2.2

6. $\cos\left(\frac{\pi}{2} - u\right) = \sin(u)$ Proof by graphing.

Verifying a Trigonometric Identity In Exercises 9–50, verify the identity.

See identities in e-mail.

14. $\cos^2\beta - \sin^2\beta = 2\cos^2\beta - 1$

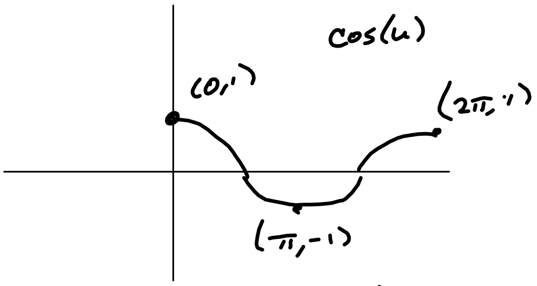
$\cos^2\beta - \sin^2\beta$
$= \cos^2\beta - (1 - \cos^2\beta)$
$= \cos^2\beta - 1 + \cos^2\beta$
$= 2\cos^2\beta - 1$ ■

Check your online.aims.edu e-mail on D2L. I sent you a grade report.

16. $\sin^2\alpha - \sin^4\alpha = \cos^2\alpha - \cos^4\alpha$

$\sin^2\alpha - \sin^4\alpha$
$= \sin^2\alpha - (\sin^2\alpha)^2$
$= 1 - \cos^2\alpha - (1 - \cos^2\alpha)^2$
$= 1 - \cos^2\alpha - (1 - 2\cos^2\alpha + \cos^4\alpha)$
$= 1 - \cos^2\alpha - 1 + 2\cos^2\alpha - \cos^4\alpha$
$= \cos^2\alpha - \cos^4\alpha$ ■

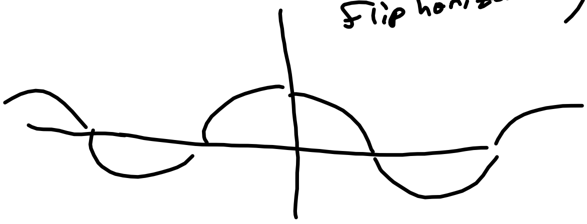
$(a-b)^2 = a^2 - 2ab + b^2$



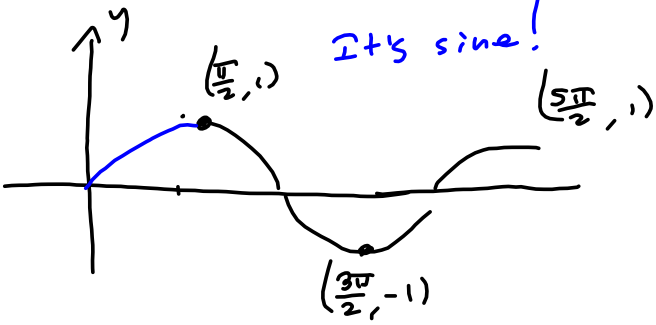
$$\frac{\pi}{2} - u = \underbrace{-}_{\text{Flip}}\underbrace{\left(u - \frac{\pi}{2}\right)}_{\text{RIGHT } \frac{\pi}{2}}$$

$\cos(-u) = \cos(u)$

Flip horizontally



It's sine!



$\cos\left(\frac{\pi}{2} - u\right) =$

$\cos\left(-\left(u - \frac{\pi}{2}\right)\right)$

$= \cos\left(u - \frac{\pi}{2}\right)$

RIGHT $\frac{\pi}{2}$

& we get sine.

20. $\dfrac{1}{\tan\beta} + \tan\beta = \dfrac{\sec^2\beta}{\tan\beta}$

$$\frac{1}{\tan\beta} + \tan\beta \cdot \frac{\tan\beta}{\tan\beta}$$

$$= \frac{1}{\tan\beta} + \frac{\tan^2\beta}{\tan\beta}$$

$$= \frac{1+\tan^2\beta}{\tan\beta}$$

$$= \frac{\sec^2\beta}{\tan\beta} \quad \blacksquare$$

41. $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \dfrac{1+\sin\theta}{|\cos\theta|}$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}}$$

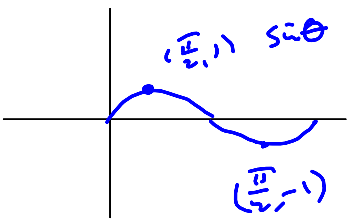
$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

