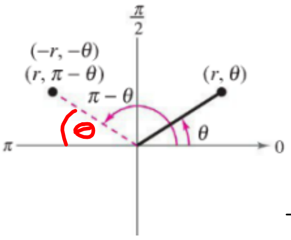
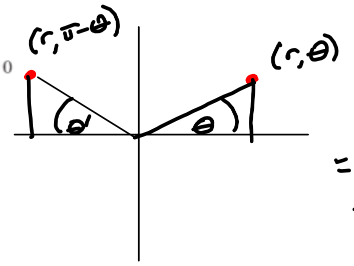
Homework Questions?

As usual, graphing by just plotting points is the last-ditch effort of a person who has no clue what the thing looks like. We want as much insight/intuition as possible to help guide us, starting with the different kinds of symmetry.



Replace (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$.

Symmetric about $\theta = \frac{\pi}{2}$



$r = 1 + \sin\theta$

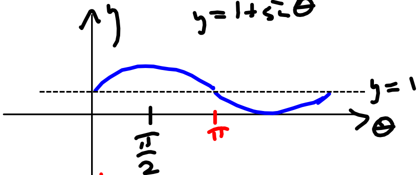
$r = 1 + \sin(\pi - \theta)$

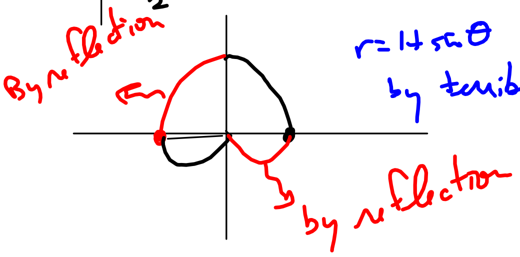
$= 1 + \sin\pi\cos(-\theta) + \sin(-\theta)\cos(\pi)$

$= 1 + 0\cos(-\theta) + (-\sin\theta)(-1)$

$= 1 + \sin\theta$ = SAME!

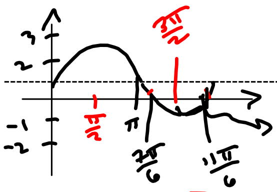
Rectangular coords



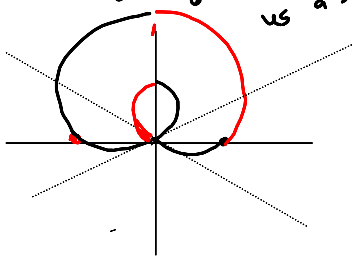


$r = 1 + \sin\theta$ by terrible artist.

$r = 1 + 2\sin\theta$ will have a loop in it!



→ Negative part gives us a loop!



Has same symmetry

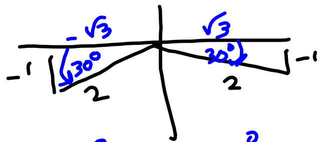
$r = 1 + 2\sin(\pi - \theta) = 1 + 2\sin\theta$ equiv.

Scratch:

$1 + 2\sin\theta = 0$

$2\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$



210^0 or 330^0

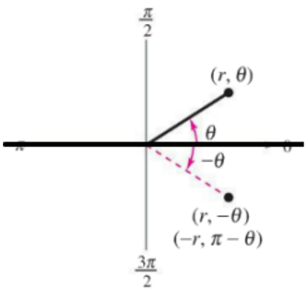
$\frac{7\pi}{6}$ or $\frac{11\pi}{6}$

3-petal rose

$r = \sin(3\theta)$

$r = \cos(3\theta)$ — Symmetric about polar axis

See next page



Symmetry with Respect to the Polar Axis

2. The polar axis:

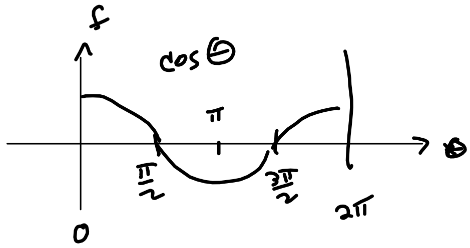
Replace (r, θ) by $(r, -\theta)$ or $(-r, \pi - \theta)$.

