I re-scheduled 2.4 and 2.5. They were too early on WebAssign.

2.4 - Angle Sum and Power-Reducting Identities

S 2.4 Due Wed.
S 2.5 " Sun.

Sum and Difference Formulas

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\boxed{\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}}$

Also, sine and tangent are odd.

Cosine is even.

Click Here for cheat sheet for Test 2 (and beyond?)

Not on Cheat Sheet?!
Add this!

$\sin(u-v) = \sin(u+(-v)) = \sin(u)\cos(-v) + \sin(-v)\cos(u)$
$= \sin(u)\cos(v) - \sin(v)\cos(u)$

so you don't need $\sin(u-v)$ separately

~~$\frac{\pi}{4} + \frac{\pi}{3} = \frac{3\pi + 4\pi}{12} = \frac{7\pi}{12}$.~~

Build the Question

FIND THE EXACT VALUE OF $\sin\left(\frac{7\pi}{12}\right)$

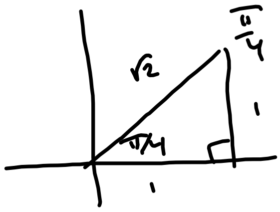
$\frac{7\pi}{12} = \frac{6\pi}{12} + \frac{\pi}{12}$

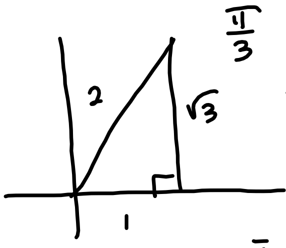
$= \frac{5\pi}{12} + \frac{2\pi}{12}$

Breaking down the question.

$= \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$!

$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$

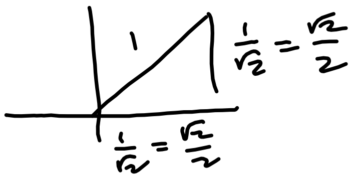




$= \boxed{\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}}$ is fine for me, but "simplified radical form would be same

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2} + \sqrt{3}\sqrt{2}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$



$$\frac{\pi}{3} - \frac{\pi}{4} = \frac{(4-3)\pi}{12} = \frac{\pi}{12}$$

Find $\tan\left(\frac{\pi}{12}\right)$, exactly!

$$\frac{\pi}{12} = \frac{2\pi}{12} - \frac{1\pi}{12}$$

$$= \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$$ use this!

$$= \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$$

$$\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \tan\left(\frac{\pi}{4} + \left(-\frac{\pi}{6}\right)\right)$$

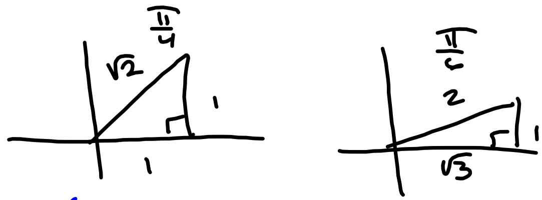
$$= \frac{\tan\left(\frac{\pi}{4}\right) + \tan\left(-\frac{\pi}{6}\right)}{1 - \tan\left(\frac{\pi}{4}\right)\tan\left(-\frac{\pi}{6}\right)} = \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)}$$

Sum and Difference Formulas

$\sin(u + v) = \sin u \cos v + \cos u \sin v$

$\cos(u + v) = \cos u \cos v - \sin u \sin v$

$\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$



slick way $= \dfrac{\left(1 - \frac{1}{\sqrt{3}}\right)}{\left(1 + \frac{1}{\sqrt{3}}\right)} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

my way $= \dfrac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \dfrac{\sqrt{3}-1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}+1} =$

Alternate for tangents:

$$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)} = \frac{\sin(u)\cos(v) + \sin(v)\cos(u)}{\cos(u)\cos(v) - \sin(u)\sin(v)}$$

Because we do $\sin(u+v)$ & $\cos(u+v)$, 1st.

Find the exact values of

$\sin\left(\frac{\pi}{12}\right)$, $\cos\left(\frac{\pi}{12}\right)$ & $\tan\left(\frac{\pi}{12}\right)$

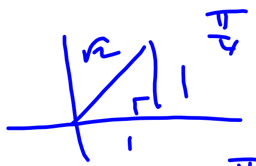
$$\cos\left(\tfrac{\pi}{4}\right) \qquad -\sin\left(\tfrac{\pi}{6}\right)$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

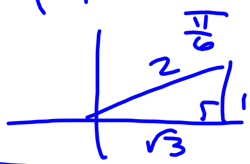
$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4} = \sin\left(\frac{\pi}{12}\right)}$$

