Writing Project #1 is due Friday!

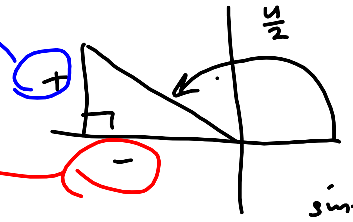
11. (5 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u) = \frac{3}{4}$ and $\frac{3\pi}{2} \le u < 2\pi$.

$$\sin\left(\frac{u}{2}\right) = + \sqrt{\frac{1-\cos(u)}{2}}$$

$$\frac{3\pi}{4} < \frac{u}{2} < \pi$$

QII

$$\cos\left(\frac{u}{2}\right) = - \sqrt{\frac{1+\cos(u)}{2}}$$



sine > 0

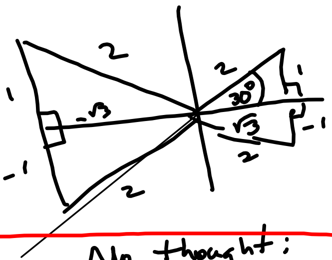
cosine < 0

12. Consider the equation $2\sin^2(x)-1=0$.

a. (5 pts) Find all solutions x, in radians *and* degrees, to the equation in the interval $[0,2\pi)$.

b. (5 pts) Find all real solutions x, in radians *and* degrees.

$$4\cos^2(x)-3=0$$
$$4\cos^2(x)=3$$
$$\cos^2(x)=\frac{3}{4}$$
$$\cos(x)=\pm\sqrt{\frac{3}{4}}=\pm\frac{\sqrt{3}}{2}$$



$x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$

$=30°,150°,210°,330°$

(a)

No thought: (b)

$\frac{\pi}{6}+2n\pi,\ \frac{5\pi}{6}+2n\pi,\ \frac{7\pi}{6}+2n\pi,\ \frac{11\pi}{6}+2n\pi \quad \forall n\in\mathbb{Z}$

I prefer a set:

$\{x+2n\pi \mid x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6},\ n\in\mathbb{Z}\}$

$\{x+2n\pi \mid x\in\{\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}\},\ n\in\mathbb{Z}\}$

Clever:

$\{x+n\pi \mid x\in\{\frac{\pi}{6},\frac{5\pi}{6}\},\ n\in\mathbb{Z}\}$

$\frac{\pi}{6}+n\pi,\ \frac{5\pi}{6}+2n\pi,\ n\in\mathbb{Z}$

$\{x+360°n \mid x=30°,150°,210°,330°,\ n\in\mathbb{Z}\}$

$\{x+180°n \mid x=30°,150°,\ n\in\mathbb{Z}\}$

$x=30°+360°n,\ 150°+360°n,\ 210°+360°n,\ 330°+360°n,\ n\in\mathbb{Z}$

$x=30°+180°n,\ 150°+180°n,\ n\in\mathbb{Z}$

13. (5 pts) Re-write $\sin\left(\cos^{-1}\left(\frac{2}{x}\right)\right)$ as an algebraic expression.

14. Find the *exact* value of $\sin\left(\frac{5\pi}{6}\right)$ in two ways: (Hint: If degrees are easier for you, *use degrees*.)

a. (5 pts) Use a Sum identity.
b. (5 pts) Use a Half-Angle identity

15. (5 pts) Re-write $\cos\left(\arcsin(x)+\arccos(x)\right)$ as an algebraic expression. (Hint: Use Sum identity.)

See 3/4 notes & video

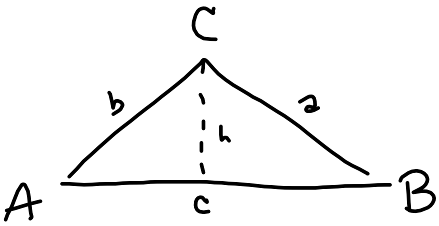
16. (5 pts) Find $\sin(2u), \cos(2u)$ and $\tan(2u)$, given that $\sin(u) = \frac{4}{7}$ and $\cos(u) < 0$.

17. (5 pts) Find the arc length on a circle of radius $r = 8$ that is intercepted by an angle of 1356^0.

Area $= \frac{1}{2}r^2\theta$

Arc Length: $s = r\theta$

3.1 **Law of Sines**



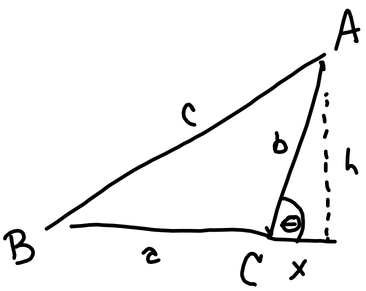
$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$

$$\frac{h}{b} = \sin A$$

$$h = b\sin A = a\sin B$$

$$\boxed{\begin{aligned} A &= \tfrac{1}{2}ab\sin C \\ &= \tfrac{1}{2}bc\sin A \\ &= \tfrac{1}{2}ca\sin B \end{aligned}}$$

$$\text{Area} = \frac{1}{2}cb\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin(C)$$



$$\theta = \pi - C$$

$$\frac{h}{b} = \sin(\pi - C) = \sin\pi\cos(-C) + \sin(-C)\cos\pi$$

$$= 0\cdot\cos(C) - \sin(C)(-1)$$

$$= \sin C$$

$$\Rightarrow h = b\sin C$$

$$\text{base} = a$$

$$\text{Area} = \frac{1}{2}ab\sin C$$

3.1

Use the Law of Sines to solve the triangle. Round your answers to two decimal places.

