14. 0/2 points

Write the polynomial as a product of linear factors.

$f(x) = x^2 - x + 42$

$f(x) =$ ☐ ✗ $\left(x+\frac{1}{2}\left(-1-i\sqrt{167}\right)\right)\left(x+\frac{1}{2}\left(-1+i\sqrt{167}\right)\right)$

Find all the zeros of the function. (Enter your answers as a comma-separated list.)

$x =$ ☐ ✗ $\frac{1}{2}\left(1-i\sqrt{167}\right), \frac{1}{2}\left(1+i\sqrt{167}\right)$

Does 167 factor & simplify?

167

$x^2 - x + 42 = 0$

$a = 1, b = -1, c = 42$

$b^2 - 4ac = 1^2 - 4(1)(42) = -167$

```
                 167
167/2
                83.5
Ans*2
                 167
Ans/3
         55.66666667
```

```
                33.4
Ans*5
                 167
Ans/7
         23.85714286
Ans*7
                 167
```

```
         15.18181818
Ans*11
                 167
Ans^.5
         12.92284798
13²
                 169
```

Stop @ greatest Prime # less than $\sqrt{167}$

2, 3, 5, 7, 11, 13, 17, 19, 23, 29,
31, 37,

$11^2 = 121$ — Last one to check

$13^2 = 169$ — Too big to work if $\sqrt{167}$ 2, 3, 5, 7, 11 didn't work.

So 167 is prime, just in case

you need to do these on a desert island.

Factor by Grouping

$$3x^3 - 15x^2 - x + 5$$

$$= 3x^2(x-5) - 1(x-5) \qquad 3x^2y - 15xy = 3xy[x-5]$$

$$= (x-5)\underbrace{[3x^2 - 1]}_{(\sqrt{3}x)^2 - 1^2} = (x-5)(\sqrt{3}x - 1)(\sqrt{3}x + 1)$$

or, just $3x^2 - 1 = 0$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{-167}}{2(1)} = \frac{1 \pm i\sqrt{167}}{2} = 0 \Longrightarrow$$

$$f(x) = \left(x - \left(\frac{1-i\sqrt{167}}{2}\right)\right)\left(x - \left(\frac{1+i\sqrt{167}}{2}\right)\right)$$

$x=c$ makes it zero $\longleftrightarrow$

$x-c$ is a factor!

$x^2+3x+2 = 0$

$a=1,\ b=3,\ c=2$

$b^2-4ac = 3^2-4(1)(2)$

$= 9-8 = 1 \rightsquigarrow \sqrt{1} = 1$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-3 \pm 1}{2(1)} = \frac{-3 \pm 1}{2} \quad \begin{cases} \frac{-2}{2} = -1 \\ \frac{-4}{2} = -2 \end{cases}$$

$\Longrightarrow f(x) = (x-(-1))(x-(-2))$

$= (x+1)(x+2) \overset{\text{CHECK}}{=\!=} x^2+2x+1x+2$

$= x^2+3x+2$ ✓

$\exists$ cubic & quartic formulas, just like quadratic formula

18. 0/1 points

Find a cubic polynomial function f with real coefficients that has the given complex zeros and x-interce
correct answers.)

Complex Zeros	*x-Intercept*
$x = -2 \pm \sqrt{2}i$	$(-3, 0)$

$f(-3) = 0$!

$f(x) =$ ✖ $x^3 + 7x^2 + 18x + 18$

$$(x - (-2 + \sqrt{2}\,i))(x - (-2 - \sqrt{2}\,i))(x - (-3))$$

11. 0/2 points

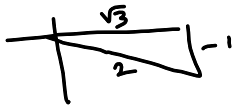
Write the standard form of the complex number.

$\sqrt{48}[\cos(-30°) + i\sin(-30°)]$ $= 4\sqrt{3}\left(\frac{\sqrt{3}}{2} + (-\frac{1}{2})i\right) = z^*$

$(x,y) = (\frac{\sqrt{3}}{2}, -\frac{1}{2})$ is on the unit circle!

$\cos^2(-30°) + \sin^2(-30°) = 1$

✗ $6 - 2\sqrt{3}i$

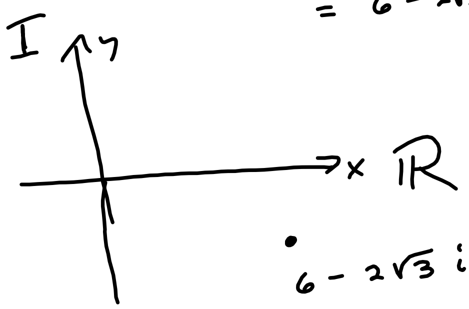


Plot the complex number.

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$\sqrt{48} = 2\cdot 2\sqrt{3}$
$= 4\sqrt{3}$

$* \quad z = \frac{4\sqrt{3}\sqrt{3}}{2} - \frac{4\sqrt{3}}{2}i = \frac{12}{2} - 2\sqrt{3}i$
$= 6 - 2\sqrt{3}i$



$z = \cos\alpha + i\sin\alpha$ is a point on the unit circle,
b/c $\cos^2\alpha + \sin^2\alpha = 1$

$|z| = \sqrt{\cos^2\alpha + \sin^2\alpha}$

$z = r(\cos\theta + i\sin\theta)$ is the trigonometric form of the complex # z.

θ = the angle a line segment from 0 to z makes with the real (x-) axis.