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$$



$$= \cos\left(\frac{\pi}{4}\right)\cos\left(-\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(-\frac{\pi}{6}\right)$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$



$$= \boxed{\frac{\sqrt{6}+\sqrt{2}}{4} = \cos\left(\frac{\pi}{12}\right)}$$

$$\tan\left(\frac{\pi}{12}\right) = \frac{\sin\left(\frac{\pi}{12}\right)}{\cos\left(\frac{\pi}{12}\right)} = \frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \tan\left(\frac{\pi}{12}\right)}$$

$$= \left(\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) = \frac{(\sqrt{6}-\sqrt{2})^2}{6-2} = \frac{6-2\sqrt{2}\sqrt{6}+2}{4}$$

$$= \frac{8-2\sqrt{12}}{4} = \frac{4-\sqrt{4\cdot 3}}{2} = \frac{4-2\sqrt{3}}{2}$$

$$= \boxed{\frac{2-\sqrt{3}}{2} = \tan\left(\frac{\pi}{12}\right)}$$

using formula for tangent:

$$\tan(u+v) = \frac{\tan(u) + \tan(v)}{1 - \tan(u)\tan(v)} = \tan\left(\frac{\pi}{4} + \left(-\frac{\pi}{6}\right)\right)$$

$$\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6} \qquad = \frac{\tan\frac{\pi}{4} + \tan\left(-\frac{\pi}{6}\right)}{1 - \tan\left(\frac{\pi}{4}\right)\tan\left(-\frac{\pi}{6}\right)}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \boxed{\frac{\sqrt{3}-1}{\sqrt{3}+1} = \tan\left(\frac{\pi}{12}\right)}$$

I'm having a tough time showing you these two methods are the same, but the point is you may use either method:

1. 3 different formulas for sine, cosine and tangent.

2. 2 formulas for sine and cosine and just form the quotient to get the tangent (and simplify it).

Find exact value of $\sin(75^\circ)$

$75^\circ = 45^\circ + 30^\circ$

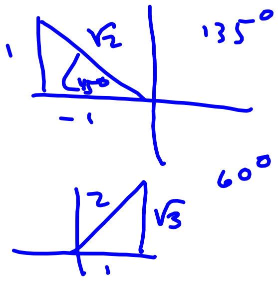
$\sin(75^\circ) = \sin(45^\circ)\cos(30^\circ) + \sin(30^\circ)\cos(45^\circ)$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}} = \sin(75^\circ)$

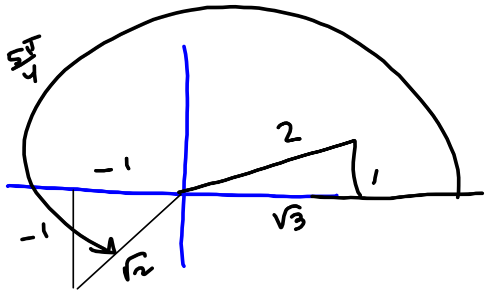
$75^\circ = 135^\circ - 60^\circ$

$\sin(135^\circ)\cos(-60^\circ) + \sin(-60^\circ)\cos(135^\circ)$

$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot \frac{-\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$



$$\frac{17\pi}{12} = \frac{15\pi}{12} + \frac{2\pi}{12} = \frac{5\pi}{4} + \frac{\pi}{6}$$



2.5 - Multiple-Angle and Product-to-Sum Identities.

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= 2 \cos^2 u - 1$$
$$= 1 - 2 \sin^2 u$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Half-Angle Formulas

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}} \qquad \cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

The signs of $\sin \frac{u}{2}$ and $\cos \frac{u}{2}$ depend on the quadrant in which $\frac{u}{2}$ lies.

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$

$= 2\cos^2 u - 1$

$= 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2 \tan u}{1 - \tan^2 u}$

Proof:

$\sin(2u) = \sin(u+u) = \sin(u)\cos(u) + \sin(u)\cos(u)$

$= 2\sin(u)\cos(u)$

$\cos(2u) = \cos(u+u) = \cos(u)\cos(u) - \sin(u)\sin(u)$

$\boxed{= \cos^2(u) - \sin^2(u)}$

$= \cos^2(u) - (1 - \cos^2(u)) = 1 - \sin^2(u) - \sin^2(u)$

$= \boxed{2\cos^2(u) - 1}$ $= \boxed{1 - 2\sin^2(u)}$

Power-Reducing Formulas

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$

$\cos^2 u = \dfrac{1 + \cos 2u}{2}$

$\tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$

By the above:

$\cos(2u) = 2\cos^2(u) - 1 \longrightarrow$

$2\cos^2(u) = \cos(2u) + 1$

$\boxed{\cos^2(u) = \dfrac{\cos(2u)+1}{2}} = \dfrac{1 + \cos(2u)}{2}$

$\cos(2u) = 1 - 2\sin^2(u) \longrightarrow$

$-2\sin^2(u) = \cos(2u) - 1$

$\sin^2(u) = \dfrac{\cos(2u) - 1}{-2} = \boxed{\dfrac{1 - \cos(2u)}{2} = \sin^2(u)}$

This is the source for the half-angle formulas

$$\sin^2(u) = \frac{1-\cos(2u)}{2} \Longrightarrow \qquad \cos^2(u) = \frac{1+\cos(2u)}{2}$$

$$\sin(u) = \pm\sqrt{\frac{1-\cos(2u)}{2}} \qquad \cos(u) = \sqrt{\frac{1+\cos(2u)}{2}}$$

Re-label:

HALF-ANGLE

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}} \qquad \cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

You decide → KNOW what quadrant $\frac{u}{2}$ is in!

Find the exact value of $\sin(15^\circ)$

$$\sin(15^\circ) = \pm\sqrt{\frac{1-\cos(30^\circ)}{2}} = \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}}$$

$\frac{u}{2} = 15^\circ$

$u = 30^\circ$

$$= \pm\frac{\sqrt{2-\sqrt{3}}}{2}$$

$15^\circ \in Q\,I$, so $\sin 15^\circ$

$$= +\sqrt{\frac{2-\sqrt{3}}{2}}$$