Polar Equations of Conics

The graph of a polar equation of the form

1. $r = \dfrac{ep}{1 \pm e \cos\theta}$ or **2.** $r = \dfrac{ep}{1 \pm e \sin\theta}$

is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

$r = \dfrac{ep}{1 \pm e \sin\theta}$ Horizontal directrix

$r = \dfrac{ep}{1 \pm e \cos\theta}$ Vertical directrix

1. Horizontal directrix above the pole: $r = \dfrac{ep}{1 + e \sin\theta}$

2. Horizontal directrix below the pole: $r = \dfrac{ep}{1 - e \sin\theta}$

3. Vertical directrix to the right of the pole: $r = \dfrac{ep}{1 + e \cos\theta}$

4. Vertical directrix to the left of the pole: $r = \dfrac{ep}{1 - e \cos\theta}$

17. 0/1 points LarTrig10 6.9.0

Find a polar equation of the indicated conic in terms of *r* with the given characteristics and focus at the pole.

Conic	*Eccentricity*	*Directrix*
Parabola	$e = 1$	$x = -1$

$r = \dfrac{1}{1 - \cos(\theta)}$

To the left

$e = 1,\ p = 1.$

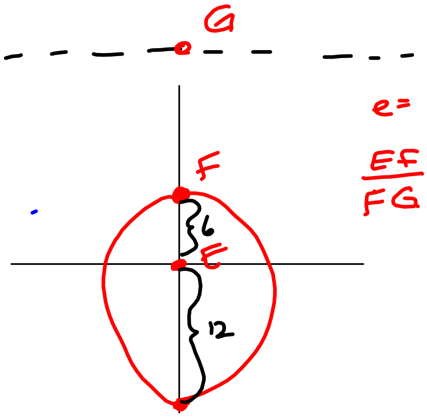
$r = \dfrac{ep}{1 - e\cos\theta} = \dfrac{1 \cdot 1}{1 - \cos\theta}$

21. 0/1 points LarTrig10 6.9.048. [3884566]

Find a polar equation of the indicated conic in terms of r with the given characteristics and focus at the pole.

Conic	Vertices
Ellipse	$\left(6, \frac{\pi}{2}\right), \left(12, \frac{3\pi}{2}\right)$

$r = \dfrac{24}{\sin(\theta) + 3}$



$y = 24?$

$e = \frac{1}{3}$

$\frac{EF}{FG} = \frac{1}{3}$

Want $r\left(\frac{\pi}{2}\right) = 6$

$r\left(\frac{3\pi}{2}\right) = 12$

$$r = \frac{ep}{1 + e\sin\theta}$$

$$r\left(\frac{\pi}{2}\right) = \frac{ep}{1 + e\sin\frac{\pi}{2}} = 6$$

$$\frac{ep}{1+e} = 6$$

$$ep = 6(1+e)$$

$$e = \frac{6(1+e)}{p}$$

$$r\left(\frac{3\pi}{2}\right) = \frac{ep}{1 + e\sin\left(\frac{3\pi}{2}\right)} = 12$$

$$\frac{ep}{1-e} = 12$$

$$\frac{\frac{6(1+e)}{p}\,p}{1-e} = 12$$

$$\frac{6+6e}{1-e} = 12$$

$$6 + 6e = 12 - 12e$$

$$18e = 6$$

$$e = \frac{1}{3}$$

$$\frac{ep}{1+e} = \frac{\frac{1}{3}p}{1+\frac{1}{3}} = 6$$

$$\frac{\frac{1}{3}p}{\frac{4}{3}} = 6$$

$$\frac{1}{4}p = 6$$

$$p = 24$$

$$r = \frac{\left(\frac{1}{3}\right)(24)\,(3)}{\left(1 + \frac{1}{3}\sin\theta\right)(3)} = \frac{24}{3 + \sin\theta}$$

20. 0/1 points

Find a polar equation of the conic in terms of r with its focus at the pole.

