Taken from

https://harryzaims.com/public_html/122/122-spring-22/notes/220407-notes.pdf

S 4.4 Trigonometric Form of a Complex Number

$(a+bi)(a-bi)$
$= a^2+b^2$

Trigonometric Form of a Complex Number

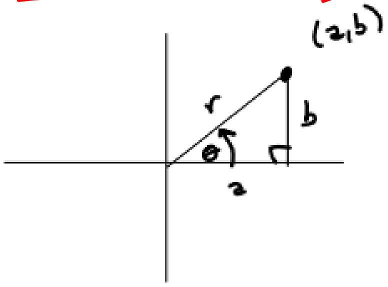
The **trigonometric form** of the complex number $z = a + bi$ is

$$z = r(\cos\theta + i\sin\theta)$$

$r = \sqrt{a^2+b^2} = \sqrt{z\bar{z}}$

Not "the"

where $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2+b^2}$, and $\tan\theta = b/a$. The number r is the **modulus** of z, and θ is an **argument** of z.



$\frac{b}{r} = \sin\theta$

$b = y = r\sin\theta$

$a = x = r\cos\theta$

$r = \text{modulus} = \sqrt{a^2+b^2}$

In the 10th Edition, DeMoivre's Theorem is a separate section.

Previous editions put all the trigonometric form of complex #s into

Section 4.4.

What this means is that 4.5 questions occur on the 4.4 assignment. It's all there, but it's not where you might expect. I added 4.5 to the Course Schedule, just now, but we don't really have a separate 4.5 assignment. In fact, most of the 4.4 work looks like 4.5 stuff.

Find all solutions of $x^4-1=0$ (we find the 4th roots of unity.)

$x^4=1$

Let $u=x^2$.

Then $u^2=1$ from $\sqrt{u^2}=|u|=\sqrt{1} \Rightarrow u=\pm 1$

$u=\pm 1$

$\Rightarrow (u-1)(u+1)$ by Factor Theorem or jus by factoring skills $(u^2-1=(u-1)(u+1))$ $(\Longleftrightarrow)$

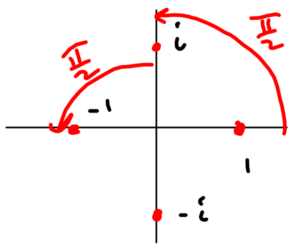
Now, we have $(x^2-1)(x^2+1)=0$ iff

$x^2-1=0$ OR $x^2+1=0$

$x^2=1$ $\quad$ $x^2=-1$

$x=\pm 1$ $\quad$ $x=\pm\sqrt{-1}=\pm i$

IOW $x^4-1=(x-1)(x+1)(x-i)(x+i)$



All the 4th roots of $z=1$.

$1=\sqrt[4]{1}$ = its own principal 4th root.

Notice $\arg(1)=\theta=0$

& $\arg(\sqrt{1})=\frac{\theta}{2}=0$

$= \arg(1^{\frac{1}{2}})$

"Increment" between the 4 4th roots is $\frac{\pi}{2}$, (but think of it as $\frac{2\pi}{4}$)

In trig terms:

$1 = \sqrt[4]{1}(\cos(0) + i\sin(0))$

$i = \sqrt[4]{1}(\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}))$

$-1 = \sqrt[4]{1}(\cos(\pi) + i\sin(\pi))$

$-i = \sqrt[4]{1}(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2}))$

The 4 4th roots of unity.

4th roots of $z=5$

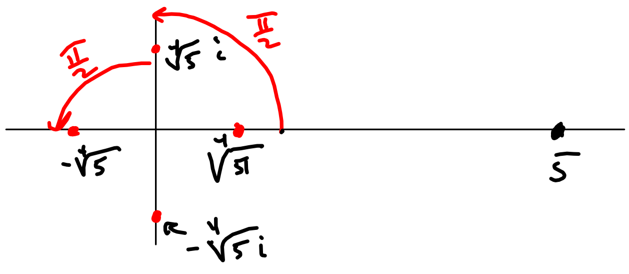
$\sqrt[4]{5}(\cos(0) + i\sin(0))$

$\sqrt[4]{5}(\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}))$

$\sqrt[4]{5}(\cos(\pi) + i\sin(\pi))$

$\sqrt[4]{5}(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2}))$

~~$\sqrt[4]{5}(\cos(2\pi) + i\sin(2\pi))$ went too far.~~



10. 0/1 points

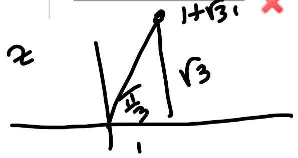
Find the square roots of the complex number. (Enter your answers as a comma-separated list.)