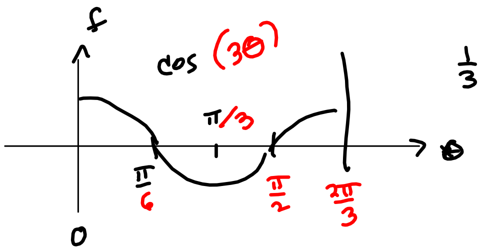
$r = \cos(3\theta) \rightsquigarrow r = \cos(3(-\theta)) = \cos(3\theta)$,

b/c cosine is even

$f(3x)$ from graph of $f(x)$

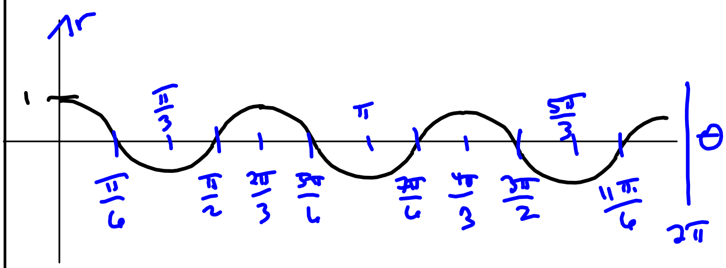
$(\frac{1}{3}x, y) \leftarrow (x, y)$

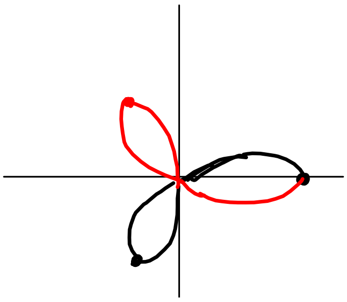




$\frac{1}{3}$ of the picture for a full circuit

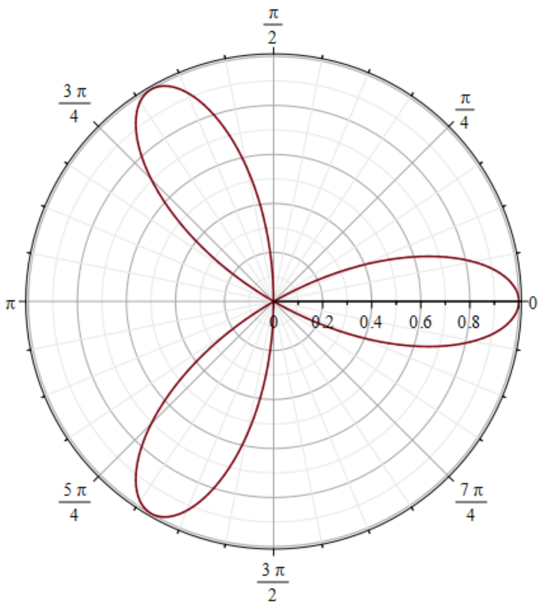
$r = \cos(3\theta)$



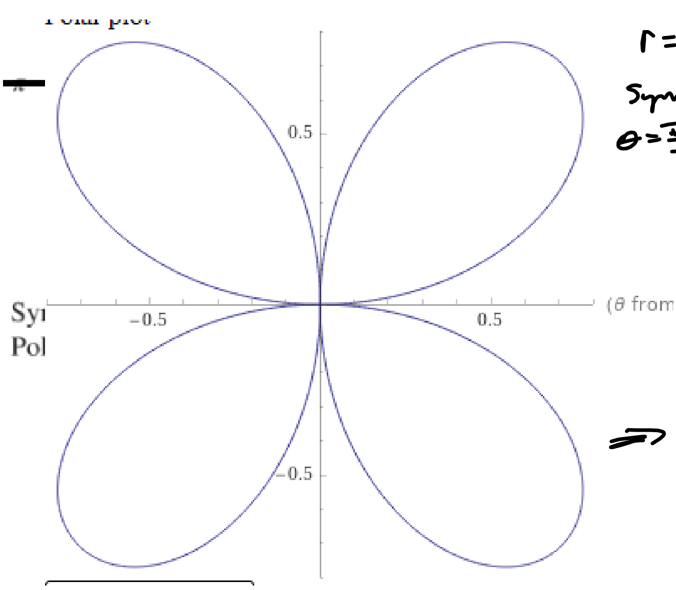


Traces itself out 3 times.

