Please Check your E-Mail.

Kindly give me a chat as you come in.

It's also nice if you say "Adios" on your way out.

Writing Project #0 Late Edition is Open on D2L

Section 2.1 - Trig Identities:

Basically word puzzles with algebra thrown in.

Reciprocal Identities

$\sin u = \frac{1}{\csc u}$ $\cos u = \frac{1}{\sec u}$ $\tan u = \frac{1}{\cot u}$

$\csc u = \frac{1}{\sin u}$ $\sec u = \frac{1}{\cos u}$ $\cot u = \frac{1}{\tan u}$

Quotient Identities

$\tan u = \frac{\sin u}{\cos u}$ $\cot u = \frac{\cos u}{\sin u}$

$\tan(u) = \frac{y}{x}$

$\frac{\sin(u)}{\cos(u)} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x}$, see?

Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$ $\boxed{1 + \tan^2 u = \sec^2 u}$ $1 + \cot^2 u = \csc^2 u$

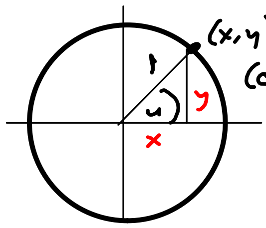
↳ $\sec^2(u) - 1 = \tan^2(u)$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - u\right) = \cos u$ $\cos\left(\frac{\pi}{2} - u\right) = \sin u$

$\tan\left(\frac{\pi}{2} - u\right) = \cot u$ $\cot\left(\frac{\pi}{2} - u\right) = \tan u$

$\sec\left(\frac{\pi}{2} - u\right) = \csc u$ $\csc\left(\frac{\pi}{2} - u\right) = \sec u$



$(x, y) = (\cos(u), \sin(u))$

$1 + \frac{\sin^2(u)}{\cos^2(u)} = \frac{\cos^2(u) + \sin^2(u)}{\cos^2(u)}$

$= \frac{1}{\cos^2(u)} = \left(\frac{1}{\cos(u)}\right)^2$

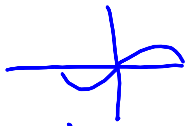
$= \sec^2(u)$

Even/Odd Identities

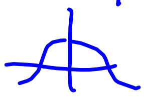
$\sin(-u) = -\sin u$ $\cos(-u) = \cos u$ $\tan(-u) = -\tan u$

$\csc(-u) = -\csc u$ $\sec(-u) = \sec u$ $\cot(-u) = -\cot u$

sine is odd



cosine is even



EXAMPLE 1 Using Identities to Evaluate a Function

Use the conditions $\sec u = -\frac{3}{2}$ and tan $u > 0$ to find the values of all six trigonometric functions.

→ The 2nd given

Solution Using a reciprocal identity, you have

$$\cos u = \frac{1}{\sec u} = \frac{1}{-3/2} = -\frac{2}{3}.$$

Using a Pythagorean identity, you have

$$\begin{aligned} \sin^2 u &= 1 - \cos^2 u && \text{Pythagorean identity} \\ &= 1 - \left(-\tfrac{2}{3}\right)^2 && \text{Substitute } -\tfrac{2}{3} \text{ for } \cos u. \\ &= \tfrac{5}{9}. && \text{Simplify.} \end{aligned}$$

I wouldn't even think of using a Pythagorean identity, here, as I would have the triangle already drawn, with all 3 sides:

(i) $\cos(u) = \frac{1}{\sec(u)} = \frac{1}{(-\frac{3}{2})} = -\frac{2}{3} = \cos(u)$. There are 2 triangles fitting the bill

