10. 0/3 points LarTrig

Consider the following.

$x = 1 + \cos\theta$
$y = 1 + 7\sin\theta$

(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

(b) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

[ ] ✗ $\frac{(x-1)^2}{1} + \frac{(y-1)^2}{49} = 1$

Adjust the domain of the rectangular equation, if necessary.

- ○ [−6, 8]
- ○ (−8, 6)
- ○ (0, 2)
- ○ [0, 2]
- ○ not necessary

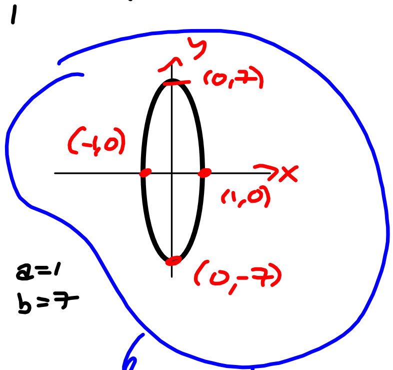
?

$x = 1 + \cos\theta \qquad y = 1 + 7\sin\theta \quad \Longrightarrow \quad x - 1 = \cos\theta \qquad y - 1 = 7\sin\theta \qquad \frac{y-1}{7} = \sin\theta$

$\sin^2\theta + \cos^2\theta = (x-1)^2 + \left(\frac{y-1}{7}\right)^2 = 1$

$\Longleftrightarrow (x-1)^2 + \frac{(y-1)^2}{49}$

$49 = 7^2$



$a = 1$
$b = 7$

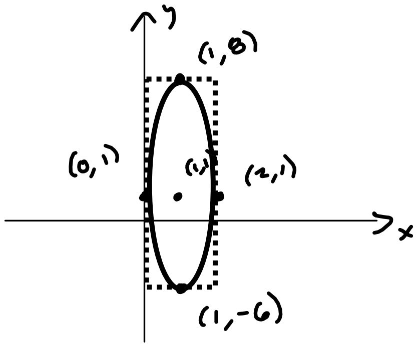
Right shape.
wrong center!
$(h,k) = (1,1)$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$(h,k) = (h,k)$

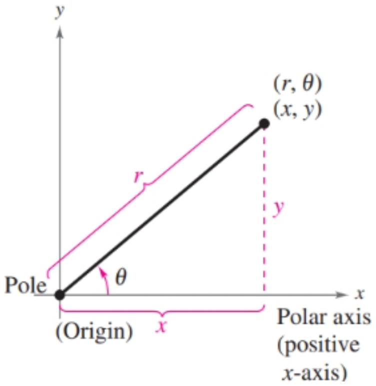
$a = \longleftrightarrow$ from (h,k)

$b = \updownarrow$ " "



Coordinate Conversion

The polar coordinates (r, θ) and the rectangular coordinates (x, y) are related as follows.



Polar-to-Rectangular

$x = r \cos\theta$

$y = r \sin\theta$

$\frac{x}{r} = \cos\theta$

$\frac{y}{r} = \sin\theta$

Rectangular-to-Polar

$\tan\theta = \frac{y}{x}$

$r^2 = x^2 + y^2$

$\tan^{-1}\left(\frac{y}{x}\right)$

$= \arctan\left(\frac{y}{x}\right)$ isn't necessarily in the right quadrant.

7. 0/7 points S6.7

Plot the point given in polar coordinates.

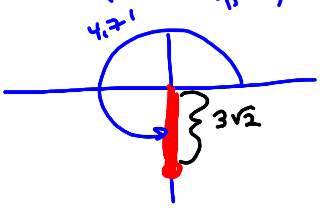
$(3\sqrt{2}, 4.71) = (r, \theta)$

$r = 3\sqrt{2} \approx 4.2$ ish $(3(1.4)$ ish$)$

$4.71 \approx \frac{3\pi}{2}$ How?

$(4.71)\left(\frac{180^\circ}{\pi}\right) \approx 270^\circ$

```
4.71*180/π
       269.8631215
```



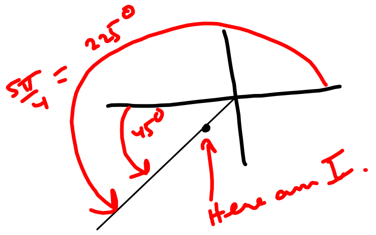
5.6.7

5. 0/7 points

Plot the point given in polar coordinates.

