$

$= x^3+25x-x^2-25$

$= \boxed{x^3-x^2+25x-25}$

$(a+b)(a-b) = a^2-b^2$

$(2+i)(2-i) = 4-i^2 = 4-(-1) = 5$

$(a+bi)(a-bi) = a^2+b^2$

WebAssign will make you expand these out. On written tests, all I need to see is the factored form!

(6) $x = 2, 2, 1+i$:

$(x-2)^2\underline{(x-(1+i))(x-(1-i))} = (x-2)^2(x-1-i)(x-1+i)$

$(x^2-4x+4)(x^2-(1-i)x-(1+i)x+(1+i)(1-i))$

or $(x^2-4x+4)(x^2-x+ix-x+1-i-ix+i-i^2)$ $+1$

$= (x^2-4x+4)(x^2+2) = (x^2+2)(x^2-4x+4)$

$$
\begin{array}{r}
= \quad x^4 - 4x^3 + 4x^2 \qquad\qquad \\
2x^2 - 8x + 8 \\
\hline
x^4 - 4x^3 + 6x^2 - 8x + 8
\end{array}
$$

Finding all the zeros of a polynomial and factoring that polynomial.

$x^4 - 6x^2 - 7$ is quadratic in form.

$u^2 - 6u - 7$, where $u = x^2$

$(-i)^2 = i^2$!

$u^2 - 6u - 7 = (u-7)(u+1)$

$= (x^2 - 7)(x^2 + 1) = (x^2 - (\sqrt{7})^2)(x^2 - (-i)^2)$

$= (x - \sqrt{7})(x + \sqrt{7})(x - i)(x + i)$

$= ($

zeros: $\pm\sqrt{7}$, $\pm i$

$x^3 - 4x^2 + 16x - 64$ **factors by grouping!**

$x^2(x-4) + 16(x-4)$

$= (x-4)(x^2+16) = (x-4)(x-4i)(x+4i)$

$x^2 - (-16)$

$= x^2 - 16i^2$

$= (x-4i)(x+4i)$

$x^2 + 16 = 0$

$x^2 = -16$

$x = \pm\sqrt{-16} = \pm 4i$

54. $g(x) = 4x^3 + 23x^2 + 34x - 10$ $\quad -3 + i$

That means $x-(-3+i)$ is a factor

$x+3-i \overline{\big) 4x^3+23x^2+34x-10}$

Book way: multiply $(x-(-3+i))(x-(-3-i))$

$= (x+3-i)(x+3+i) = x^2+3x+ix+3x+9+3i-ix-3i-i^2$

$= x^2+6x+10$

$$\begin{array}{r} 4x-1 \\ x^2+6x+10 \overline{\big) 4x^3+23x^2+34x-10} \\ -(4x^3+24x^2+40x) \\ \hline -x^2-6x-10 \\ -(-x^2-6x-10) \\ \hline 0 \end{array}$$

$\frac{4x^3}{x^2} = 4x$

$g(x) = (4x-1)(x+3-i)(x+3+i)$