Dry boring part of 3.4

$\bar{u} \cdot \bar{v} = \bar{v} \cdot \bar{u}$

$2\bar{u} = 2\langle u_1, u_2 \rangle = \langle 2u_1, 2u_2 \rangle$

$2\bar{u} \cdot \bar{v} = \bar{v} \cdot 2\bar{u}$

$\bar{u} \cdot (\bar{v} + \bar{w}) = \bar{u} \cdot \bar{v} + \bar{u} \cdot \bar{w}$. Also, $(\bar{u} + \bar{v}) \cdot \bar{w} = \bar{u} \cdot \bar{w} + \bar{v} \cdot w$

(Left & right Distributive Laws)

$\|\bar{u}\| = \sqrt{u_1^2 + u_2^2} = \sqrt{\bar{u} \cdot \bar{u}}$

$\|\bar{u} \pm \bar{v}\|^2 = (\bar{u} \pm \bar{v}) \cdot (\bar{u} \pm \bar{v})$

$= \bar{u} \cdot \bar{u} \pm \bar{u} \cdot \bar{v} \pm \bar{v} \cdot \bar{u} + \bar{v} \cdot \bar{v}$ (FOIL!) (Distributive Law)

$= \bar{u} \cdot \bar{u} \pm 2\bar{u} \cdot \bar{v} + \bar{v} \cdot \bar{v}$

3.4 Exercises of Note

\#9 $\bar{u} = 3\bar{i} - 2\bar{j} = \langle 3, -2 \rangle$

$\bar{v} = \bar{i} - \bar{j} = \langle 1, -1 \rangle$

Find $\bar{u} \cdot \bar{v} = 3 + 2 = 5$

$\bar{0} = \langle 0, 0 \rangle$

$\bar{0} \cdot \bar{v} = 0$

\#17 Find $\|\bar{u}\|$, if $u = \langle -8, 15 \rangle$, using dot product:

$\|\bar{u}\| = \sqrt{\bar{u} \cdot \bar{u}} = \sqrt{(-8)^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = \boxed{17 = \|\bar{u}\|}$

20. [–/1 Points] DETAILS LARTRIG10 3.4.030.

Find the angle θ (in radians) between the vectors. (Round your answer to two decimal places.)

$\mathbf{u} = \langle 6, 9 \rangle$

$\mathbf{v} = \langle 6, 0 \rangle$

$\theta =$ ✗ 0.98

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{6(6)+9(0)}{\sqrt{6^2+9^2}\sqrt{6^2+0^2}}$$

$$= \frac{36}{\sqrt{117}\sqrt{36}} = \frac{6}{\sqrt{117}} \Longrightarrow$$

$$\theta = \cos^{-1}\left(\frac{6}{\sqrt{117}}\right) \approx 0.9827937232$$

$$\approx 56.30993247^{\circ}$$

19. [–/1 Points] DETAILS LARTRIG10 3.4.029.

Find the angle θ (in radians) between the vectors. (Round your answer to two decimal places.)

$\mathbf{u} = \langle 4, 0 \rangle$

$\mathbf{v} = \langle 0, -8 \rangle$

Orthogonal.

$$\langle 4, 0\rangle \cdot \langle 0, -8\rangle = 0 \Longrightarrow \text{orthogonal}$$

$\theta =$ ✗ 1.57

3.4 #29 Find $\text{proj}_{\bar v}\bar u$ if $\bar u = \langle 4,4\rangle$ & $\bar v = \langle 0,1\rangle$

$$\text{proj}_{\bar v}\bar u = \frac{\bar u\cdot\bar v}{\|\bar v\|^2}\bar v = \frac{0+4}{1}\langle 0,1\rangle = \langle 0,4\rangle \text{ ?!}$$

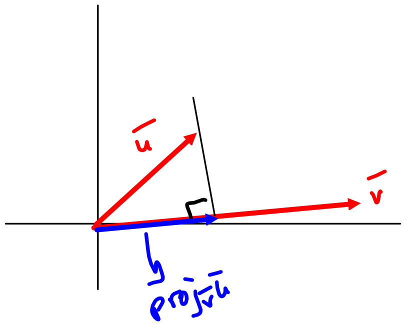
Now do it for $\bar v = \langle 9,1\rangle$, b/c you can read!