$\left(1, \frac{5\pi}{4}\right) = (r, \theta) = (1, 225^\circ) \in Q\,\overline{III}$

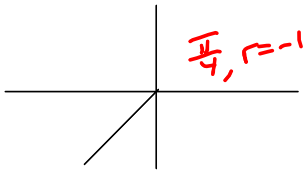
$\uparrow$ 45° reference angle

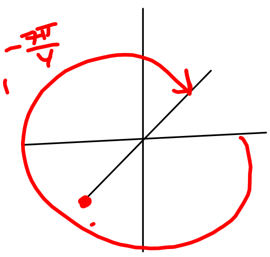


For more representations

$r = 1,\ \theta = \frac{5\pi}{4}$

$r = -1,\ \theta = \frac{\pi}{4}$





Find three additional polar representations of the point, using $-2\pi < \theta < 2\pi$. (Enter your answers in order from smallest to largest first by r-value, then by θ-value.)

$(r, \theta) = \left(\quad ✗\ -1,\ \quad ✗\ \frac{-7\pi}{4} \right)$

$(r, \theta) = \left(\quad ✗\ -1,\ \quad ✗\ \frac{\pi}{4} \right)$

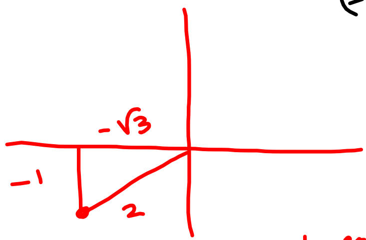
$(r, \theta) = \left(\quad ✗\ 1,\ \quad ✗\ \frac{-3\pi}{4} \right)$

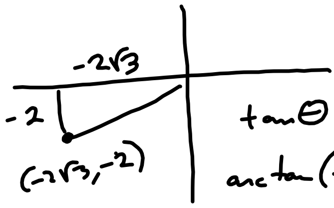
$\boxed{-\frac{3\pi}{4} = \theta \quad r = 1}$

Tricky ones involving arctangent

Convert

$(-2\sqrt{3}, -2)$ to Polars



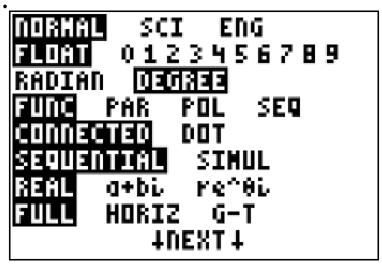


$\tan\theta = \frac{-2}{-2\sqrt{3}}$

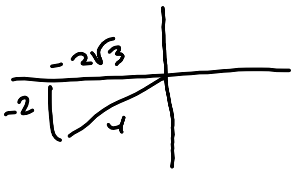
$\arctan(\tan\theta) = \arctan\left(\frac{1}{\sqrt{3}}\right)$

$= 30^\circ$

I prefer to use degrees, because they're easier to see.



r=4 version



$\tan\theta = \frac{-2}{-2\sqrt{3}}$

$\arctan(\tan\theta) = \arctan\left(\frac{1}{\sqrt{3}}\right)$

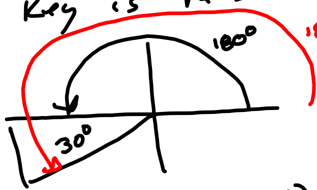
$= 30^\circ$

```
4.71*180/π
        269.8631215
tan⁻¹(1/3^.5)
                 30
```

But we know we're in QIII

$180^\circ < \theta < 270^\circ$

Key is reference angle will be 30°



$180^\circ + 30^\circ = 210^\circ = \boxed{\frac{7\pi}{6} = \theta}$

$r^2 = 2^2 + (2\sqrt{3})^2$
$= 4 + 4 \cdot 3 = 4 + 12 = 16$
$\Rightarrow r = \pm\sqrt{16} = \pm 4.$
We take $r = 4.$

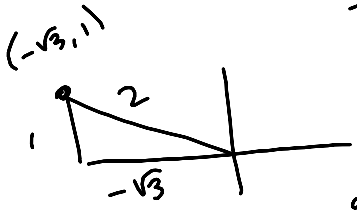
$(4, \pi + \arctan(\frac{1}{\sqrt{3}}))$

Also: $\frac{\pi}{4}$ & $r = -4$ would put you there.