$1 + \sqrt{3}i$

$-\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$

$\sqrt{1^2+\sqrt{3}^2} = \sqrt{1+3} = 2$

$2^{\frac{1}{2}} = \sqrt{2}$ = modulus of the 2^{nd} roots.

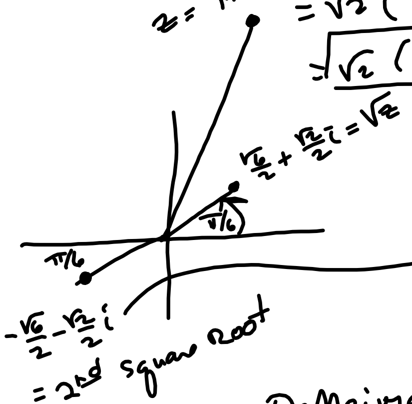


So, $\sqrt{z} = \sqrt{2}\left(\cos\left(\frac{\theta}{2}\right) + i\sin\left(\frac{\theta}{2}\right)\right)$

$z = 1+\sqrt{3}i$

$= \sqrt{2}\left(\cos\left(\frac{\frac{\pi}{3}}{2}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = (\sqrt{2})\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \boxed{\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i}$



$\boxed{-\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i} = \sqrt{2}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$= \sqrt{2}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

DeMoivre's Theorem:

In General, The n^{th} roots of a complex #

n^{th} Roots of $z = r(\cos\theta + i\sin\theta)$ are as follows:

$w = r^{\frac{1}{n}}\left(\cos\left(\frac{\theta}{n} + \frac{2\pi}{n}k\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi}{n}k\right)\right)$

for $k = 0, 1, 2, \ldots, n-1$

The Principal root

$1+\sqrt{3}i$, $r=2$, $\theta=\frac{\pi}{3}$

$n=2$ (2^{nd} root), so

increment $= \frac{2\pi}{2} = \pi$

so $z_1 = \sqrt{2}\left(\cos\left(\frac{\frac{\pi}{3}}{2}\right) + i\sin\left(\frac{\frac{\pi}{3}}{2}\right)\right)$

$= \sqrt{2}\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$

$\frac{\pi}{6} + \frac{2\pi}{2}\cdot\frac{3}{3} = \frac{7\pi}{6}$

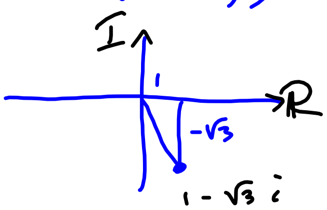
$z_2 = \sqrt{2}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = .$

& now convert to rectangular,

because (apparently) that's what +

$= -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$, S4.3 #s 8, 13, 14, 18, 19

Visually, think of $1-\sqrt{3}\,i$ as $\langle 1, -\sqrt{3}\rangle$ or $(1, -\sqrt{3})$ } in $\mathbb{R}^2$



13. 0/1 points LarTrig10 4.4.037. [3

Use a graphing utility to write the complex number in standard form. (Round all numerical values to four decimal plac

$5\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$

$4.0451 + 2.9389i$

```
NORMAL  SCI  ENG
FLOAT  0123456789
RADIAN  DEGREE
FUNC  PAR  POL  SEQ
CONNECTED  DOT
SEQUENTIAL  SIMUL
REAL  a+bi  re^θi
FULL  HORIZ  G-T
←NEXT→
```

```
cos(π/5)
          .8090169944
Ans*5
          4.045084972
sin(π/5)
          .5877852523
```

```
Ans*5
          2.938926261
```

```
5(cos(π/5)+isin(
π/5))
4.045084972+2.9…
```

```
5(cos(π/5)+isin(
π/5))
…72+2.938926261i
```

18. 0/4 points

Consider the following.

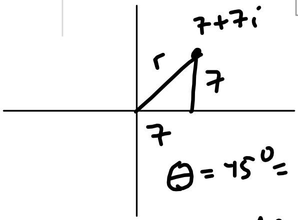
$(7 + 7i)(4 - 4i)$

(a) Write the trigonometric forms of the complex numbers. (Let $0 \le \theta < 2\pi$.)