Conic	*Vertices*
Ellipse	$(4, 0)$, $(12, \pi)$

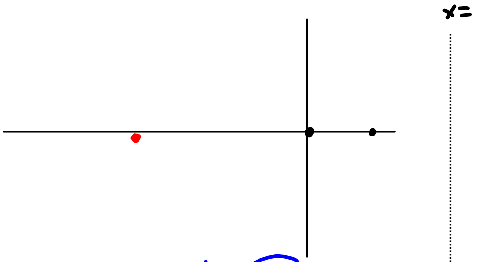
$r = \dfrac{12}{\cos(\theta) + 2}$

$\dfrac{ep}{1 + e\cos\theta}$

$r(0) = 4 \Rightarrow$

$\dfrac{ep}{1+e} = 4$

$r(\pi) = \dfrac{ep}{1-e} = 12$



$ep = 4 + 4e$
$ep = 12 - 12e$ $\Rightarrow 4 + 4e = 12 - 12e$

$16e = 8$

$\boxed{e = \frac{1}{2}}$

$\Rightarrow \frac{1}{2}p = 4 + 2 = 6$

$\Rightarrow \boxed{p = 12}$

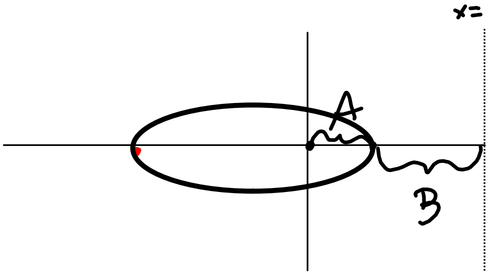
$r = \dfrac{\frac{1}{2}(12)}{1 + \frac{1}{2}\cos\theta} = \dfrac{12}{2 + \cos\theta}$

$= \dfrac{6}{1 + \frac{1}{2}\cos\theta}$

Directrix for hand sketch?

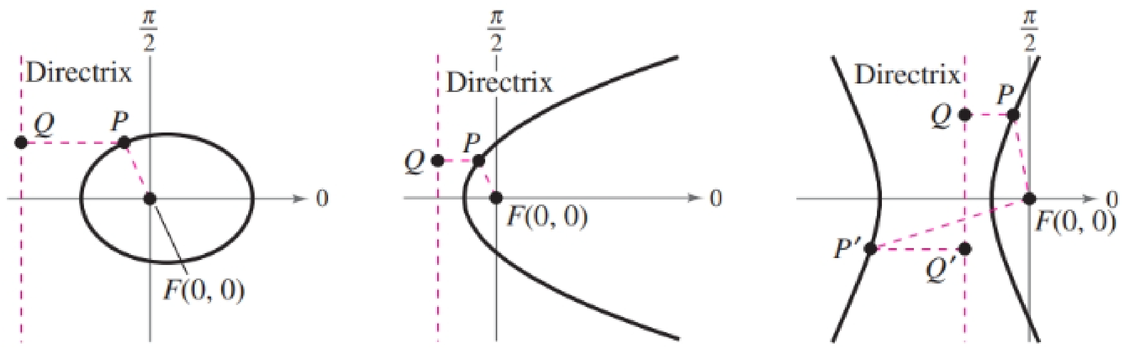
p = Distance from pole to directrix $= 12$

$e = \frac{1}{2}$



$\dfrac{A}{B} = e = \dfrac{1}{2}$?

$A = 4$
$B = 8$ $\Big\}$ $\dfrac{4}{8} = \dfrac{1}{2}$ ✓



Ellipse: $0 < e < 1$ $\quad \frac{PF}{PQ} < 1$

Parabola: $e = 1$ $\quad \frac{PF}{PQ} = 1$

Hyperbola: $e > 1$ $\quad \frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

$r = \frac{1}{1-\sin\theta}$ -vs- $\frac{-1}{1-\sin\theta}$

parabola, H.D. BELOW POLE

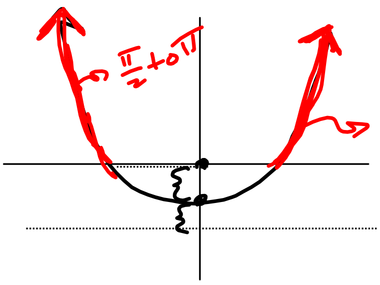
$\frac{1}{1-1\sin\theta}$ ← $ep=1$

$e=1$

$p=1$ gives $y=-1$

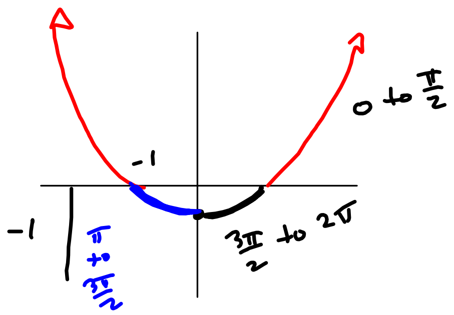
and

$e=1$ means vertex @ $(x,y) = (0,-\frac{1}{2})$



$r(\frac{\pi}{2}) = \frac{1}{1-\sin\frac{\pi}{2}} = \frac{1}{0} = \infty$?!

