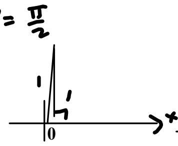
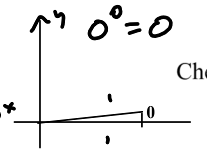
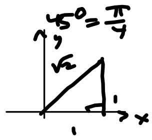
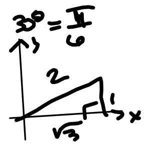


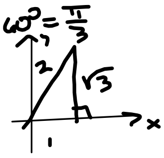
MAT 1420



Cheat Sheet







Arc Length: $s = r\theta$, **Area of a Sector:** $A = \frac{1}{2}r^2\theta$. **Convert degrees to radians:** $\frac{\pi}{180^0}$

Convert radians to degrees: $\frac{180^0}{\pi}$

Pythagorean Identities

$\boxed{\sin^2(x)+\cos^2(x)=1}$ → $1-\cos^2\theta = \sin^2\theta$

$\tan^2(x)+1=\sec^2(x)$

$\cot^2(x)+1=\csc^2(x)$

Angle Sum Formulas

$$\sin(u+v)=\sin(u)\cos(v)+\sin(v)\cos(u)$$

$$\cos(u+v)=\cos(u)\cos(v)-\sin(u)\sin(v)$$

$$\tan(u+v)=\frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)}$$

Radians without π

1.570796327

3.141592654 — 6.283185308

4.712388981

Double-Angle Formulas: $\sin(2u)=2\sin(u)\cos(u)$,

$$\cos(2u)=\cos^2(u)-\sin^2(u)=2\cos^2(u)-1=1-2\sin^2(u)$$

$$\tan(2u)=\frac{2\tan(u)}{1-\tan^2(u)}=\frac{\sin(2u)}{\cos(2u)}!$$

Half-Angle Formulas:

$$\sin\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1-\cos(u)}{2}}$$

$$\cos\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\tan\left(\frac{u}{2}\right)=\frac{1-\cos(u)}{\sin(u)}=\frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)}$$

Power-Reducing Formulas:

$$\sin^2(u)=\frac{1-\cos(2u)}{2}$$

$$\cos^2(u)=\frac{1+\cos(2u)}{2}$$

$$\tan^2(u)=\frac{1-\cos(2u)}{1+\cos(2u)}=\frac{\sin^2(u)}{\cos^2(u)}$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v)-\cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v)+\cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v)+\sin(u-v)]$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

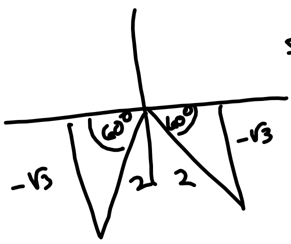
Solve the multiple-angle equation. (Enter your answers as a comma-separated list. Use *n* as an integer constant. Enter your response in radians.)

$2\sin(5x) + \sqrt{3} = 0$

Practice Test #22

$2\sin(5x) = -\sqrt{3}$

$\sin(5x) = -\frac{\sqrt{3}}{2}$



$5x = 180° + 60° = (240°)\left(\frac{\pi}{180°}\right) = \frac{4\pi}{3}$

$5x = 360° - 60° = (300°)\left(\frac{\pi}{180°}\right) = \frac{5\pi}{3}$

So $5x = \frac{4\pi}{3} + 2n\pi$

$5x = \frac{5\pi}{3} + 2n\pi$

$x = \boxed{\frac{4\pi}{15} + \frac{2n\pi}{5}}$

$x = \frac{5\pi}{15} + \frac{2n\pi}{5} = \boxed{\frac{\pi}{3} + \frac{2n\pi}{5}}$

#23

Solve the equation. (Enter your answers as a comma-separated list. Use n as an integer constant. Enter your response in radians.)