Arctangent's in the wrong quadrant for points in QII

Recall

$\mathcal{R}(\text{arctangent}) = (-\frac{\pi}{2}, \frac{\pi}{2}) = (-90^\circ, 90^\circ)$



$\arctan(\frac{1}{-\sqrt{3}}) = -30^\circ \rightsquigarrow$ QII w/ 30°

reference is:

$(2, 180^\circ - 30^\circ) = (2, \pi - \frac{\pi}{6})$

$= (2, \pi + \arctan(-\frac{1}{\sqrt{3}}))$

Don't be formulaic.

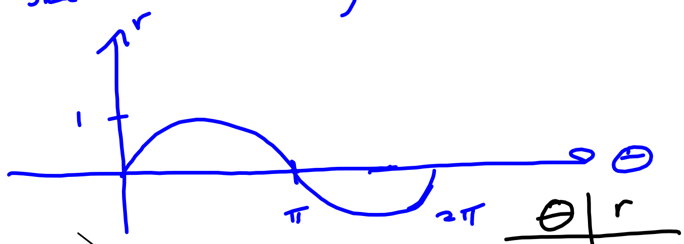
Section 6.8 - Graphs in Polar Coordinates.

I want you to READ the symmetry tests. We'll use them next time.

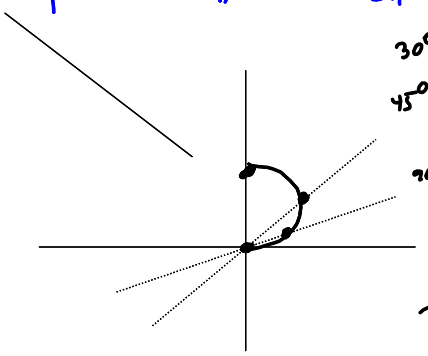
For now, I'm looking for the basic skill.

Graph $r = \sin\theta$

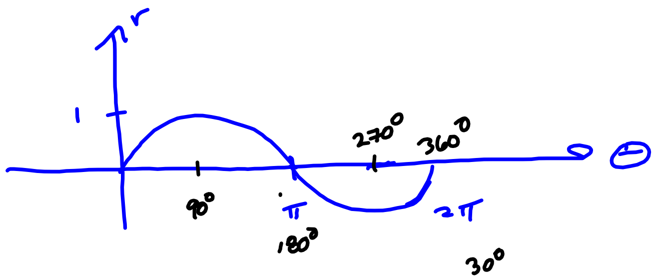
Sketch in Rectangular Form:



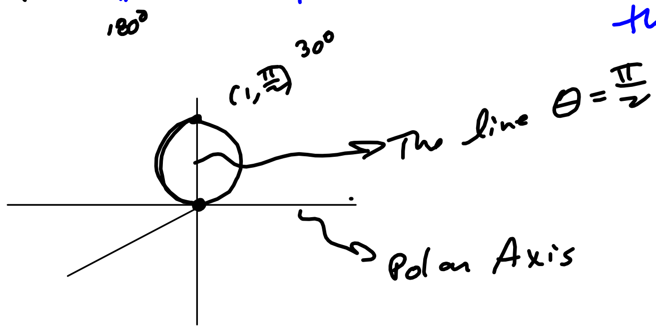
	θ	r
30°	$\frac{\pi}{6}$	$\frac{1}{2}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
90°	$\frac{\pi}{2}$	1
	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$



Turns out to be a circle.

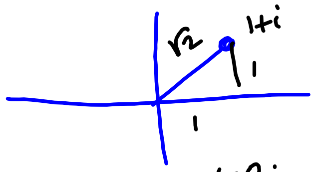


0 to 2π traces out the circle twice!



