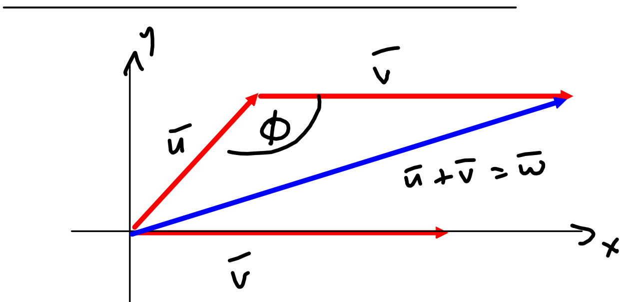


What's the angle between $\bar{u}$ & $\bar{v}$?

$\bar{u}+\bar{v}=\bar{w}$ = RESULTANT.

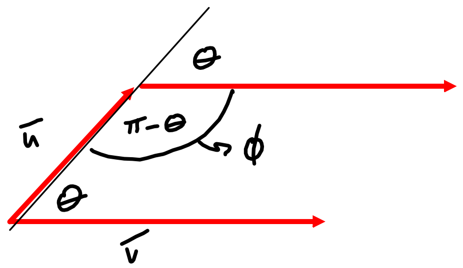
$\|\bar{u}\| = 40$

$\|\bar{v}\| = 70$

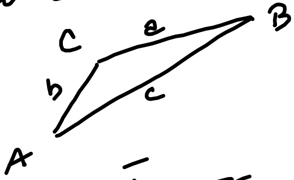
$\|\bar{u}+\bar{v}\| = \|\bar{w}\| = 95$

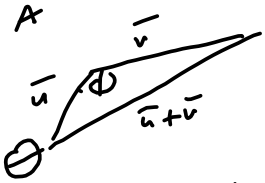
$\theta = ?$

$\theta = \pi - \phi$



Law of Cosines:





$a^2=b^2+c^2-2bc\cos A$

$c^2=a^2+b^2-2ab\cos C$

$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\cos\phi = \dfrac{\|\bar{u}\|^2+\|\bar{v}\|^2-\|\bar{w}\|^2}{2\|\bar{u}\|\|\bar{v}\|}$

$\cos\phi = \dfrac{40^2+70^2-95^2}{2(40)(70)}$

9025

$= \dfrac{1600+4900-9025}{5600}$

=

$= \dfrac{6500-9025}{5600}$

≈ -0.4508928571

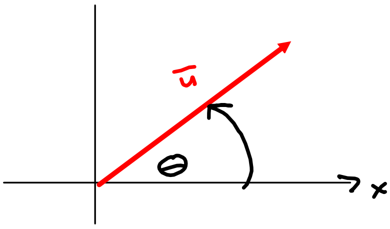
$\phi \approx \cos^{-1}(-0.4508928571)$

≈ 116.8009832

$\theta = 180° - \phi \approx 180° - 116.8009832°$

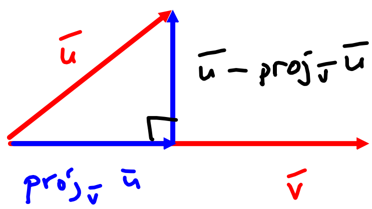
$\boxed{63.19901682° \approx \theta}$

Vector $\bar{u}$ = $\|\bar{u}\|$ unit vector in direction of $\bar{u}$



θ = Direction Angle

Then $\bar{u} = \|\bar{u}\| \langle \cos\theta, \sin\theta \rangle$



$\bar{u} - \text{proj}_{\bar{v}}\,\bar{u}$ = your 1st exposure to GRAM-SCHMIDT ORTHOGONALIZATION in Linear Algebra.

BASIS: $\bar{i} = \langle 1, 0 \rangle$ &
$\bar{j} = \langle 0, 1 \rangle$ form an orthogonal basis for the plane.

ORTHOGONAL: $\bar{i}$ & $\bar{j}$ are $\perp$ ← perpendicular

angle is 90°:

cosine is 0

$$\cos\theta = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \frac{\langle 1,0 \rangle \cdot \langle 0,1 \rangle}{\sqrt{\langle 1,0 \rangle \cdot \langle 1,0 \rangle}\,\sqrt{\langle 0,1 \rangle \cdot \langle 0,1 \rangle}} = \frac{\bar{i} \cdot \bar{j}}{\|\bar{i}\|\|\bar{j}\|}$$

$\|\bar{u}\| = \sqrt{\bar{u}\cdot\bar{u}}$ $\qquad = \frac{0}{\sqrt{1}\sqrt{1}} = 0$

so $\bar{u} \perp \bar{v}$ ✓

They <u>span</u> the plane: $\langle 3,2\rangle = 3\bar{i} + 2\bar{j}$

Let $\bar{v}$ in the plane.