(b) Perform the indicated operation using the trigonometric forms. (Let $0 \le \theta < 2\pi$.)

$(7 + 7i) =$ ✗ $7\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

$(4 - 4i) =$ ✗ $4\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right)$



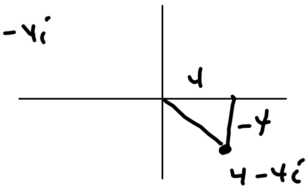
$r = \sqrt{7^2+7^2} = 7\sqrt{2}$

$\theta = 45^\circ = \frac{\pi}{4}$

arctan$(7/7)$

arcsin$(7/$

$4-4i$



$\sqrt{2(4^2)}$
$= 4\sqrt{2}$

$\theta = -\frac{\pi}{4}$ or $\frac{7\pi}{4}$

$(7+7i)(4-4i)$

$= \left(7\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)\left(4\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)\right)$

$= (28)(\sqrt{2})(\sqrt{2})\left(\cos(0) + i\sin(0)\right)$

$\frac{\pi}{4} - \frac{\pi}{4} = 0$!

$= 56(1 + i\cdot 0) = 56$!

Gettin' cute:

$(7+7i)(4-4i) = 7(1+i)(4)(1-i) = 28(1+i)(1-i)$
$= 28(1^2+1^2)$
$= 28(2) = 56$

19. 0/4 points LarTrig10 4.4.057. [3883325]

Consider the following.

$$\frac{8+15i}{1-\sqrt{3}i}$$

(a) Write the trigonometric forms of the complex numbers. (Let $0 \le \theta < 2\pi$. Round your angles to three decimal places.)

$8 + 15i =$ ✗ $\boxed{17(\cos(1.081) + i\sin(1.081))}$

$1 - \sqrt{3}i =$ ✗ $\boxed{2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)}$

Think conjugates

(b) Perform the indicated operation using the trigonometric forms. (Let $0 \le \theta < 2\pi$. Round your angles to three decimal places.)

✗ $\boxed{\frac{17}{2}(\cos(2.128) + i\sin(2.128))}$

(c) Perform the indicated operation using the standard forms, and check your result with that of part (b). (Round all numerical values to three decimal places.)

✗ $\boxed{-4.495 + 7.214i}$

$$\left(\frac{8+15i}{1-\sqrt{3}i}\right)\left(\frac{1+\sqrt{3}i}{1+\sqrt{3}i}\right) = \frac{8+8\sqrt{3}i+15i+15\sqrt{3}i^2}{1^2+\sqrt{3}^2}$$

$$= \frac{8-15\sqrt{3}+(8\sqrt{3}+15)i}{4}$$

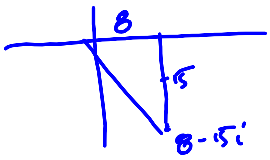
$$= \frac{8-15\sqrt{3}}{4} + \frac{(8\sqrt{3}+15)}{4}i = w$$

$$|w| = \sqrt{\left(\frac{8-15\sqrt{3}}{4}\right)^2 + \left(\frac{8\sqrt{3}+15}{4}\right)^2}$$

owie!

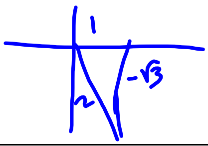
Follow Instructions, Doc Mills

(b) $8-15i$: $\sqrt{8^2+15^2} = \sqrt{64+225} = \sqrt{289} = 17$



$\arctan\left(-\frac{15}{8}\right) \approx$

$1-\sqrt{3}\,i$: $\sqrt{1^2+\sqrt{3}^2} = \sqrt{4} = 2$



$\theta = -\frac{\pi}{3}$

```
…72+2.938926261i
tan⁻¹(-15/8)
     -1.080839001
tan⁻¹(-√(3))
     -1.047197551
π/3
      1.047197551
```

$17(\cos(-1.08\ldots) + i\sin(-1.08\ldots))$

etc.

WATCHOUT!

Don't use rounded #s from answer to part (a) to do calculations of part (b).

```
Ans→A
      1.047197551
tan⁻¹(-15/8)
     -1.080839001
Ans→B
     -1.080839001
17(cos(
Rcl B
```

Ugh!

Terrible! messing up w/ STO & RCL inside the cosine.