Desmos can do polar graphs.



A plot using Maple.



$r = \sin(2\theta)$

Symm:

$\theta = \frac{\pi}{2}$: $r = \sin(2(\pi - \theta))$

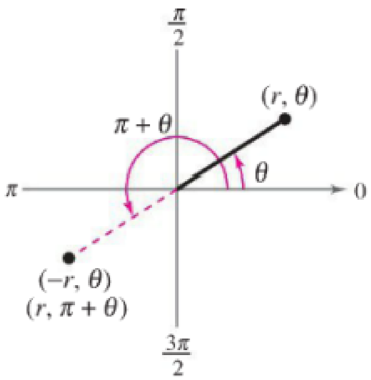
$= \sin(2\pi - 2\theta)$

$= \sin(2\pi)\cos(-2\theta) + \sin(-2\theta)\cos(2\pi)$

$= -\sin(2\theta)$ doesn't work

$-r = \sin(2(-\theta)) = -\sin(2\theta)$

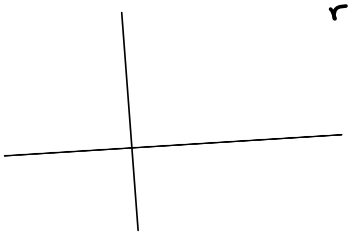
$\Rightarrow r = \sin(2\theta)$ so symm about $\theta = \frac{\pi}{2}$



Symmetry with Respect to the Pole

3. The pole:

Replace (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$.



$r = \sin(2\theta) = f(\text{sine})$

$\theta = \frac{\pi}{2}$ symmetry

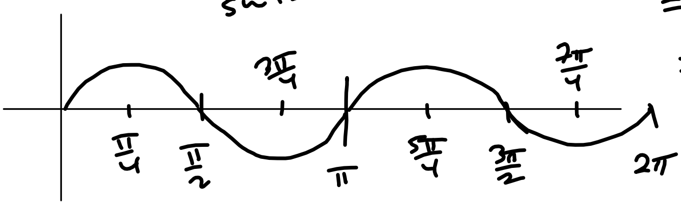
Through the pole

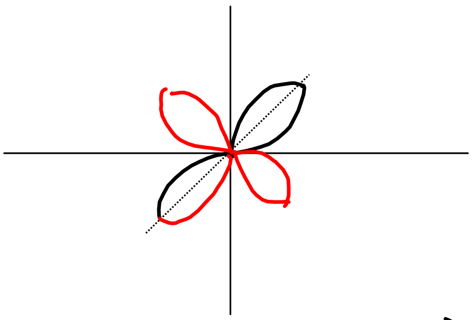
$r = \sin(2(\pi+\theta))$

$= \sin(2\pi + 2\theta)$

$= \sin 2\pi \cos 2\theta + \sin 2\theta \cos 2\pi$

$= 0 + \sin(2\theta)$, so symmetry thru pole.





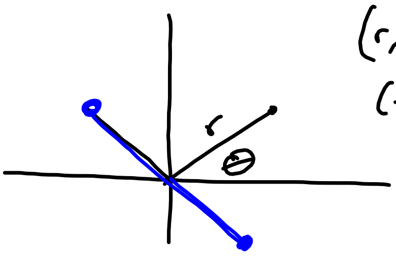
Symmetry about $\theta = \frac{\pi}{2}$:

$r = \sin(2\theta)$

$r = \sin(2(\pi - \theta))$

$= \sin 2\pi \cos(-2\theta) + \sin(-2\theta)\cos(-2\pi)$

$= -\sin(2\theta)$ Nope!



$(r, \pi - \theta)$

$(-r, -\theta)$

$r = \sin(2\theta)$

$-r = \sin(2(-\theta)) = -\sin(2\theta)$

$r = \sin(2\theta)$, so it IS symmetric about $\theta = \frac{\pi}{2}$

(Basically the vertical axis.)