$\tan^2(x) + \tan(x) - 6 = 0$. Let $u = \tan(x)$. Then

$u^2 + u - 6 = (u+3)(u-2) = 0 \longrightarrow$

$u+3=0$ OR $u-2=0 \longrightarrow$

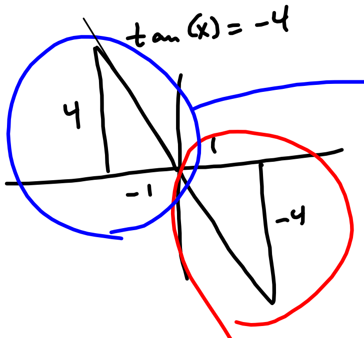
$u = \tan(x) = -3$ $\quad$ $u = \tan(x) = 2$

No "to 3 decimal places" specified. They want EXACT ANSWERS.

$\tan^2(x) + \tan(x) - 12 = 0$

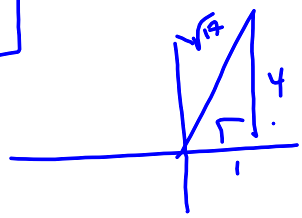
$(\tan(x)+4)(\tan(x)-3) = 0$

$\tan(x) = -4$ OR $\tan(x) = 3$



$\boxed{\pi + \arctan(-4)}$

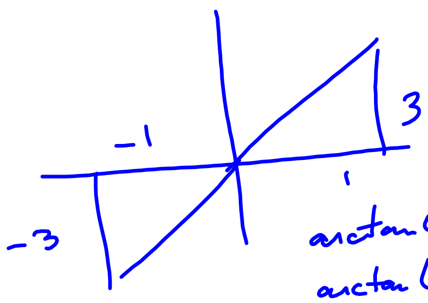
$\pi - \arctan(4)$



$\arctan(-4) = \boxed{\tan^{-1}(-4)}$

$\boxed{\tan^{-1}(-4) + \pi n,\ n \in \mathbb{Z}}$

Now, $\tan(x) = 3$:



$\left.\begin{array}{l}\arctan(3) + 2\pi n \\ \arctan(3) + \pi + 2\pi\end{array}\right\}$

$\boxed{\arctan(3) + \pi n,\ n \in \mathbb{Z}}$

#15

$$\sin(x)\cos(4x) + \cos(x)\sin(4x) = \sin(5x)$$
$$= \sin(x+4x)$$

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$$
$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{\sin^2\theta}} = \frac{|1+\cos\theta|}{|\sin\theta|} = \frac{1+\cos\theta}{|\sin\theta|}$$

#30

$$\cos\left(\underbrace{\arccos(3x)}_{u} - \underbrace{\arctan(x)}_{v}\right) = \cos(u - v)$$

, where

$u = \arccos(3x)$ &

$v = \arctan(x)$.

I just do $\cos(u+v)$

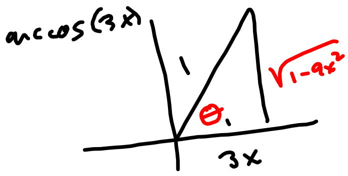
cosine even & sine odd

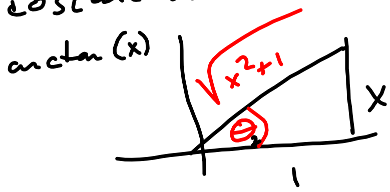
$$\cos(u-v) = \cos(u + (-v))$$

$$= \cos(u)\cos(-v) - \sin(u)\sin(-v)$$

$$= \cos(u)\cos(v) + \sin(u)\sin(v)$$

$$= \cos(\arccos(3x))\cos(\arctan(x)) + \sin(\arccos(3x))\sin(\arctan(x))$$





$$\sqrt{1-(3x)^2} = \sqrt{1-9x^2}$$

$$= \boxed{\frac{3x}{1} \cdot \frac{1}{\sqrt{x^2+1}} + \frac{\sqrt{1-9x^2}}{1} \cdot \frac{x}{\sqrt{x^2+1}}}$$

#32

$\cancel{3}\sin(2x)\sin(x) = \cancel{3}\cos(x) \Rightarrow$ (Double-Angle)