Find sol'ns of $x^3-(1+i)=0$

$\Rightarrow$ $x^3 = 1+i$, i.e., find all cube roots of $1+i$



$\theta = \frac{\pi}{4}$

Increment $= \frac{2\pi}{3}$

$k=0$: $\sqrt[3]{1+i} = r^{\frac{1}{3}}\left(\cos\left(\frac{\theta}{3}\right) + i\sin\left(\frac{\theta}{3}\right)\right)$

$= \sqrt[3]{\sqrt{2}}\left(\cos\left(\frac{\frac{\pi}{4}}{3}\right) + i\sin\left(\frac{\frac{\pi}{4}}{3}\right)\right)$

$= \sqrt[6]{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ cool! Yes.

$k=1$: $\frac{\pi}{12} + \frac{2\pi}{3}\cdot\frac{4}{4} = \frac{9\pi}{12} = \frac{3\pi}{4}$

$\sqrt[6]{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

$k=2$: $\frac{9\pi}{12} + \frac{8\pi}{12} = \frac{17\pi}{12}$

$\sqrt[6]{2}\left(\cos\left(\frac{17\pi}{12}\right) + i\sin\left(\frac{17\pi}{12}\right)\right)$

$k=3$: Do we end up back home?

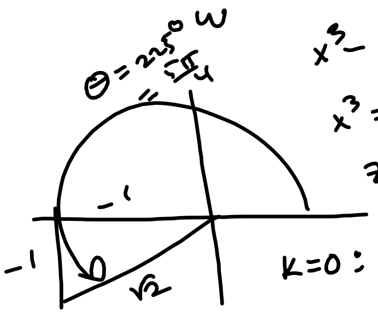
$\frac{(17+8)\pi}{12} = \frac{25\pi}{12} = \frac{24\pi}{12} + \frac{\pi}{12} = 2\pi + \frac{\pi}{12} \rightsquigarrow \frac{\pi}{12}$

7. 0/4 points LarTrig10 4.5.069. [3883150

Use the formula $z_k = \sqrt[n]{r}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right)$ to find all solutions of the equation. (Enter your answers in trigonometric form. Let $0 \le \theta < 2\pi$.)

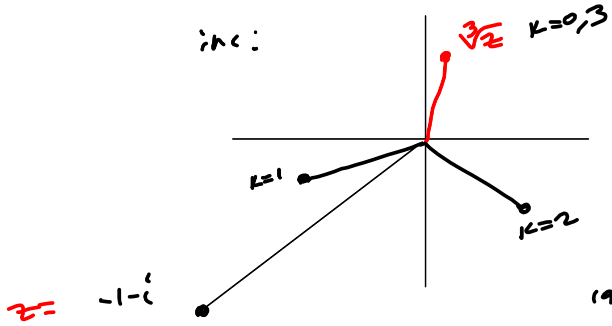
$x^3 - (1 + i) = 0$

Key.



$$x^3 - (-1-i) = 0$$
$$x^3 = -1-i$$
$$z = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$
$$k=0: \sqrt[3]{z} = \sqrt[6]{2}\left(\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right)$$

inc:



$$\frac{2\pi}{3}\cdot\frac{4}{4} = \frac{8\pi}{12}$$
$$5+8=13$$
$$\sqrt[6]{2}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right)$$
$$13+8$$
$$195+120 = 315$$
$$13+8 = 21$$
$$\frac{21\pi}{12}$$
$$315+120 = 435$$
$$360 \quad \frac{-360}{75}$$

Messing up my check

$$\frac{13\pi}{12}\cdot\frac{180}{\pi} = \frac{13\cdot 30\pi}{2\pi}$$
$$= \frac{13\cdot 15}{\cancel{\ }}$$
$$\begin{array}{r}150\\ 45\\ \hline 195°\end{array}$$
$$\frac{13\pi}{12} + \frac{2\pi}{3}\cdot\frac{4}{4} = \frac{(13+8)\pi}{12} = \frac{21\pi}{12}$$
$$\frac{21\pi}{12} + \frac{8\pi}{12} = \frac{29\pi}{12} = 2\pi + \frac{5\pi}{12} \rightarrow \frac{5\pi}{12}\checkmark$$