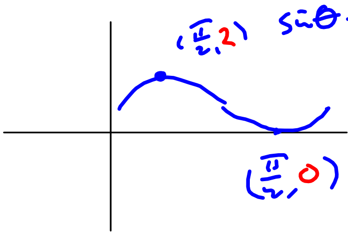
$$= \frac{\sqrt{(1+\sin\theta)^2}}{\sqrt{\cos^2\theta}}$$

$$= \frac{|1+\sin\theta|}{|\cos\theta|}$$

$$= \frac{1+\sin\theta}{|\cos\theta|} \quad \blacksquare$$

$$|1+\sin\theta| = \begin{cases} 1+\sin\theta & \text{if } 1+\sin\theta \geq 0 \\ -(1+\sin\theta) & \text{if } 1+\sin\theta < 0 \end{cases}$$



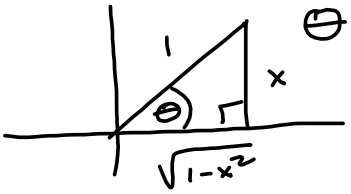


$\sin\theta+1$ is ≥ 0!

1. An equation that is true for all real values in its domain is called an identity.
2. An equation that is true for only some values in its domain is called a conditional equation.

47. $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1 - x^2}}$

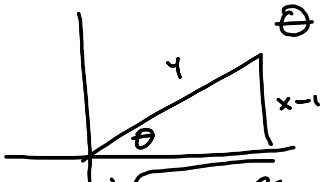
$\tan(\arcsin(x)) = \tan\theta$



$\tan\theta = \dfrac{x}{\sqrt{1-x^2}}$

49. $\tan\left(\sin^{-1}\dfrac{x-1}{4}\right) = \dfrac{x-1}{\sqrt{16-(x-1)^2}}$

$= \tan\left(\arcsin\left(\frac{x-1}{4}\right)\right) = \tan\theta$



$\sqrt{4^2-(x-1)^2}$

$= \sqrt{16-(x-1)^2}$

$\Rightarrow \tan\theta = \boxed{\dfrac{x-1}{\sqrt{16-(x-1)^2}}}$

Work on §2.2.

§2.3 solve trig equations

From LAST TIME.

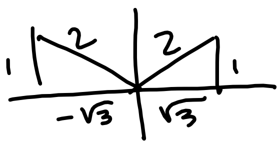
§2.3: Solve

$$4\sin^2\theta - 1 = 0$$

Find all solutions in $[0, 2\pi]$

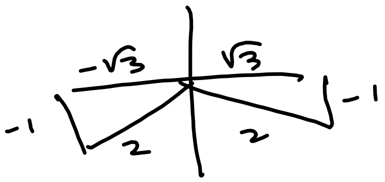
$$(2\sin\theta + 1)(2\sin\theta - 1) = 0$$

$$\Rightarrow \sin\theta = -\tfrac{1}{2} \quad \text{OR} \quad \sin\theta = \tfrac{1}{2}$$



$30°, 150°$

$\frac{\pi}{6}, \frac{5\pi}{6}$



$\frac{7\pi}{6}, \frac{11\pi}{6}$

$x \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

Find ALL Solutions:

$$\frac{\pi}{6} + 2n\pi, \quad \frac{5\pi}{6} + 2n\pi, \quad \frac{7\pi}{6} + 2n\pi, \quad \frac{11\pi}{6} + 2n\pi$$

$$\frac{5\pi}{6} + n\pi$$

$$\frac{\pi}{6} + n\pi$$

Set-builder:

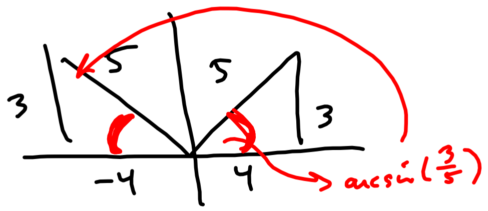
Sol'n set: $\left\{ x + n\pi \;\middle|\; x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6},\ n \in \mathbb{Z} \right\}$

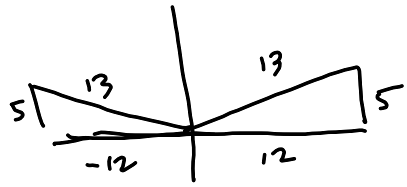
$65\sin^2\theta - 64\sin\theta + 15 = 0 \Longrightarrow$

$(5\sin u - 3)(13\sin(u) - 5) = 0 \Longrightarrow$

$5\sin u = 3$ or $13\sin u = 5 \Longrightarrow$

$\sin(u) = \frac{3}{5}$ or $\sin u = \frac{5}{13}$





$13^2 - 5^2 = 169 - 25 = 144 = 12^2$

Write the answer:

EXACT:

$u = \arcsin(\frac{3}{5})$ or $\pi - \arcsin(\frac{3}{5})$

SAME REFERENCE ANGLE.