$2\sin(x)\cos(x)\sin(x) = \cos(x) \Rightarrow$

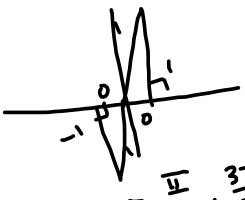
$2\sin^2(x)\cos(x) - \cos(x) = 0 \Rightarrow$

$\cos(x)\left(2\sin^2(x) - 1\right) = 0 \Rightarrow$

$\cos(x) = 0$ or $2\sin^2(x) = 1$

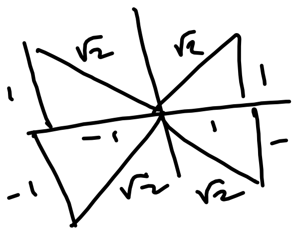
$\sin^2(x) = \frac{1}{2}$

$\sin(x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

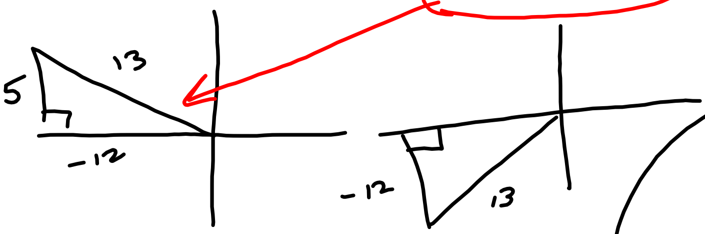
$90°, 270°$



$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

#33

$\cos(u) = -\frac{12}{13}$ $\quad \frac{\pi}{2} < u < \pi$



HERE:

$\sin(2u) = 2\sin(u)\cos(u)$

$= 2\left(\frac{5}{12}\right)\left(-\frac{12}{13}\right) = \boxed{-\frac{10}{13}}$

$\cos(2u) = \cos^2(u) - \sin^2(u)$

$= \left(-\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144-25}{169} = \boxed{\frac{119}{169}}$

$\tan(2u) = \frac{\sin(2u)}{\cos(2u)} = \left(-\frac{120}{169}\right)\left(\frac{169}{119}\right)$

$= \frac{-10(13)}{119} = \boxed{-\frac{120}{119}}$

$13^2 - 12^2 = 169 - 144 = 25 = 5^2$

$\sin(2u) = 2\sin(u)\cos(u) = 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$

$= \boxed{-\frac{120}{169}}$

That's not what #33 is asking. It's asking you to find the sine, cosine and tangent of the DOUBLE ANGLE.

$\cancel{\sin\theta = \frac{5}{13}}$ $\quad \cancel{\csc\theta = \frac{13}{5}}$

$\cancel{\cos\theta = -\frac{12}{13}}$ $\quad \cancel{\sec\theta = -\frac{13}{12}}$

$\cancel{\tan\theta = -\frac{5}{12}}$ $\quad \cancel{\cot\theta = -\frac{12}{5}}$

Read the question more carefully, Steve.

$$\cos^4(x) = \left(\cos^2(x)\right)^2 = \left(\frac{1+\cos(2x)}{2}\right)^2 = \frac{(1+\cos(2x))^2}{2^2}$$

2x 4x 4x

$$= \frac{1+2\cos(2x)+\cos^2(2x)}{4} = \frac{1+2\cos(2x)+\frac{1+\cos(4x)}{2}}{4}$$

4x 4x 4x 8x

$$= \frac{\frac{2+4\cos(2x)+1+\cos(4x)}{2}}{4}$$

4x 8x

$$= \frac{3+4\cos(2x)+\cos(4x)}{8}$$

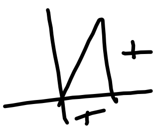
4x 8x Dummy

75° $\frac{1}{2}$-angle of 150°

$$\sin(75^\circ) = \pm\sqrt{\frac{1-\cos(150^\circ)}{2}}$$

$$\sin(75^\circ) = \sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}} = +\sqrt{\frac{1-\cos(150^\circ)}{2}}$$

$75^\circ \in QI$



sine >0
cosine >0
tangent >0

$$= \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$