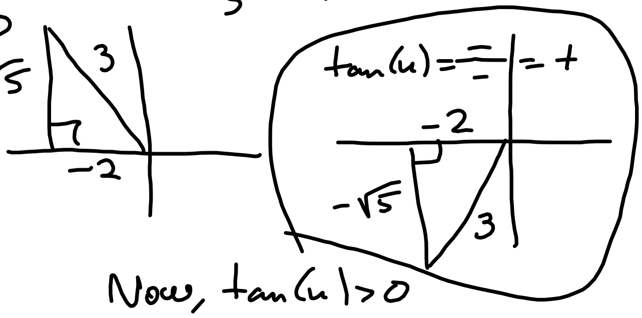
(ii) $\sec(u) = -\frac{3}{2} \Rightarrow \frac{1}{\cos(u)} = -\frac{3}{2} \longrightarrow -\frac{2}{3} = \cos(u)$

(iii) $\sec(u) = -\frac{3}{2} \rightarrow \cos(u) = -\frac{2}{3}$

$\cos(u) = -\frac{2}{3} = \frac{x}{r}$

$\tan(u) < 0$

NO $\sqrt{5}$



Pythagorus

$\sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

$= 6$

Now, $\tan(u) > 0$ says

$\sin(u) = \frac{-\sqrt{5}}{3}$	$\csc(u) = -\frac{3}{\sqrt{5}} = \frac{-3\sqrt{5}}{5}$
$\cos(u) = -\frac{2}{3}$	$\sec(u) = -\frac{3}{2}$
$\tan(u) = \frac{\sqrt{5}}{2}$	$\cot(u) = \frac{2}{\sqrt{5}}$ OR $\frac{2\sqrt{5}}{5}$

Simplify $\cos^2 x \csc x - \csc(x)$

$= \csc(x)(\cos^2(x) - 1) =$

CHECKPOINT
After EXAMPLE 2.

$= -\csc(x)(1 - \cos^2(x))$

$\sin^2(u) + \cos^2(u) = 1 \longrightarrow$
$\sin^2(u) = 1 - \cos^2(u)$

$= -\frac{1}{\sin(x)}\left(\sin^2(x)\right) = \boxed{-\sin(x)}$

$$\frac{\overset{a^2 - b^2}{\sec^2(x) - 1^2}}{\sec(x) - 1} = \frac{\overset{(a-b)}{(\sec(x) - 1)}\overset{(a+b)}{(\sec(x) + 1)}}{\sec(x) - 1} = \sec(x) + 1$$

$$\frac{\sec^2(x) - 1}{\sec(x) - 1} = \frac{\tan^2(x)}{\sec(x) - 1} \cdot \frac{(\sec(x) + 1)}{(\sec(x) + 1)}$$

$$= \frac{\tan^2(x)(\sec(x) + 1)}{\sec^2(x) - 1^2} = \frac{\tan^2(x)(\sec(x) + 1)}{\tan^2(x)}$$

$$= \sec(x) + 1$$

Factoring Trig Expressions

FACTOR $4\sin^2\theta - 1$, Let $u = \sin\theta$. Then $4u^2 - 1 = (2u-1)(2u+1)$

$= (2\sin\theta - 1)(2\sin\theta + 1)$

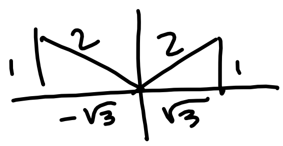
§2.3: Solve

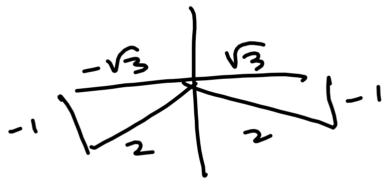
$4\sin^2\theta - 1 = 0 \longrightarrow$

$(2\sin\theta - 1)(2\sin\theta + 1) = 0 \longrightarrow$

$2\sin\theta = 1$ OR $2\sin\theta = -1$

$\sin\theta = \frac{1}{2}$ OR $\sin\theta = -\frac{1}{2}$





$2^2 - 1^2 = 3$

$\rightsquigarrow \sqrt{3}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$30^\circ, 150^\circ$

$\frac{7\pi}{6}, \frac{11\pi}{6}$

$\boxed{\text{Sol'ns in } [0, 2\pi]: \ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

All sol'ns:

$\frac{\pi}{6} + 2n\pi, n \in \mathbb{Z}$

$\frac{5\pi}{6} + 2n\pi, n \in \mathbb{Z}$

$\frac{7\pi}{6} + 2n\pi, n \in \mathbb{Z}$

$\frac{11\pi}{6} + 2n\pi, n \in \mathbb{Z}$

$\rightarrow \frac{\pi}{6} + n\pi, n \in \mathbb{Z}$

$\frac{5\pi}{6} + n\pi, n \in \mathbb{Z}$

§2.3.