$|r|$ = length of that segment.

$z = r\cos\theta + ir\sin\theta$

We want to go back and forth between rectangular and trigonometric forms.

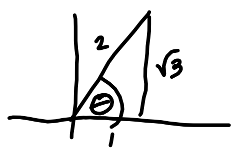
Recall, last time, I spent 15 minutes trying to convince you that multiplication in the complex plane involves length and a rotation.

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right) = \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4} i + \frac{\sqrt{6}}{4} i - \frac{\sqrt{6}}{4} i^2$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} + \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) i$$

$$= \frac{\sqrt{2} + \sqrt{6}}{4} + \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) i$$

$z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

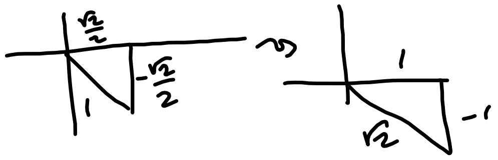


So, $\theta = 60^\circ = \frac{\pi}{3}$

$|z| = 1$

So, $z = 1(\cos 60^\circ + i \sin 60^\circ)$ is trig form.

$w = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$ $\quad |w| = 1$ $\quad$ & $w = 1(\cos 315^\circ + i \sin 315^\circ)$



$z = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$

$w = \cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)$

or $\cos\left(\frac{7\pi}{4}\right) + i \sin\left(\frac{7\pi}{4}\right)$

$\alpha = \text{angle} = -45^\circ = -\frac{\pi}{4}$

or $315^\circ = \frac{7\pi}{4}$

Multiply it out :

$$\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

$$= \cos\frac{\pi}{3}\cos\frac{7\pi}{4} + i\cos\frac{\pi}{3}\sin\frac{7\pi}{4}$$

$$+ i\sin\frac{\pi}{3}\cos\frac{7\pi}{4}\boxed{+ i^2\sin\frac{\pi}{3}\sin\frac{7\pi}{4}}$$

$$= \cos\frac{\pi}{3}\cos\frac{7\pi}{4} - \sin\frac{\pi}{3}\sin\frac{7\pi}{4} + \left(\sin\frac{7\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{7\pi}{4}\right)i$$

$$= \cos\left(\frac{\pi}{3} + \frac{7\pi}{4}\right) + i\sin\left(\frac{\pi}{3} + \frac{7\pi}{4}\right)$$

from angle sum identity (in reverse)

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$= \cos\left(\frac{4\pi + 21\pi}{12}\right) + i\sin\left(\frac{25\pi}{12}\right) = \cos\frac{25\pi}{12} + i\sin\frac{25\pi}{4}$$

$$\cos\left(\frac{25\cdot \mathrm{Pi}}{12}\right) + I\cdot \sin\left(\frac{25\cdot \mathrm{Pi}}{12}\right)$$

$$\cos\left(\frac{\pi}{12}\right) + \mathrm{I}\sin\left(\frac{\pi}{12}\right)$$

$evalf(\%)$

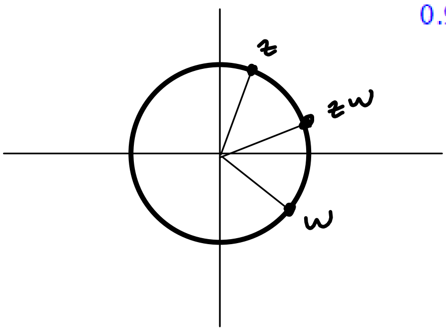
$$0.9659258263 + 0.2588190451\,\mathrm{I}$$

$$\frac{(\mathrm{sqrt}(2) + \mathrm{sqrt}(6))}{4} + I\cdot \frac{(-\mathrm{sqrt}(2) + \mathrm{sqrt}(6))}{4}$$

$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} + \frac{\mathrm{I}\left(-\sqrt{2} + \sqrt{6}\right)}{4}$$

$evalf(\%)$

$$0.9659258263 + 0.2588190452\,\mathrm{I}$$



$3(\cos(17^\circ)+i\sin(17^\circ))(7(\cos(57^\circ)+i\sin(57^\circ))$

$= 21(\cos(74^\circ)+i\sin(74^\circ))$

This result has ramifications for roots and powers that will blow your mind!

$x^6+1=0$

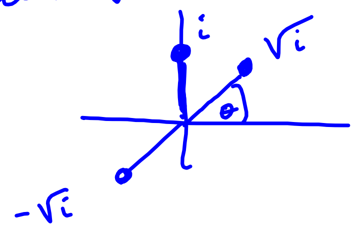
$x^6=-1$

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1} = \pm i$

Principal Square root is the one with the smallest positive argument (angle)



$\theta+\frac{2\pi}{2} = \theta+\pi$

$\theta+\frac{2\pi}{n}$

5th root?

$\sqrt{9} = 3$

5 of 'em

$\frac{\theta}{5}$ Principal (1st)

$\frac{\theta}{5}+\frac{2\pi}{5}$

$\frac{\theta}{5}+\frac{2\pi}{5}\cdot 2$

$\frac{\theta}{5}+\frac{2\pi}{5}\cdot 3$

$\frac{\theta}{5}+\frac{2\pi}{5}\cdot 4$

$\frac{\theta}{5}+\frac{2\pi}{5}\cdot 5 = \frac{\theta}{5}+2\pi$

brings me home.