$\bar{v} = \langle v_1, v_2\rangle = v_1\langle 1,0\rangle + v_2\langle 0,1\rangle$

$= v_1\bar{i} + v_2\bar{j}$

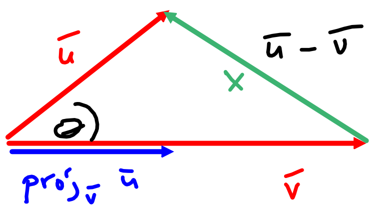
This gives $\bar{v}$ a representation as a *linear combination* of $\bar{i}$ and $\bar{j}$.

This means the set $\{\bar{i}, \bar{j}\}$ is an orthogonal spanning set.

For this to be a *basis*, it has to be a *minimal* spanning set.

any more vectors would be redundant.

Plane is 2-D. Any <u>Basis</u> will have 2 vectors.



We want $\text{proj}_{\bar v}\bar u$:

$$x^2 = \|\bar u\|^2 + \|\bar v\|^2 - 2\|\bar u\|\|\bar v\|\cos\theta$$

u^2+v^2-2uvcos θ

NOTE: $\|\text{proj}_{\bar v}\bar u\| = \|\bar u\||\cos\theta|$

& $\text{proj}_{\bar v}\bar u = \|\text{proj}_{\bar v}\bar u\| \cdot \frac{1}{\|\bar v\|}\bar v$

= **magnitude of the projection times a unit vector in the right direction.**

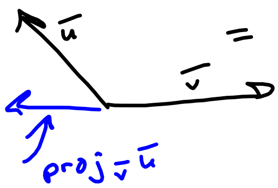
If the angle is greater than 90 degrees, then the cosine will be negative and the projection will point in the opposite direction from v-bar.

$$= \|\bar u\|\cos\theta\,\frac{\bar v}{\|\bar v\|}$$

$$= \|\bar u\|\,\frac{\bar u\cdot\bar v}{\|\bar u\|\|\bar v\|}\cdot\frac{\bar v}{\|\bar v\|\|\bar u\|}$$

$\rightarrow$ NO!

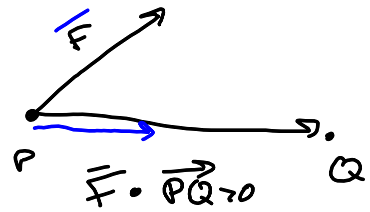
I don't know why I put that in there, but it doesn't belong!



$$= \boxed{\frac{\bar u\cdot\bar v}{\|\bar v\|^2}\,\bar v = \text{proj}_{\bar v}\bar u}$$

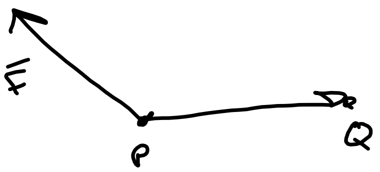
$$= \frac{\bar u\cdot\bar v}{\bar v\cdot\bar v}\,\bar v$$

§3.4 #6



$\vec{F} \cdot \overrightarrow{PQ} > 0$

Acute



$\vec{F} \cdot \overrightarrow{PQ} < 0$

Obtuse

$\text{proj}_{\overrightarrow{PQ}} \vec{F}$

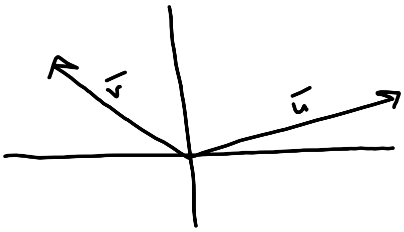
$$\text{WORK} = \|\text{proj}_{\overrightarrow{PQ}} \vec{F}\| \, \|\overrightarrow{PQ}\|$$

$$= \frac{|\vec{F} \cdot \overrightarrow{PQ}|}{\|\overrightarrow{PQ}\|^2} \|\overrightarrow{PQ}\|$$

$$= |\vec{F} \cdot \overrightarrow{PQ}|$$

$\bar{u} = \langle 8, 2 \rangle$, $\bar{v} = \langle -4, 3 \rangle$

$\Rightarrow \bar{u} \cdot \bar{v} = 8(-4) + (2)(3) = -32 + 6 = -26$



#30 Find $\text{proj}_{\bar{v}}\bar{u}$:

$$\bar{u} = \langle 4, 3 \rangle$$

$$\bar{v} = \langle 6, -8 \rangle$$

$$\text{proj}_{\bar{v}}\bar{u} = \frac{\bar{u} \cdot \bar{v}}{\|\bar{v}\|^2}\,\bar{v}$$

$$= \frac{-8}{36+64}\langle 6, -8 \rangle$$

$$= \frac{-8}{100}\langle 6, -8 \rangle = \frac{-2}{25}\langle 6, -8 \rangle = \text{proj}_{\bar{v}}\bar{u}$$

$$= \left\langle \frac{-12}{25}, \frac{16}{25} \right\rangle$$