Factor $65\sin^2\theta - 64\sin\theta + 15$ We'll need this in

SLEDGEHAMMER!?

§2.3 question: Solve $65\sin^2\theta - 64\sin\theta + 15 = 0$
with only a scientific calculator. Find all solutions
in $[0, 2\pi]$ in Radians, to 3 decimal places.

You can factor ANY polynomial with integer coefficients up to degree 4, by radicals.

Factor $65\sin^2\theta - 64\sin\theta + 15$

$65u^2 - 64u + 15$

$65u^2 - 39u - 25u + 15$

$= 13u(5u-3) - 5(5u-3)$

$= (5u-3)(13u-5)$

$= (5\sin(u) - 3)(13\sin(u) - 5)$

$15 = 3 \cdot 5$

$65 = 13 \cdot 5$

$13 \cdot 3 + 5 \cdot 5$

$= 39 + 25 = 64$

$(15)(65) = 975$

-64	$= -63 - 1$	63
	$= -54 - 10$	540
	$= -44 - 20$	880
	$= -40 - 24$	960
	$= -39 - 25$	975

48
480
528

I'm done fooling with these messy big factors, because my teacher sucks at arithmetic.

$65u^2 - 64u + 15 = 0$

$a = 65, b = -64, c = 15$

$b^2 - 4ac = 64^2 - 4(65)(15) = 196$

$\sqrt{196} = \sqrt{2^2 \cdot 7^2} = 2 \cdot 7 = 14$

$$\begin{array}{r|l} 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ & 7 \end{array}$$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{64 \pm 14}{2(65)} =$

$= \dfrac{64+14}{130}$ OR $\dfrac{64-14}{130}$

$= \dfrac{78}{130} = \dfrac{39}{65} = \dfrac{3}{5}$ $\quad = \dfrac{50}{130} = \dfrac{5}{13}$

So $u = \frac{3}{5}, \frac{5}{13} \longrightarrow$

$65\left(u - \frac{3}{5}\right)\left(u - \frac{5}{13}\right)$

$= 13 \cdot 5 \left(u - \frac{3}{5}\right)\left(u - \frac{5}{13}\right)$

$= 5\left(u - \frac{3}{5}\right)(13)\left(u - \frac{5}{13}\right) = (5u - 3)(13u - 5)$