$r(0) = \frac{1}{1-0} = 1$



$\frac{1}{1-\sin\frac{5\pi}{4}} = \frac{1}{1-(-\frac{\sqrt{2}}{2})}$

$\frac{1}{1-\sin\frac{3\pi}{2}} = \frac{1}{1-(-1)} = \frac{1}{2}$

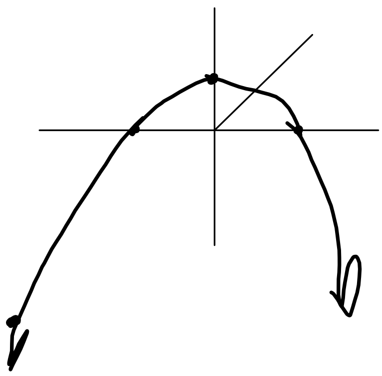
$\frac{1}{1+\frac{\sqrt{2}}{2}} \approx \frac{1}{1+.4} = \frac{1}{2.4} = \frac{10}{24}$

.707...

$= \frac{5}{12}$

.5857864376

$r = \frac{1}{1-\sin\theta}$ -vs- $\frac{-1}{1-\sin\theta}$



$r(\frac{\pi}{4}) = \frac{-1}{1-\sin\frac{\pi}{4}} = \frac{-1}{.3} = -1.\overline{3}$

Same parabola, up-side-down

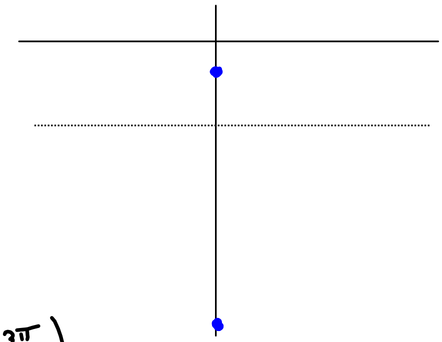
50. Hyperbola $(2, 0), (8, 0)$

51. Hyperbola $(1, 3\pi/2), (9, 3\pi/2)$

52. Hyperbola $(4, \pi/2), (1, \pi/2)$

51 $\left(1, \frac{3\pi}{2}\right), \left(9, \frac{3\pi}{2}\right) = \left(-9, \frac{\pi}{2}\right)$?

$\left(1, \frac{3\pi}{2}\right) = \left(-1, \frac{\pi}{2}\right)$?



$\frac{ep}{1 - e\sin\theta}$

$\theta = \frac{3\pi}{2}$

$\frac{ep}{1 - e\sin\left(\frac{3\pi}{2}\right)} = \frac{ep}{1+e} = 1$

-1

$\frac{ep}{1 - e\sin\frac{3\pi}{2}} = 9$?

$\left(1, \frac{3\pi}{2}\right)$

or is it

$\left(-1, \frac{\pi}{2}\right)$

$\frac{ep}{1+e} = 9$

$\frac{ep}{1-e} = -1$ $\left(\text{from } \frac{ep}{1 - e\sin\frac{\pi}{2}} = -1\right)$

$\frac{ep}{1+e} = 9 \Rightarrow ep = 9 + 9e$

$\frac{ep}{1-e} = -1 \Rightarrow ep = -1 + e$

$9 + 9e = -1 + e$

$8e = -10$

$e = \frac{-10}{8}$?!

$= -\frac{5}{4}$?