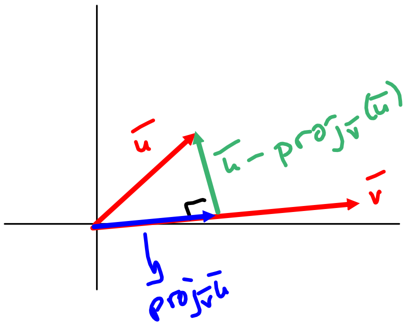
$$\text{proj}_{\bar v}\bar u = \frac{\bar u\cdot\bar v}{\|\bar v\|^2}\bar v = \frac{36+4}{9^2+1^2}\langle 9,1\rangle = \frac{40}{82}\langle 9,1\rangle$$

$$= \left\langle \frac{360}{82}, \frac{40}{82}\right\rangle$$

$$= \left\langle \frac{180}{41}, \frac{20}{41}\right\rangle$$

$$\approx \langle 4.39, 0.49\rangle$$





$\bar u - \text{proj}_{\bar v}\bar u$ & $\text{proj}_{\bar v}\bar u$ are two orthogonal vectors whose sum (resultant) is $\bar u$. so we want

$$\bar u - \text{proj}_{\bar v}\bar u = \langle 4,4\rangle - \left\langle \frac{180}{41}, \frac{20}{41}\right\rangle$$

$$= \left\langle \frac{164-180}{41}, \frac{164-20}{41}\right\rangle$$

$$= \boxed{\left\langle \frac{-16}{41}, \frac{144}{41}\right\rangle = \bar u - \text{proj}_{\bar v}\bar u}$$

This is the basic technique for Gram-Schmidt Orthogonalization process. You'll see this in Linear Algebra if you take the course.

$\mathbb{R}^2$ has basis $\{\bar i, \bar j\}$

$= \{\langle 1,0\rangle, \langle 0,1\rangle\}$

$\mathbb{R}^3 = \{\bar i, \bar j, \bar k\} = \{\langle 1,0,0\rangle, \langle 0,1,0\rangle, \langle 0,0,1\rangle\}$

$\langle 3,2,1\rangle = 3\bar i + 2\bar j + 1\bar k.$

25. [–/3 Points] DETAILS LARTRIG10 3.4.044.

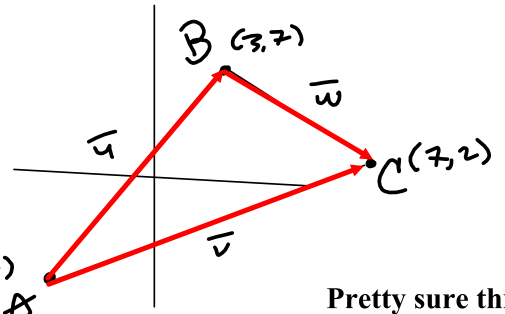
Use vectors to find the interior angles of the triangle with the given vertices. (Round your answers to two decimal places.)

(−8, −9), (3, 7), (7, 2)

19.24 ° (smallest value)

73.17 °

87.59 ° (largest value)



Pretty sure this is how it's done, but I made some mistakes in my calculations, most likely...

$\bar{u} = \langle 3-(-8), 7-(-9)\rangle = \langle 11, 16\rangle = \bar{u}$

$\bar{v} = \langle 7-(-8), 2-(-9)\rangle = \langle 15, 11\rangle = \bar{v}$

$\bar{w} = \langle 7-3, 2-7\rangle = \langle 4, -5\rangle = \bar{w}$

$\|\bar{u}\| = \sqrt{11^2+16^2} = \sqrt{121+256} = \sqrt{377} = \|\bar{u}\|$

$\|\bar{v}\| = \sqrt{15^2+11^2} = \sqrt{225+121} = \sqrt{346} = \|\bar{v}\|$

$\|\bar{w}\| = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41} = \|\bar{w}\|$

$\cos A = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \dfrac{\langle 11,16\rangle\cdot\langle 15,11\rangle}{\sqrt{377}\sqrt{346}} = \dfrac{165+176}{\sqrt{130,442}} = \dfrac{341}{\sqrt{130442}}$

$\rightarrow A = \cos^{-1}(\text{prev}) \approx$

$\cos B =$ use $-\bar{u}$ & $\bar{w}$ (tail-to-tail) $= \dfrac{-\bar{u}\cdot\bar{w}}{\|\bar{u}\|\|\bar{w}\|} = \dfrac{\langle -11,-16\rangle\cdot\langle 4,-5\rangle}{\sqrt{377}\sqrt{41}}$

$= \dfrac{-44+80}{\sqrt{377}\sqrt{41}} = \dfrac{36}{\sqrt{15457}} \approx$

$\cos C = \dfrac{-\bar{w}\cdot(-\bar{v})}{\|\bar{w}\|\|\bar{v}\|} = \dfrac{\langle 4,-5\rangle\cdot\langle 15,11\rangle}{\sqrt{41}\sqrt{346}} = \dfrac{60-55}{\sqrt{14186}} = \dfrac{5}{\sqrt{14186}} \approx$

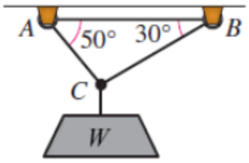
2. [–/2 Points] DETAILS LARTRIG10 3.3.515.XP.