Different Quadrants

$\approx 36.86989765^\circ$ in Desmos. & $180^\circ - 36.86989765^\circ \approx 143.1301023^\circ$

≈ 0.6435011089 radians & $\pi - 0.6435011089 \approx 2.498091545$

Rounded to 4 places, all sol'ns in $[0, 2\pi]$ are

$u \approx .6435, 2.4981$

Find All sol'ns:

$\boxed{u \approx .6435 + 2n\pi, \ 2.4981 + 2n\pi}$

In degrees:

$\boxed{u \approx 36.8699^\circ + 360^\circ n, \ 143.1301^\circ + 360^\circ n}$

https://www.desmos.com/scientific

Solve $4\sin^2(3\theta)-1=0 \Longrightarrow (2\sin(3\theta)-1)(2\sin(3\theta)+1)$

Find all sol'ns $\theta \in [0,2\pi]$

$0 \le \theta \le 2\pi \Longrightarrow$

$0 \le 3\theta \le 6\pi$

we must find all $3\theta \in [0,6\pi]$

to capture all θ in $[0,2\pi]$

$\sin(3\theta)=\frac{1}{2}$ or $\sin(3\theta)=-\frac{1}{2}$ by previous work.

Trig polynomials with multiple-angle arguments.

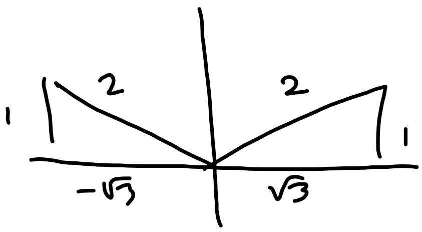
3θ

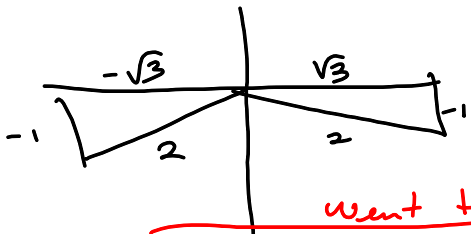
multiple.

$\frac{4\pi}{3}$

$\frac{4\pi}{3}$

$\sin 3\theta = \frac{1}{2}$





went too far!

$3\theta = \frac{\pi}{6}, \frac{\pi}{6}+2\pi, \frac{\pi}{6}+4\pi, \boxed{\frac{\pi}{6}+6\pi > 6\pi ?!}$

$\Rightarrow \theta = \frac{\pi}{18}, \frac{\pi}{18}+\frac{2\pi}{3}, \frac{\pi}{18}+\frac{4\pi}{3}$ $\quad \frac{\pi}{18}+2\pi \notin [0,2\pi]$

Likewise, $\frac{\pi}{18}, \frac{13\pi}{18}, \frac{25\pi}{18}$

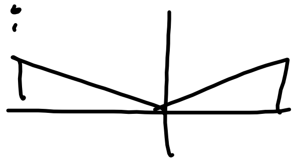
$3\theta = \frac{5\pi}{6}, \frac{5\pi}{6}+2\pi, \frac{5\pi}{6}+4\pi$

$\theta = \frac{5\pi}{18}, \frac{5\pi}{18}+\frac{2\pi}{3}, \frac{5\pi}{18}+\frac{4\pi}{3}$

$= \frac{5\pi}{18}, \frac{17\pi}{18}, \frac{29\pi}{18}$

$\theta \in \boxed{\left\{\frac{\pi}{18}, \frac{13\pi}{18}, \frac{25\pi}{18}, \frac{5\pi}{18}, \frac{17\pi}{18}, \frac{29\pi}{18}\right\} = A}$

Handles:



Still need

$\frac{7\pi}{18}, \frac{7\pi}{18}+\frac{2\pi}{3}, \frac{7\pi}{18}+$

$\frac{11\pi}{18}, \frac{11\pi}{18}+\frac{2\pi}{3}, \frac{11\pi}{18}+$

$\boxed{\theta \in \left\{\frac{11\pi}{18}, \text{etc.} \right\} = B}$

Solution Set is $\boxed{A \cup B}$

Please check your Grade Report.

Make sure the idiot didn't get anything wrong.