3. $C =$ 105° 105 °

$a =$ 4.75

$b =$ 5.32

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$C = 180^\circ - 35^\circ - 40^\circ = 105^\circ$$

$A = 35^\circ$

$B = 40^\circ$

$c = 8$

ASA

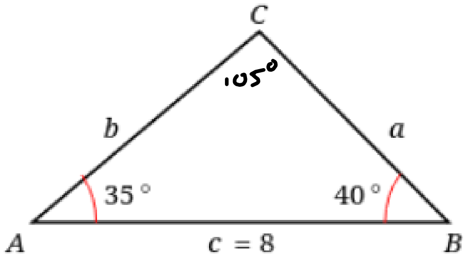
angle side angle

$$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow$$

$$\frac{\sin(35^\circ)}{a} = \frac{\sin(105^\circ)}{8}$$

$$\Rightarrow a = \frac{8\sin(35^\circ)}{\sin(105^\circ)}$$

$$\approx \boxed{4.750480178 \approx a}$$



$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

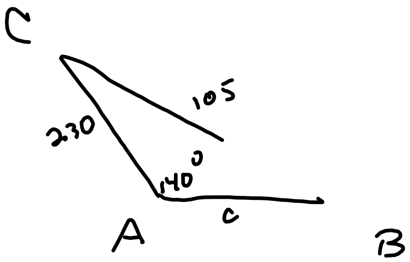
$$a = \frac{c\sin A}{\sin C} =$$

3.1

5. Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places. (If a triangle is not possible, enter IMPOSSIBLE in each corresponding answer blank.)

$A = 140°$, $a = 105$, $b = 230$

A is obtuse.



ASS : can be ambiguous.
may not have solution

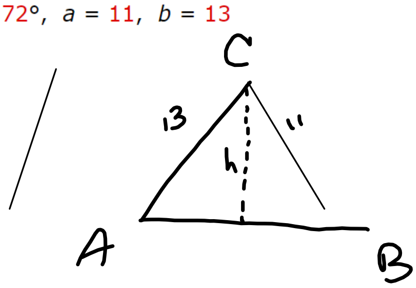
See anything wrong?
Side a is too short
to reach side c !
IMPOSSIBLE.

3.1

Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decima places. (If a triangle is not possible, enter IMPOSSIBLE in each corresponding answer blank.)

$A = 72°$, $a = 11$, $b = 13$

6.



Does $a = 11$ reach?
Is side a long enough?
It needs to be bigger than the height h

$\frac{h}{b} = \sin A$

$h = b \sin A = 13 \sin(72°) \approx 12.36373471$

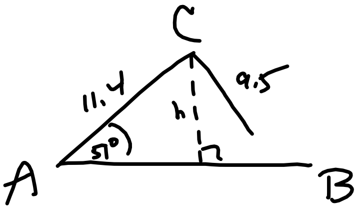
Side $a < h \Rightarrow$

IMPOSSIBLE

3.1

Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places. (If a triangle is not possible, enter IMPOSSIBLE in each corresponding answer blank.) **8.**

$A = 51°$, $a = 9.5$, $b = 11.4$



$\frac{h}{11.4} = \sin A$

$h = 11.4 \sin(51°) \approx 8.859463961 < a = 9.5$,

So $b > a > h \Longrightarrow$
2 SOLUTIONS!

$$\frac{\sin B}{11.4} = \frac{\sin(51°)}{9.5}$$

$$\sin B = \frac{11.4 \sin(51°)}{9.5} \approx 0.9325751537 \quad \& \quad \arcsin(0.9325751537)$$

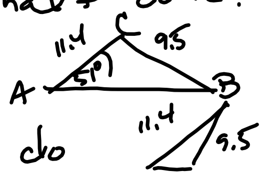
$$\approx 68.83986003 \approx \boxed{68.84° \approx B}$$

$$C = 180° - 51° - B \approx 60.16013997° \approx \boxed{60.16° \approx C}$$

Then do $c = \frac{a \sin(C)}{\sin(A)}$ to find c.

But we're only half done!

We did

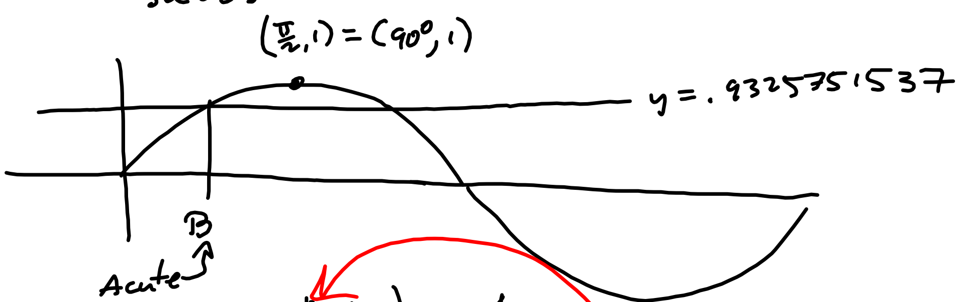


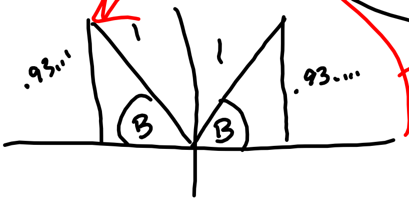
C is obtuse

Now we do

B is obtuse

$\sin(B) \approx 0.9325751537$





$B = 180° - 60.16013997°$

$\approx 119.83986° \approx B$

for B obtuse case

$C = 180° - 51° - B$

$\approx 180° - 51° - 119.83980° \approx 9.160139975° \approx B$ for B obtuse.