$= (5\sin\theta - 3)(13\sin\theta - 5)$

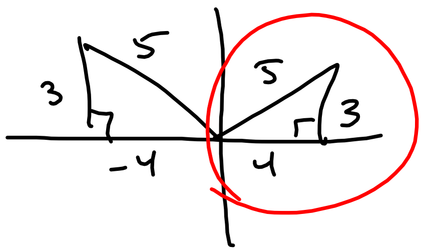
§2.3 says

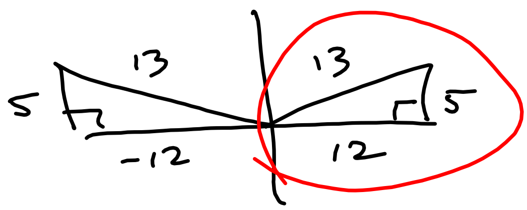
Solve this thing $= 0$

$5\sin\theta - 3 = 0$ or $13\sin\theta - 5 = 0$

$5\sin\theta = 3$ $\qquad$ $13\sin\theta = 5$

$\sin\theta = \frac{3}{5}$ or $\sin\theta = \frac{5}{13}$





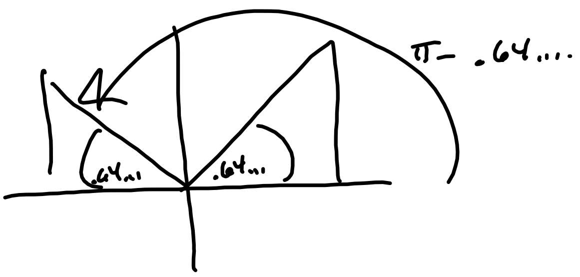
$\arcsin\left(\frac{3}{5}\right)$ $\qquad$ $\arcsin\left(\frac{5}{13}\right)$

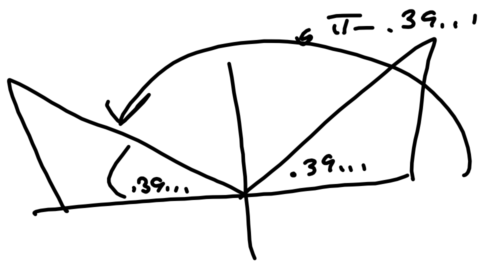
$= \sin^{-1}\left(\frac{3}{5}\right) \approx 36.86989765^\circ$ $\qquad$ $\sin^{-1}\left(\frac{5}{13}\right) \approx 22.61986495^\circ$

≈ 0.6435011089 $\qquad$ ≈ 0.3947911197

&

$\pi - .39\ldots$





$(2\sin\theta - \sqrt{3})(2\sin\theta + \sqrt{3})$

Solve: $4\sin^2\theta - 3 = 0$

$4u^2 - 3 = 0$

$4u^2 = 3$

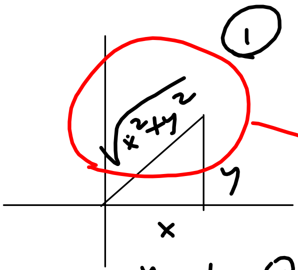
$u^2 = \frac{3}{4}$

$u = \pm\frac{\sqrt{3}}{2}$

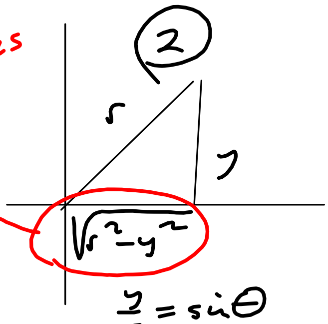
$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$4\left(u - \frac{\sqrt{3}}{2}\right)\left(u + \frac{\sqrt{3}}{2}\right)$

Trig Substitution:



Pythagorean structures

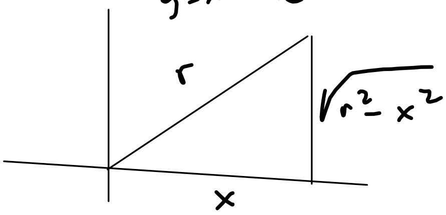


$\frac{y}{x} = \tan\theta$

$y = x\tan\theta$

$\frac{y}{r} = \sin\theta$

$y = r\sin\theta$



$\frac{x}{r} = \cos\theta$

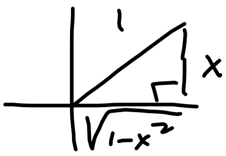
$x = r\cos\theta$

$\frac{r}{x} = \sec\theta$

$r = x\sec\theta$

misc:

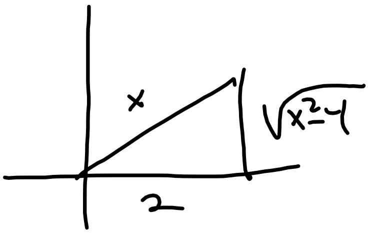
$$\sec(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