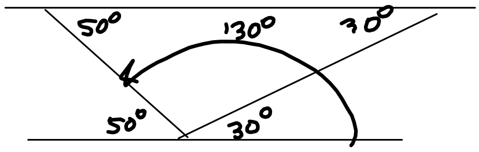
Use the figure to determine the tension in each cable supporting the load. (Round your answers to one decimal place.)

W = 2158 lb

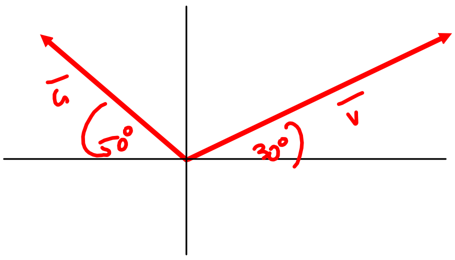
tension in $\overline{AC}$ ✗ 1897.7 lb

tension in $\overline{BC}$ ✗ 1408.5 lb





$180° - 50° = 130°$



$\bar{u} + \bar{v} = \bar{w} = \langle 0, 2158 \rangle$

Want $x = \|\bar{u}\|$

$y = \|\bar{v}\|$

$\bar{u} = \|\bar{u}\| \langle \cos 150°, \sin 150° \rangle$

$\bar{v} = \|\bar{v}\| \langle \cos 30°, \sin 30° \rangle$

Let $a = \cos 150°$, $b = \sin 150°$

$c = \cos 30°$, $d = \sin 30°$

$\bar{u} + \bar{v} = \langle 0, 2158 \rangle \Longrightarrow$

$\bar{u} = \langle ax, bx \rangle \Longrightarrow ax + cy = 0$

$\bar{v} = \langle cy, dy \rangle \qquad bx + dy = 2158$

Use substitution:

$ax + cy = 0 \Longrightarrow$

$ax = -cy \Longrightarrow$

$\boxed{x = -\frac{cy}{a}} \Longrightarrow bx + dy = b\left(-\frac{cy}{a}\right) + dy = 2158 \Longrightarrow$

$-\frac{bcy}{a} + dy = \left(-\frac{bc}{a} + d\right)y = 2158$

$\Longrightarrow y = \dfrac{2158}{-\frac{bc}{a} + d}$

$= \dfrac{2158}{\frac{(-\sin 130^\circ)\cos(30^\circ)}{\cos(130^\circ)} + \sin(30^\circ)} \approx \boxed{1408.534465 \text{ lbs} \approx \|\bar{v}\|}$

& $x = -\frac{cy}{a} \approx -\dfrac{\cos(30^\circ)(1408.534465)}{\cos(130^\circ)}$

$\approx \boxed{1897.713351 \text{ lbs}} \approx \|\bar{u}\|$

$$\frac{2158}{-\frac{\sin(130)\cos(30)}{\cos(130)} + \sin(30)} = 1408.534465$$

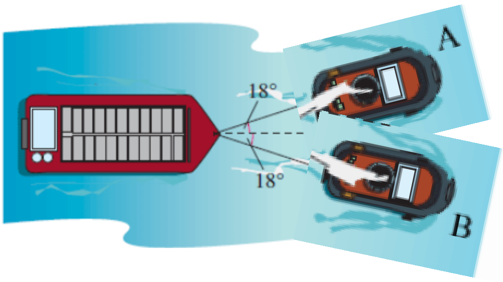
$$-\cos(30) \cdot \frac{1408.534465}{\cos(130)} = 1897.713351$$

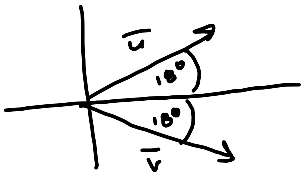
4. [–/2 Points] DETAILS LARTRIG10 3.3.087.MI.

Two tugboats are towing a loaded barge and the magnitude of the resultant is 8000 pounds directed along the axis of the barge (see figure). Find the tension (in pounds) in the tow lines when they each make an 18° angle with the axis of the barge. (Round your answers to one decimal place.)

tension in tow line from tugboat A ✗ 4205.8 lb

tension in tow line from tugboat B ✗ 4205.8 lb





$\bar{u} = \|\bar{u}\| \langle \cos 18^\circ, \sin 18^\circ \rangle$

$\bar{v} = \|\bar{v}\| \langle \cos(-18^\circ), \sin(-18^\circ) \rangle$

Symmetry $\Rightarrow$ $\|\bar{u}\| = \|\bar{v}\|$