S6.9?!

https://harryzaims.com/public_html/122/122-fall-22/

Polar Equations of Conics

The graph of a polar equation of the form

1. $r = \dfrac{ep}{1 \pm e\cos\theta}$ or **2.** $r = \dfrac{ep}{1 \pm e\sin\theta}$

is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

$r = \dfrac{ep}{1 \pm e\sin\theta}$ Horizontal directrix

$r = \dfrac{ep}{1 \pm e\cos\theta}$ Vertical directrix

1. Horizontal directrix above the pole: $r = \dfrac{ep}{1 + e\sin\theta}$

2. Horizontal directrix below the pole: $r = \dfrac{ep}{1 - e\sin\theta}$

3. Vertical directrix to the right of the pole: $r = \dfrac{ep}{1 + e\cos\theta}$

4. Vertical directrix to the left of the pole: $r = \dfrac{ep}{1 - e\cos\theta}$

17. 0/1 points LarTrig10 6.9.0

Find a polar equation of the indicated conic in terms of *r* with the given characteristics and focus at the pole.

Conic	Eccentricity	Directrix
Parabola	$e = 1$	$x = -1$

$e=1,\ p=1.$

To the left

$r = \dfrac{1}{1-\cos(\theta)}$

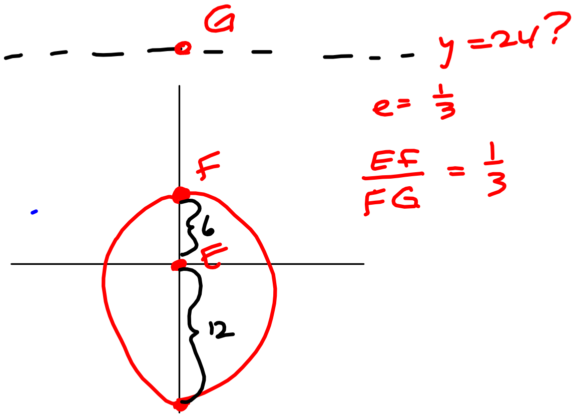
$r = \dfrac{ep}{1-e\cos\theta} = \dfrac{1\cdot 1}{1-\cos\theta}$

21. 0/1 points LarTrig10 6.9.048. [3884566]

Find a polar equation of the indicated conic in terms of r with the given characteristics and focus at the pole.

Conic	Vertices
Ellipse	$\left(6, \frac{\pi}{2}\right), \left(12, \frac{3\pi}{2}\right)$

$r = \dfrac{24}{\sin(\theta) + 3}$



$e = \frac{1}{3}$

$\frac{EF}{FG} = \frac{1}{3}$

Want $r\left(\frac{\pi}{2}\right) = 6$

$r\left(\frac{3\pi}{2}\right) = 12$

$r = \dfrac{ep}{1 + e\sin\theta}$

$r\left(\frac{\pi}{2}\right) = \dfrac{ep}{1 + e\sin\frac{\pi}{2}} = 6$

$\dfrac{ep}{1+e} = 6$

$ep = 6(1+e)$

$e = \dfrac{6(1+e)}{p}$

$r\left(\frac{3\pi}{2}\right) = \dfrac{ep}{1 + e\sin\left(\frac{3\pi}{2}\right)} = 12$

$\dfrac{ep}{1-e} = 12$

$\dfrac{\frac{6(1+e)}{p}\,p}{1-e} = 12$

$\dfrac{6+6e}{1-e} = 12$

$6 + 6e = 12 - 12e$

$18e = 6$

$e = \frac{1}{3}$

$\dfrac{ep}{1+e} = \dfrac{\frac{1}{3}p}{1+\frac{1}{3}} = 6$

$\dfrac{\frac{1}{3}p}{\frac{4}{3}} = 6$

$\frac{1}{4}p = 6$

$p = 24$

$r = \dfrac{\left(\frac{1}{3}\right)(24)\,(3)}{\left(1 + \frac{1}{3}\sin\theta\right)(3)} = \dfrac{24}{3 + \sin\theta}$

7. 0/3 points LarTrig10 6.6.023. [388

Consider the following.

$x = \sqrt{t} - 3 \Rightarrow x+3 = \sqrt{t} \Rightarrow (x+3)^2 = \sqrt{t}^2 = t$

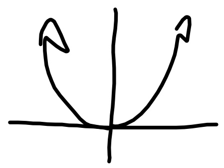
$y = t^3$

$\Rightarrow t \geq 0$

$y = \left((x+3)^2\right)^3 = (x+3)^6$

(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

x^6



$(x+3)^6$