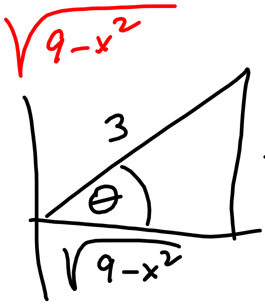


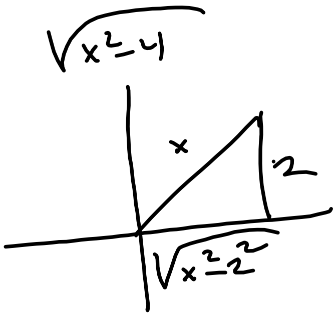
Trigonometric Substitution In Exercises 55–58, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

55. $\sqrt{9 - x^2}$, $x = 3 \cos \theta$
56. $\sqrt{49 - x^2}$, $x = 7 \sin \theta$
57. $\sqrt{x^2 - 4}$, $x = 2 \sec \theta$
58. $\sqrt{9x^2 + 25}$, $3x = 5 \tan \theta$

$\frac{x}{2} = \sec\theta$







$\frac{x}{3} = \sin\theta$

$x = 3\sin\theta$

is standard.

Now, substitute

$\sqrt{9-x^2} = \sqrt{9 - 9\sin^2\theta}$

$= \sqrt{9(1-\sin^2\theta)}$

$= 3\sqrt{1-\sin^2\theta} = 3\sqrt{\cos^2\theta}$

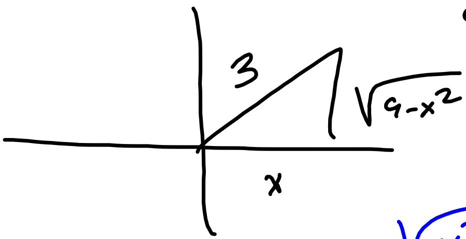
$= 3|\cos\theta|$

$= 3\cos\theta$, due to $0 < \theta < \frac{\pi}{2}$ condition

so $|\cos\theta| = \cos\theta$.

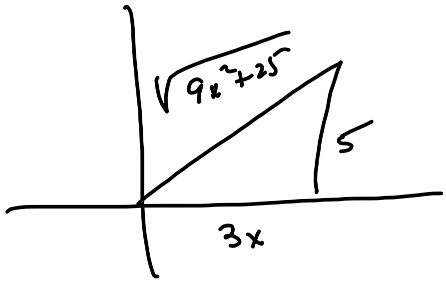
THEY WANT $x = 3\cos\theta$

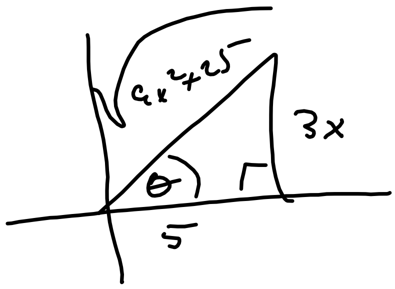
$\cos\theta = \frac{x}{3}$



$\sqrt{x^2} = |x|$

$\sqrt{9x^2+25}$





$\frac{3x}{5} = \tan\theta$

$x = \frac{5}{3}\tan\theta$

is your substitution.

Add

$$\frac{\sin(x)}{1+\cos(x)} + \frac{\sin(x)}{1-\cos(x)}$$

$$LCD = (1+\cos(x))(1-\cos(x)) = 1-\cos^2(x) = \sin^2(x)$$

$$\left(\frac{\sin(x)}{1+\cos(x)}\right)\cdot\left(\frac{1-\cos(x)}{1-\cos(x)}\right) + \left(\frac{\sin(x)}{1-\cos(x)}\cdot\right)\left(\frac{1+\cos(x)}{1+\cos(x)}\right)$$

$$= \frac{\sin(x) - \sin(x)\cos(x) + \sin(x) + \sin(x)\cos(x)}{1-\cos^2(x)}$$

$$= \frac{2\sin(x)}{\sin^2(x)} = \frac{2}{\sin(x)} = \boxed{2\csc(x)}$$

$$\cos(\arctan(x)) = \cos\theta = \boxed{\frac{1}{\sqrt{x^2+1}}}$$