$e < 0$?

alternate

$\left(9, \frac{3\pi}{2}\right) = \left(-9, \frac{\pi}{2}\right)$

$$r\left(\frac{\pi}{2}\right) = -9$$

$$\frac{ep}{1 - e\sin\left(\frac{\pi}{2}\right)} = -9$$

$$\frac{ep}{1 - e\cdot 1} = \frac{ep}{1-e} = -9$$

$$ep = -9 + 9e$$

$$-9 + 9e = 1 + e$$

$$8e = 10$$

$$e = \frac{10}{8} = \frac{5}{4}$$

$$r\left(\frac{3\pi}{2}\right) = 1$$

$$\frac{ep}{1 - e\sin\left(\frac{3\pi}{2}\right)} = 1$$

$$\frac{ep}{1+e} = 1$$

$$ep = 1 + e$$

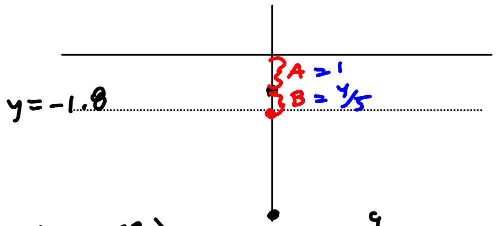
$$\left(\frac{5}{4}\right)p = 1 + \frac{5}{4} = \frac{9}{4}$$

$$p = \left(\frac{9}{4}\right)\left(\frac{4}{5}\right) = \boxed{\frac{9}{5} = p}$$

$$\frac{A}{B} = \frac{5}{4}$$

$$\frac{1}{B} = \frac{5}{4}$$

$$\frac{4}{5} = B$$

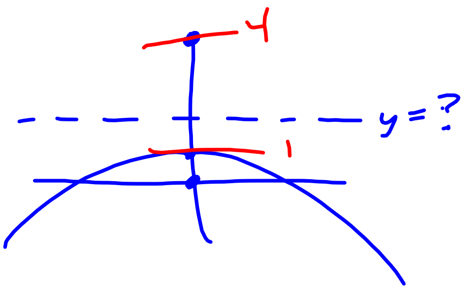


$$r = \frac{ep}{1 - e\sin\theta} = \frac{(5/4)\left(\frac{9}{5}\right)}{1 - \frac{5}{4}\sin\theta} = \frac{\frac{9}{4}}{1 - \frac{5}{4}\sin\theta} = \frac{9}{4 - 5\sin\theta}$$

$$r\left(\frac{\pi}{2}\right) = \frac{\frac{9}{4}}{1 - \frac{5}{4}} = \frac{\frac{9}{4}}{-\frac{1}{4}} = -9$$

$$r\left(\frac{3\pi}{2}\right) = \frac{\frac{9}{4}}{1 - \frac{5}{4}(-1)} = \frac{\frac{9}{4}}{1 + \frac{5}{4}} = \frac{\frac{9}{4}}{\frac{9}{4}} = 1$$

$\left(4, \frac{\pi}{2}\right), \left(1, \frac{\pi}{2}\right)$ OR $\left(-4, \frac{3\pi}{2}\right), \left(1, \frac{\pi}{2}\right)$



Above pole

$$\frac{ep}{1+e\sin\theta}$$

$r\left(\frac{3\pi}{2}\right) = -4$ $\qquad$ $r\left(\frac{\pi}{2}\right) = 1$

$$\frac{ep}{1+e\sin\left(\frac{3\pi}{2}\right)} = -4 \qquad \frac{ep}{1+e\sin\left(\frac{\pi}{2}\right)} = 1$$

$$\frac{ep}{1-e} = -4 \qquad \frac{ep}{1+e} = 1$$

$$ep = -4+4e = 1+e$$

$$3e = 5$$

$$\boxed{e = \frac{5}{3}} \Longrightarrow \frac{\frac{5}{3}p}{1-\frac{5}{3}} = -4$$

$$\frac{\frac{5}{3}p}{1+\frac{5}{3}\sin\theta} \qquad \frac{\frac{5}{3}p}{-\frac{2}{3}} = -4$$

$$\frac{5}{3}p = \left(\frac{2}{3}\right)(4)$$

$$p = \frac{3}{5}\left(\frac{2}{3}\right)(4) = \boxed{\frac{8}{5} = p}$$

$$r = \frac{ep}{1+e\sin\theta} = \frac{\left(\frac{5}{3}\right)\left(\frac{8}{5}\right)}{1+\frac{5}{3}\sin\theta}$$

$$\frac{\frac{40}{5}}{3+5\sin\theta} = \frac{8}{3+5\sin\theta}$$